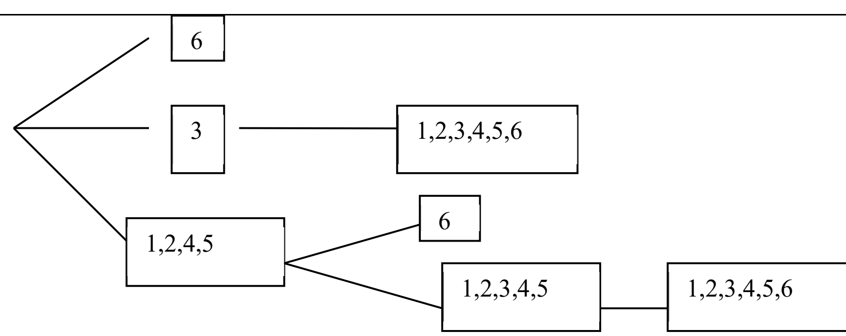
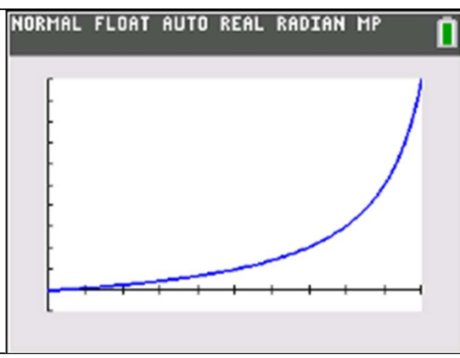
<u>Probability</u>

1(a)(i)	P(all three cards even) $=\frac{10}{20}\frac{9}{19}\frac{8}{18}=\frac{2}{19}$ or $=\frac{\binom{10}{3}}{\binom{20}{3}}=\frac{2}{19}$
(ii)	P(exactly one even) $=\frac{10}{20}\frac{10}{19}\frac{9}{18}\times\binom{3}{1}=\frac{15}{38}$ or $=\frac{\binom{10}{1}\binom{10}{2}}{\binom{20}{3}}=\frac{15}{38}$
(b)(i)	$\mathrm{P}(B)=\frac{4}{20}\frac{16}{19}+\frac{16}{20}\frac{15}{19}=\frac{4}{5}$
(ii)	$\mathrm{P}(A\cap B)=\mathrm{P}(\text{1st card}\le 5\ \ \&\ \ \text{2nd card}\ge 5)$ $=\mathrm{P}(\text{1st card}=5\ \ \&\ \ \text{2nd card}>5)+\mathrm{P}(\text{1st card}<5\ \ \&\ \ \text{2nd card}\ge 5)$ $=\frac{1}{20}\frac{15}{19}+\frac{4}{20}\frac{16}{19}=\frac{79}{380}$
(iii)	$\mathrm{P}(B\mid A)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}=\frac{\frac{79}{380}}{\frac{1}{4}}=\frac{79}{95}$

2(a)	$\frac{1}{4}=\frac{P(B\cap A')}{1-\frac{1}{4}}\quad \therefore P(B\cap A')=\frac{3}{16}$ $P(B\cap A)=P(B)-P(B\cap A')=\frac{1}{3}-\frac{3}{16}=\frac{7}{48}\neq 0$ Hence A and B are not mutually exclusive.
(b)(i)	 P(score is 5) = P[(3,2), (1,1,3), (1,2,2), (1,3,1), (2,1,2), (2,2,1)] $=\frac{1}{36}+\frac{5}{216}=\frac{11}{216}$
(ii)	$P(score\ 3)=P[(1,1,1,)]=\frac{1}{216}$

	$P(score\ 4) = P[(3,1), (1,1,2),(1,2,1), (2,1,1)\ ] = \dfrac{9}{216}$ $P\left(score\ 4 \middle\vert score \le 5\right) = P\left(\dfrac{score\ 4 \cap score \le 5}{score \le 5}\right)$ $= P\left(\dfrac{score\ 4}{score \le 5}\right)$ $= \dfrac{9}{1+9+11} = \dfrac{3}{7}$

3(i)	$(p)(0.9)\ +\ (100\ - p)(0.1)\ =\ 20$ $(p)(0.9) + (100 - p)(0.1) = 20$ $0.9p + 10 - 0.1p = 20$ $p = 12.5$ The proportion of the residents infected is $\dfrac{12.5}{100} = \dfrac{1}{8}$ or 12.5 %
(ii)	P(has disease | tested negative) = $\dfrac{\text{P(has disease \& tested negative)}}{\text{P(tested negative)}}$ $= \dfrac{0.1q}{0.1q + 0.9(100-q)}$ $= \dfrac{q}{900-8q}$
(iii)	 As the proportion of people getting infected (q) increases, the probability that a person has the disease given that he has been tested negative also increases as seen in the graph. So, the test is not effective.

4(i)	P (it will take exactly n throws of the biased die to obtain a '6') = P (the first (n – 1) throws are not '6' and the nth throw is a '6') $= (1-p)^{n-1}p$
(ii)	P (it takes exactly 3 throws to obtain a '6') = $(1-p)^2 p = \frac{9}{64}$ $p^3 - 2p^2 - p - \dfrac{9}{64} = 0$ $p = 0.25, 0.4243, 1.326$ (from GC-Polynomial solver)

	$p = 0.4243 \approx 0.424$ since $\frac{1}{4} < p < 1$ P (obtain a '5') = $\frac{1-0.4243}{5} = 0.11514$ P (They obtained the same number | they obtained a number larger than 4) $= \frac{\text{P (they obtained the same number and each obtained a number greater than 4)}}{\text{P (they each obtained a number greater than 4)}} =$ $\frac{\text{P (each obtained 5 or each obtained 6)}}{\text{P(each obtained 5 or 6)}} = \frac{\text{P(5, 5) + P(6, 6)}}{[\text{P (5 or 6)}]^2}$ $= \frac{(0.11514)^2 + (0.4243)^2}{(0.11514 + 0.4243)^2} = 0.664$ (to 3 sf) **Alternative solution :** $= \frac{\text{P (each obtained 5 or each obtained 6)}}{\text{P(each obtained a number greater than 4)}} = \frac{\text{P(5, 5) + P(6, 6)}}{\text{P (5, 5) + P (6, 6) + 2 P (6, 5)}}$ $= \frac{(0.11514)^2 + (0.4243)^2}{(0.11514)^2 + (0.4243)^2 + 2(0.11514)(0.4243)}$

5(i)	P(A wins a race) = 0.2(0.9)+0.8(0.5)=0.58
(ii)	P(A wins no more than twice) = 1-P(A wins all 3 matches) = 1- 0.58^3 =0.805
(iii)	P(A wins competition|A wins 1st race) = $\frac{\text{P(A wins competition} \cap \text{A wins 1st race)}}{\text{P(A wins 1st race)}}$ P(A wins competition $\cap$ A wins 1st race) = P(WWW) + P(WWLW) + P(WLWW) + P(WLLWW) + P(WLWLW) + P(WWLLW) = $(0.58)^3 + (0.58)^3(0.42)2 + (0.58)^3(0.42)^2 3 = 0.462$ $\therefore$ Required probability = $0.462 \div 0.58 = 0.797$

6 (i)	Let U and R be the events that United and Rover scored a goal with penalty kick respectively. P(match is still undecided after 1 round)

	$= \text{P}(R \cap U) + \text{P}(R' \cap U') = 0.8 \times 0.9 + 0.2 \times 0.1 = 0.74 = \frac{37}{50}$ P(United won in less than 3 rounds| United scores a goal in the first round) $= \frac{P(\text{United won in less than 3 rounds and United scores a goal in the first round})}{P(\text{United scores a goal in the first round}}$ $= \frac{(0.8 \times 0.1) + (0.8 \times 0.9 \times 0.8 \times 0.1)}{0.8} = 0.172$
(ii)	Let X be the random variable for the number of rounds played P(match is decided in at most n rounds) > 0.98 $\text{P}(X \le n) > 0.98$ $\text{P}(X = 1) + \text{P}(X = 2) + \text{P}(X = 3) + \ldots\ldots + \text{P}(X = n) > 0.98$ Hence $\frac{13}{50} + \frac{37}{50}(\frac{13}{50}) + (\frac{37}{50})^2 \frac{13}{50} + \ldots\ldots + (\frac{37}{50})^{n-1} \frac{13}{50} > 0.98 \ldots\ldots$ (**) $\frac{13}{50}\left[1 + \frac{37}{50} + \left(\frac{37}{50}\right)^2 + \ldots\ldots + \left(\frac{37}{50}\right)^{n-1}\right] > 0.98$ $\left(\frac{37}{50}\right)^n < 0.02$ Hence $n > \frac{\ln 0.02}{\ln \frac{37}{50}} = 12.992$ Least $n = 13$ Alternative: P(match is decided in at most n rounds) > 0.98 1 – P(match is decided in more than n rounds) > 0.98 1 – P(match is undecided in the first n rounds) > 0.98 $1 - \left(\frac{37}{50}\right)^n > 0.98$

7(a)(i)	$\text{P}(A) = \frac{9}{36} = \frac{1}{4}$ $\quad \text{P}(B) = \frac{4}{36} = \frac{1}{9}$ $\quad \text{P}(A \cap B) = \frac{1}{36}$ Since $\text{P}(A \cap B) = \text{P}(A)\,\text{P}(B)$, they are independent.
(ii)	$A \cup B$ represents the event the card taken is either blue or numbered 1. $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B) = \frac{9}{36} + \frac{4}{36} - \frac{1}{36} = \frac{1}{3}$

(iii)	$P(A'\mid B')=\dfrac{P(A'\cap B')}{P(B')}=\dfrac{1-\frac{1}{3}}{\frac{32}{36}}=\dfrac{3}{4}$
(b)	Method 1: Required probability $=1\times\dfrac{3}{35}\times\dfrac{2}{34}=\dfrac{3}{595}$ or 0.00504 Method 2: Required probability $=\dfrac{4}{36}\times\dfrac{3}{35}\times\dfrac{2}{34}\times 9=\dfrac{3}{595}$ Method 3: Required probability $=\binom{4}{3}\left(\dfrac{1}{36}\right)\left(\dfrac{1}{35}\right)\left(\dfrac{1}{34}\right)3!\times 9=\dfrac{3}{595}$ Method 4: Required probability $=\dfrac{9\binom{4}{3}}{\binom{36}{3}}=\dfrac{3}{595}$
(c)	Method 1: Required probability $=\dfrac{^{32}C_{20}}{^{36}C_{20}}=\dfrac{52}{1683}=0.030897=0.0309$ Method 2: Required probability $=\dfrac{32}{36}\times\dfrac{31}{35}\times\dfrac{30}{34}\times\dfrac{29}{33}\times\dfrac{28}{32}\times\ldots\times\dfrac{17}{21}\times\dfrac{16}{20}\times\dfrac{15}{19}\times\dfrac{14}{18}\times\dfrac{13}{17}$ $=\dfrac{16}{36}\times\dfrac{15}{35}\times\dfrac{14}{34}\times\dfrac{13}{33}=0.030897=0.0309$

8(a)	P(obtain a '6') = P(R6) + P(B6) = $\dfrac{1}{3}\text{ x }\dfrac{1}{6}\text{ x }2=\dfrac{1}{9}$
(b)	P(score = 12) = $2\left[P(1,11)+P(3,9)+P(5,7)+P(2,10)+P(4,8)\right]+P(6,6)$ $=2\text{ x }\dfrac{2}{18}\text{ x }\dfrac{1}{18}\text{ x }5+\dfrac{2}{18}\text{ x }\dfrac{2}{18}=\dfrac{2}{27}$
(c)	P(score = 11|one of the die is blue) $=\dfrac{\text{P(score = 11 } \cap \text{ one of the die is blue)}}{\text{P(one of the die is blue)}}$ P(one of the die is blue)=P(RB) + P(WB) = $\dfrac{1}{3}\text{ x }\dfrac{1}{2}\text{ x }2\text{ x }2=\dfrac{2}{3}$ P(score = 11 $\cap$ one of the die is blue) = P(R1,B10) + P(R3,B8) + P(R5,B6) + P(W1,B10) + P(W3,B8) + P(W5,B6) + P(W7,B4) + P(W9, B2)

	$= \frac{1}{18} \times \frac{1}{12} \times 2 \times 8 = \frac{2}{27}$ $\therefore$ Required probability = $\frac{2}{27} \div \frac{2}{3} = \frac{1}{9}$

9(i)	P(C,C or N,N) + P(M,M or E,E) $= 2\times\left(\frac{2}{12}\cdot\frac{1}{11}\right)+2\times\left(\frac{3}{12}\cdot\frac{2}{11}\right)=\frac{4}{33}$
(ii)	P(C, C', C) + P(C', C, C) + P(C', C', C) $= 2\times\left(\frac{2}{12}\cdot\frac{10}{11}\cdot\frac{1}{10}\right)+\left(\frac{10}{12}\cdot\frac{9}{11}\cdot\frac{2}{10}\right)=\frac{1}{6}$
(iii)	$\frac{3}{12}\cdot\frac{2}{11}\cdot\frac{3}{10}=\frac{3}{220}$
(iv)	$\frac{3}{220}\times 3!=\frac{9}{110}$
	Let p = P($\le n$ more cards are drawn to get an N)

n	p
1	$\frac{2}{10} \quad < 0.75$
2	$\frac{2}{10}+\left(\frac{8}{10}\cdot\frac{2}{9}\right)=\frac{17}{45} \quad < 0.75$
3	$\frac{17}{45}+\left(\frac{8}{10}\cdot\frac{7}{9}\cdot\frac{2}{8}\right)=\frac{8}{15} \quad < 0.75$
4	$\frac{8}{5}+\left(\frac{8}{10}\cdot\frac{7}{9}\cdot\frac{6}{8}\cdot\frac{2}{7}\right)=\frac{2}{3} \quad < 0.75$
5	$\frac{2}{3}+\left(\frac{8}{10}\cdot\frac{7}{9}\cdot\frac{6}{8}\cdot\frac{5}{7}\cdot\frac{2}{6}\right)=\frac{7}{9} \quad > 0.75$

Hence, the least value of n = 5

10(a)	A and B are mutually exclusive. A and C are indept, i.e. $P(A\cap C)=P(A)\,P(C)$ $P(A)=\frac{1}{5},\quad P(B)=\frac{1}{10},\quad P(A\cup C)=\frac{7}{15}$ $P(B\cup C)=\frac{23}{60}$ $P(A\cup B)=\frac{1}{5}+\frac{1}{10}=\frac{3}{10}$

	$P(A \cup C) = P(A) + P(C) - P(A \cap C)$ $\frac{7}{15} = \frac{1}{5} + P(C) - \frac{1}{5}P(C)$ $P(C) = \frac{4}{15} \times \frac{5}{4} = \frac{1}{3}$ $\therefore P(A \cap C) = \frac{1}{5} \text{ x } \frac{1}{3} = \frac{1}{15}$ $P(B \cup C) = P(B) + P(C) - P(B \cap C)$ $\frac{23}{60} = \frac{1}{10} + \frac{1}{3} - P(B \cap C)$ $P(B \cap C) = \frac{1}{20}$ $P(B) \text{ x } P(C) = \frac{1}{10} \text{ x } \frac{1}{3} = \frac{1}{30} \neq P(B \cap C)$ Therefore, B and C are not independent.
(b)(i)	$P(1^{st} \text{ vase is flawless}) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{7}{12}$
(ii)	P(batch from X | 1^{st} vase is flawless) $= \frac{P(\text{batch from X} \cap \text{1st vase is flawless})}{P(\text{1st vase is flawless})} = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) \div \frac{7}{12} = \frac{4}{7}$
(iii)	P(2^{nd} vase is flawless | 1^{st} vase is flawless) $= \frac{P(\text{both vases are flawless})}{P(\text{1st vase is flawless})} = \left[\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2\right] \div \frac{7}{12} = \frac{25}{42}$

11(i)	P(2 balls are white) = $\binom{2}{2} \div \binom{5}{2} = \frac{1}{10}$
(ii)	P(toss once) = 1 – P(1R, 1W) = $1 - \binom{3}{1}\binom{2}{1} \div \binom{5}{2} = 1 - \frac{6}{10} = \frac{2}{5}$
(iii)	$P(X = 5)$ = P(2,3) + P(1,4) + P(5) = $\frac{3}{5} \text{ x } \frac{1}{6} \text{ x } \frac{1}{6} \text{ x } 2 \text{ x } 2 + \frac{3}{10} \text{ x } \frac{1}{6} = \frac{7}{60}$
(iv)	$P(X = 2 \mid \text{toss die once}) = \frac{P(X = 2 \cap \text{toss die once})}{P(\text{toss die once})}$

	$P(X=2 \cap \text{ toss die once}) = P(2W,1) + P(1) = \frac{1}{10} \times \frac{1}{6} + \frac{3}{10} \times \frac{1}{6} = \frac{1}{15}$ $P(\text{toss die once}) = \frac{2}{5}$ $P(X=2 \mid \text{toss die once}) = \frac{P(X=2 \cap \text{ toss die once})}{P(\text{toss die once})} = \frac{1/15}{2/5} = \frac{1}{6}$
(v)	$P(\text{2 balls are white} \mid X=2) = \frac{P(\text{2 balls are white and } X=2)}{P(X=2)}$ $P(X=2) = P(2W,1) + P(2) + P(1,1) = \frac{1}{10} \times \frac{1}{6} + \frac{3}{10} \times \frac{1}{6} + \frac{3}{5} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{12}$ $P(\text{2 balls are white and } X=2) = P(2W,1) = \frac{1}{60}$ Required prob. $= \frac{1}{60} \div \frac{1}{12} = \frac{1}{5}$

12(a)(i)	P(team wins 2 matches) = $\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \frac{4!}{2!2!} = \frac{3}{8}$
(ii)	P(team scores 4 points) = P(2D, 1L, 1W) + P(4D) = $\left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\frac{4!}{2!} + \left(\frac{1}{4}\right)^4 = \frac{25}{256}$
(b)(i)	$P(A) = \frac{1}{2}$, $P(A \cup B) = 0.65$ $P(A \cup B) = P(A) + P(B)$ $P(B) = 0.65 - 0.50 = 0.15$
(ii)	$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = 0.4$ $P(B \cap A) = 0.4(0.5) = 0.2$ $\therefore P(A \cup B) = P(A) + P(B) - P(B \cap A)$ $0.65 = 0.5 + P(B) - 0.2$ $P(B) = 0.35$ Since $P(B \mid A) \neq P(B)$, thus A and B are not independent.

13(i)	P(getting a matched pair of socks)= $\dfrac{\binom{2}{2}+\binom{4}{2}+\binom{3}{2}}{\binom{9}{2}}=\dfrac{5}{18}$
(ii)	P(getting a matched pair in 3 socks) =1-P(all 3 socks are of different colours) $=1-\dfrac{\binom{2}{1}\binom{4}{1}\binom{3}{1}}{\binom{9}{3}}=\dfrac{5}{7}$
(iii)	P(3rd sock does not give match/1st 2 not matched) $=\dfrac{\binom{2}{1}\binom{4}{1}\binom{3}{1}}{\binom{9}{3}}\div(1-\dfrac{5}{18})=\dfrac{36}{91}$

14(i)	P(C) = $p\times1+\left(1-p\right)\times\dfrac{1}{5}$
(ii)	K'∩C represents the event where Alice does not know the correct answer but she answers correctly
(iii)	P(K'|C) = $\dfrac{P(K'\cap C)}{P(C)}=\dfrac{1}{16}$ $\qquad \dfrac{\left(1-p\right)\times\frac{1}{5}}{p\times1+\left(1-p\right)\times\frac{1}{5}}=\dfrac{1}{16}$ solving, get p =0.75.
(iv)	When p = 0.3, P(C) = 0.3+0.7×0.2=0.44 P(3 consecutive correct | 3 correct answers) = $\dfrac{(0.44)^3(0.56)^2\times3}{(0.44)^3(0.56)^2\times C^5_3}=\dfrac{3}{10}$
(v)	P(negative score) = P(0 correct or 1 correct) = $(0.56)^5+C^5_1(0.56)^4(0.44)$ = 0.271

15(i)	P(Chris is late in a day) = $\frac{4}{5}+\frac{2}{5}-(\frac{4}{5})(\frac{2}{5})=\frac{22}{25}$ Let X be the number of days Chris is late out of 4 days. X~B(4, $\frac{22}{25}$). P(Chris was late exactly thrice in a week | Chris was late on Mon) = P(Chris was late twice in the remaining four days of the week) = P(X = 2) ≈ 0.0669
(ii)	P(Chris was delayed at A | Chris was late) = P(Chris was delayed at A)/P(Chris was late) $\dfrac{\frac{4}{5}}{\frac{22}{25}}=\frac{10}{11}$
(iii)	P(Chris was delayed at B | he was delayed at exactly one stop) = P(Chris was delayed at stop B only)/P(delayed at exactly one stop) $=\dfrac{(\frac{1}{5})(\frac{2}{5})}{(\frac{4}{5})(\frac{3}{5})+(\frac{1}{5})(\frac{2}{5})}=\frac{1}{7}$

16(a)(i)	Necessarily false. $P(A\|B)=0\neq P(A)$
(ii)	Necessarily false. $P(A\cap B)=P(A)P(B)\neq 0$
(b)(i)	Required probability $=0.4p+0.3\times\frac{1}{2}p+0.2\times\frac{1}{4}p+0.1\times\frac{1}{8}p=0.6125p$ or $\frac{49}{80}p$
(ii)	Required probability $=\dfrac{\left(0.4p+0.3\times\frac{1}{2}p+0.1\times\frac{1}{8}p\right)\left(\frac{2}{9}\times\frac{1}{4}p\right)+\left(\frac{2}{10}\times\frac{1}{4}p\right)\left(\frac{1}{9}\times\frac{1}{4}p\right)}{0.6125p}$ $=0.0533p$ or $\frac{47}{882}p$

(iii)	$1-\frac{49}{80}p \geq \frac{7}{10}$ $\frac{49}{80}p \leq \frac{3}{10}$ $0 < p \leq \frac{24}{49}$

17(i)	$0.2\times 0.02+0.7\times 0.05+0.1\times 0.1=0.049$
(ii)	P(slept before 11p.m. and not late for school) / P(not late for school) $=\frac{0.2\times 0.98}{1-0.049}$=0.20610
(iii)	$(1-0.049)^5 = 0.77786 = 0.778$ (3 s.f.)
(iv)	Let X be a random variable denoting the number of weeks such that the student is on time for school every day in that week. $X \sim \text{B}(n, 0.77786)$ Given, $P(X \leq 8) \leq 0.14$ From G.C., n = 14

18(i)	P(first red bead is obtained on or before the 5th draw) $=\frac{2}{5}+\frac{3}{5}.\frac{2}{5}+\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)+\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)+\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)$ or $\sum_{r=0}^{4}\left(\frac{3}{5}\right)^r\left(\frac{2}{5}\right)$ = 0.922 Or 1 – P(no red on first 5 draws) = $1-\left(\frac{3}{5}\right)^5 = 0.922$
(ii)	P(obtaining a first green bead on the 8th draw given that no green bead has been obtained after 5 draws) = P(red on 6th and 7th draws and green on 8th draw)= $\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right) = 0.096$
(iii)	P(exactly r draws are required for beads of both colours to be obtained) $=\left(\frac{2}{5}\right)^{r-1}\left(\frac{3}{5}\right)+\left(\frac{3}{5}\right)^{r-1}\left(\frac{2}{5}\right)$

	$=\left(\frac{2}{5}\right)^{r-2}\left(\frac{6}{25}\right)+\left(\frac{3}{5}\right)^{r-2}\left(\frac{6}{25}\right)$ $=\left(\frac{6}{25}\right)\left[\left(\frac{2}{5}\right)^{r-2}+\left(\frac{3}{5}\right)^{r-2}\right]$, where $r = 2,3,4,\ldots$
(iv)	P(first obtaining beads of different colours after 5 or more draws) $=\left(\frac{6}{25}\right)\left[\left(\left(\frac{2}{5}\right)^{3}+\left(\frac{3}{5}\right)^{3}\right)+\left(\left(\frac{2}{5}\right)^{4}+\left(\frac{3}{5}\right)^{4}\right)+\ldots\right]$ $=\left(\frac{6}{25}\right)\left[\frac{\left(\frac{2}{5}\right)^{3}}{1-\frac{2}{5}}+\frac{\left(\frac{3}{5}\right)^{3}}{1-\frac{3}{5}}\right]$ $=0.155$ Or P(first obtaining beads of different colours after 5 or more draws) = P(obtaining same colour in the first 4 draws) = P(first 4 red beads) + P(first 4 green beads) $=\left(\frac{2}{5}\right)^{4}+\left(\frac{3}{5}\right)^{4}=0.155$
(v)	E(no. of draws to first obtain beads of different colour) $=\sum_{r=2}^{\infty} r\left(\frac{6}{25}\right)\left[\left(\frac{2}{5}\right)^{r-2}+\left(\frac{3}{5}\right)^{r-2}\right]=3.17$ **(by GC)**

19(i)	$P(A \cap B)$ = P(rise, rise, fall) + P(rise, fall,fall) $=(0.1\times0.7\times0.3)+(0.1\times0.3\times0.9)=0.048$
(ii)	P(B) $=P(A\cap B)+P(A'\cap B)$ $=0.048+$ P(fall, fall, fall) + P(fall, rise, fall) $=0.048+(0.9\times0.9\times0.9)+(0.9\times0.1\times0.3)=0.804$
(iii)	$P(A' \cup B)=1-[P(A)-P(A\cap B)]$ $=1-[0.1-0.048]=0.948$
(iv)	$P(B\mid A)=\frac{P(A\cap B)}{P(A)}=\frac{0.048}{0.1}=0.48$

(v)	Since [P(B) = 0.804] ≠ [P($B \mid A$)= 0.48] $\therefore$ A and B are not independent

20(i)	$\mathrm{P}(A\cup B)=\mathrm{P}(B)+\mathrm{P}(A\cap B')$ $\mathrm{P}(B)=0.85-0.3=0.55$
(ii)	$\mathrm{P}(A\cap B)=\mathrm{P}(A)-\mathrm{P}(A\cap B')$ $\mathrm{P}(A\cap B)=0.45-0.3=0.15$
(iii)	$\mathrm{P}(A\mid B')=\dfrac{\mathrm{P}(A\cap B')}{1-\mathrm{P}(B)}$ $\mathrm{P}(A\mid B')=\dfrac{0.3}{1-0.55}=\dfrac{2}{3}$
(iv)	A and C are independent, $\Rightarrow A$ and C' are independent. $\mathrm{P}(A\cap C')=\mathrm{P}(A)\times\mathrm{P}(C')$ $\mathrm{P}(A\cap C')=0.45\times(1-0.6)=0.18$

21(i)	Let event A be "watched '*Jogging man*' Let event B be "watched '*Voice of me*' Given $\mathrm{P}(A)=0.7$, $\mathrm{P}(B)=0.6$, $\mathrm{P}(A\mid B')=0.4$, $\mathrm{P}(A\mid B')=\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}=0.4$ $\mathrm{P}(A\cap B')=0.4(0.4)=0.16$ $\mathrm{P}(A\cap B)=\mathrm{P}(A)-\mathrm{P}(A\cap B')=0.7-0.16=0.54$
(ii)	$\mathrm{P}(A'\cap B)+\mathrm{P}(A\cap B')=\left(\mathrm{P}(B)-\mathrm{P}(A\cap B)\right)+\mathrm{P}(A\cap B')$ $=\left(0.6-0.54\right)+0.16=0.22$
(iii)	$\mathrm{P}(B\mid A')=\dfrac{\mathrm{P}(B\cap A')}{\mathrm{P}(A')}=\dfrac{0.06}{0.3}=0.2$ Since $\mathrm{P}(A)=0.7\neq\mathrm{P}(A\mid B')=0.4$, the two events are not independent

22(i)	$\dfrac{3}{n}\cdot\dfrac{3}{n+1}+\dfrac{n-3}{n}\cdot\dfrac{4}{n+1}=\dfrac{4n-3}{n(n+1)}$

(ii)	$\dfrac{\frac{3}{n}\cdot\frac{3}{n+1}}{\frac{4n-3}{n(n+1)}}=\dfrac{9}{4n-3}$

23(i)	Number of arrangements without restrictions = $\dfrac{9!}{2!2!3!}$ Required probability = $\dfrac{8!}{2!3!}\div\dfrac{9!}{2!2!3!}=\dfrac{2}{9}$
(ii)	No. of ways to 'slot' in 'red beads' and 'green beads' = ${}^{6}C_{4}\dfrac{4!}{2!2!}$ Required probability = $\dfrac{5!}{3!}{}^{6}C_{4}\dfrac{4!}{2!2!}\div\dfrac{9!}{2!2!3!}=\dfrac{5}{42}$
(iii)	P (**2 'red beads together'** and **2 'green beads together')** $=\dfrac{7!}{3!}\div\dfrac{9!}{2!2!3!}=\dfrac{1}{18}$ Required probability $=P(A\cup B)$ $=P(A)+P(B)-P(A\cap B)$ $=\dfrac{2}{9}+\dfrac{2}{9}-\dfrac{1}{18}=\dfrac{7}{18}$
(iv)	Number of arrangements without restrictions = $\dfrac{8!}{2(2!2!3!)}$ Required probability = $\dfrac{6!}{2(3!)2!}\div\dfrac{8!}{2(2!2!3!)2!}=\dfrac{1}{14}$

24(i)	Probability = P(drawing from box A,B,A,B,A,B) =P(drawing R,W,R,W,R) $=\left(\dfrac{5}{10}\right)\left(\dfrac{6}{10}\right)\left(\dfrac{4}{9}\right)\left(\dfrac{5}{9}\right)\left(\dfrac{3}{8}\right)=\dfrac{1}{36}$
(ii)	Let event E be the event that he drew from box B on the fourth draw. Let event F be the event that he has not drawn from box C from the first to his sixth draw, including the sixth draw. $\mathrm{P}(\mathrm{F}\cap\mathrm{E})=\dfrac{1}{36}$

	$\text{P}(\text{E})$ = P(drawing from A,B,C,B or A,B,A,B or A,C,A,B) = P(drawing R,R,W or R,W,R or W,R,R) $=\left(\frac{5}{10}\right)\left(\frac{4}{10}\right)\left(\frac{7}{10}\right)+\left(\frac{5}{10}\right)\left(\frac{6}{10}\right)\left(\frac{4}{9}\right)+\left(\frac{5}{10}\right)\left(\frac{3}{10}\right)\left(\frac{5}{9}\right)=\frac{107}{300}$ $\text{P}(\text{F}\mid\text{E})$ $=\frac{\text{P}(\text{F}\cap\text{E})}{\text{P}(\text{E})}=\frac{\text{P}(\text{drawing from box A, B, A, B, A, B})}{\text{P}(\text{drawing from box B on fourth draw})}=\frac{\frac{1}{36}}{\frac{107}{300}}=\frac{25}{321}$

25(c)	Total probability $=\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{7}\right)\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{7}\right)^2\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)+\ldots$ $=\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)\left[1+\left(\frac{5}{7}\right)+\left(\frac{5}{7}\right)^2+\ldots\right]$ $=\left(\frac{1}{21}\right)\left[\frac{1}{1-\left(\frac{5}{7}\right)}\right]=\frac{1}{6}$

26(i)	Let $\text{P}(A)=\alpha$ and $\text{P}(B)=\beta$. $\text{P}(A)+\text{P}(B)=\text{P}(A\cup B)+\text{P}(A\cap B)$ $=0.55+0.05=0.6$ $\Rightarrow \alpha+\beta=0.6\cdots\cdots(1)$ Since A and B are independent, $\text{P}(A\cap B)=\text{P}(A)\text{P}(B)$ $\Rightarrow \alpha\beta=0.05$ i.e. $\alpha=\frac{0.05}{\beta}\cdots\cdots(2)$ Sub (2) into (1): $\frac{0.05}{\beta}+\beta=0.6$ $\beta^2-0.6\beta+0.05=0$ $\beta=0.5$ or $\beta=0.1\cdots\cdots(3)$ Sub (3) into (1):

	$\alpha = 0.1$ or $\alpha = 0.5$ Since $\alpha < \beta$, $\mathrm{P}(A) = 0.1$ and $\mathrm{P}(B) = 0.5$
(ii)	**<u>Method 1</u>** $\mathrm{P}(A' \cup B' \mid C)$ $= 1 - \mathrm{P}((A' \cup B')' \mid C)$ $= 1 - \mathrm{P}(A \cap B \mid C)$ $= 1 - \dfrac{\mathrm{P}(A \cap B \cap C)}{\mathrm{P}(C)}$ $= 1 - \dfrac{\mathrm{P}(A \cap B \cap C)}{0.5} = 0.95$ $\therefore \mathrm{P}(A \cap B \cap C) = 0.05 \times 0.5 = 0.025.$ **<u>Method 2</u>** $\mathrm{P}(A' \cup B' \mid C) = \dfrac{\mathrm{P}((A' \cup B') \cap C)}{\mathrm{P}(C)} = 0.95$ $\mathrm{P}((A' \cup B') \cap C) = 0.95 \times 0.5 = 0.475$ $\mathrm{P}(A \cap B \cap C) = \mathrm{P}(C) - \mathrm{P}((A' \cup B') \cap C) = 0.5 - 0.475 = 0.025$
(iii)	Since $\mathrm{P}(A \cap C) \geq \mathrm{P}(A \cap B \cap C) = 0.025 > 0$, A and C are not mutually exclusive.

27(a)(i)	$\mathrm{P}(R = 3) = \dfrac{\binom{15}{3}\binom{10}{5}}{\binom{25}{8}} = 0.10601 = 0.106$ (3 s.f) (Shown) **Alternatively: Probability method** $\mathrm{P}(R = 3) = \left(\dfrac{15}{25}\right)\left(\dfrac{14}{24}\right)\left(\dfrac{13}{23}\right)\left(\dfrac{10}{22}\right)\left(\dfrac{9}{21}\right)\left(\dfrac{8}{20}\right)\left(\dfrac{7}{19}\right)\left(\dfrac{6}{18}\right)\left(\dfrac{8!}{3!5!}\right)$ $= 0.10601 = 0.106$ (3 s.f) (Shown)
(ii)	$\mathrm{P}(R = r) > \mathrm{P}(R = r + 1)$

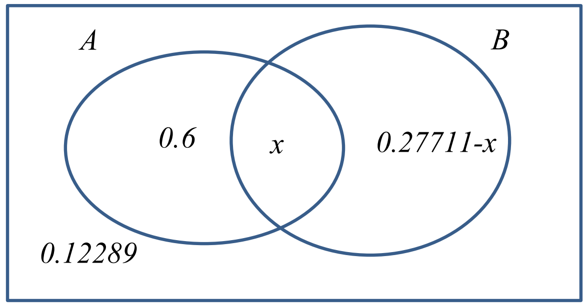
	$\frac{\binom{15}{r}\binom{10}{8-r}}{\binom{25}{8}} > \frac{\binom{15}{r+1}\binom{10}{7-r}}{\binom{25}{8}}$ $\binom{15}{r}\binom{10}{8-r} > \binom{15}{r+1}\binom{10}{7-r}$ $\frac{15!}{r!(15-r)!}\frac{10!}{(8-r)!(2+r)!} > \frac{15!}{(r+1)!(14-r)!}\frac{10!}{(7-r)!(3+r)!}$ $(r+1)!(14-r)!(7-r)!(3+r)! > r!(15-r)!(8-r)!(2+r)!$ (shown) $\frac{(r+1)!(14-r)!(7-r)!(3+r)!}{r!(15-r)!(8-r)!(2+r)!} > 1$ $\Rightarrow \frac{(r+1)(r+3)}{(15-r)(8-r)} > 1$ $(r+1)(r+3) > (15-r)(8-r)$ $r^2+4r+3 > 120-23r+r^2$ $27r > 117$ $r > 4.33,$ $r = 5, 6, 7, 8$ But **P(R=5)** > P(R=6) > P(R=7) > P(R=8) since given $\mathrm{P}(R=r) > \mathrm{P}(R=r+1)$, Therefore r = 5 since P(R=5) is the highest probability among all these values. [Similarly, $\mathrm{P}(R=r) < \mathrm{P}(R=r+1) \Rightarrow r < 4.333 \Rightarrow r = 0,1,..,4$ Hence, $\mathrm{P}(R=0) < \mathrm{P}(R=1) < ... < \mathrm{P}(R=4) < \mathrm{P}(R=5)$.] (question defined "The most probable number of red balls drawn is denoted by r.")

28(i)	<u>Method 1</u> Total number of possible codes $= 26^2 \times 10^3 = 676000$ Total number of codes with three different digits and two different letters= $= {}^{26}P_2 \times {}^{10}P_3 = 46800$ Required prob= $\frac{46800}{676000} = \frac{9}{13}$ <u>Method 2</u> Required prob $= 1 \times 1 \times \frac{9}{10} \times \frac{8}{10} \times \frac{25}{26} = \frac{9}{13}$

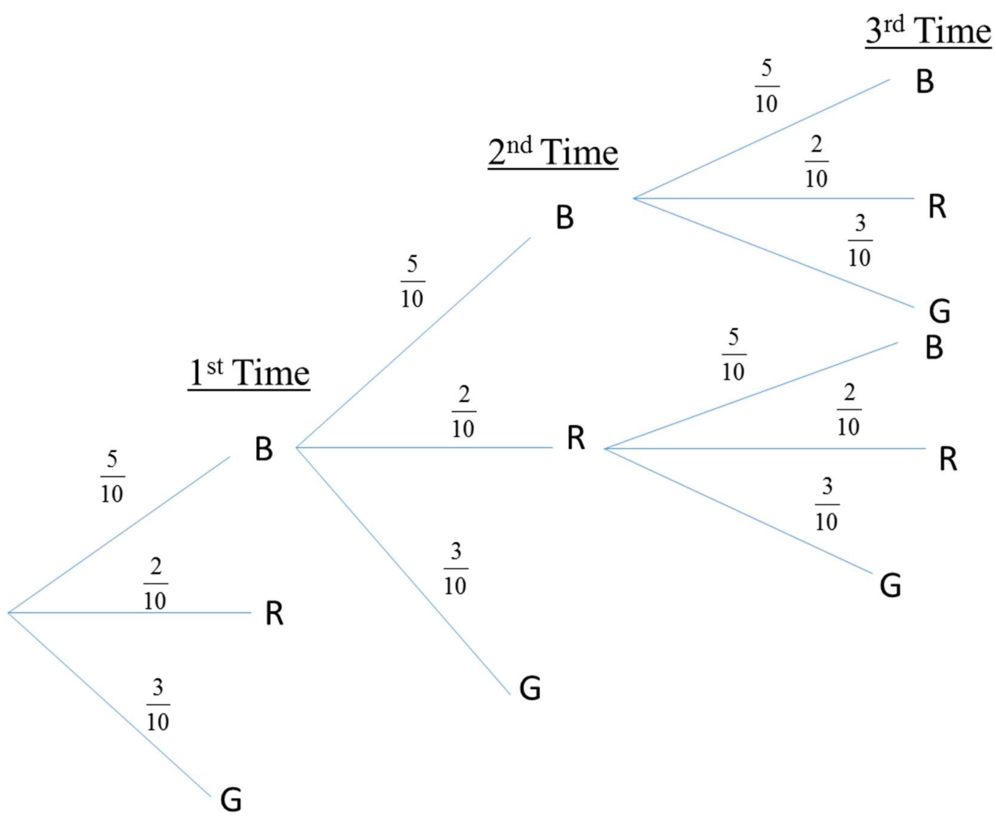
(ii)	<u>Method 1</u> Total number of palindromes $= 26\times10\times10\times1\times1 = 2600$ P(code is a Palindrome code) $= \frac{2600}{676000} = \frac{1}{260}$ <u>Method 2</u> P(code is a Palindrome code) $= 1\times1\times1\times\frac{1}{10}\times\frac{1}{26} = \frac{1}{260}$
(iii)	<u>Method 1</u> $P(\text{2 and 3 used in code} \mid \text{code is palindrome})$ $= \frac{P(\text{code is palindrome and contains 2 and 3})}{P(\text{code is palindrome})}$ $= \frac{P(\_232\_) + P(\_323\_)}{(1/260)} = \frac{2\left(1\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{26}\right)}{(1/260)} = 0.02$ <u>Method 2</u> $P(\text{2 and 3 used in code} \mid \text{code is palindrome})$ $= \frac{\text{no of palindrome codes with 2 and 3}}{\text{number of palindrome codes}}$ $= \frac{\text{no of palindrome with } (\_232\_) + \text{no of palindrome with} (\_323\_)}{2600}$ $= \frac{26+26}{2600} = \frac{1}{50}$
(iv)	Need to find the probability of a code containing 2 and 3. <u>Case 1: Code contains 2 and 3 only.</u> No. of codes with 2 and 3 only $= 2\times26^2\times{}^3C_2 = 4056$ <u>Case 2: Code contains 2, 3 and 1 other number</u> No of codes with 2, 3 and 1 other number $= {}^8C_1\times3!\times26^2 = 32448$ Prob (code contains 2 and 3) $= \frac{32448+4056}{676000} = 0.054 \neq 0.02$ Thus, the events are not independent.

29 (i)	Given $P(A \mid B') = 0.83$

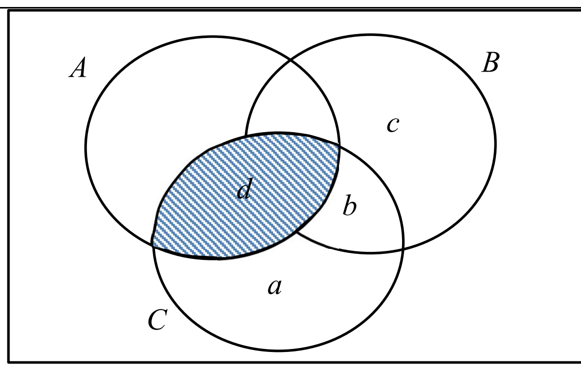
	$\Rightarrow \dfrac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')} = 0.83$ $\Rightarrow \dfrac{0.6}{1-\mathrm{P}(B)} = 0.83$ $\Rightarrow P(B) = 1 - 0.72289 = 0.27711 = 0.277$

(ii)	Let P($A \cap B$) = x  $P(A \cup B) = P(A \cap B') + P(B)$ $= 0.6 + x + 0.27711 - x$ $= 0.87711$ $P(A \cup B)' = 1 - 0.87711 = 0.12289$ Since $P(A \cup B') = 0.83$ $\therefore 0.6 + x + 0.12289 = 0.83$ $\Rightarrow x = 0.10711$ $\therefore P(A \cap B) = 0.107$
(iii)	$P(B \mid A') = \frac{P(B \cap A')}{P(A')} = \frac{0.27711 - 0.10711}{1 - (0.6 + 0.10711)} = 0.58042 = 0.580$
	Since $P(B \mid A') \neq P(B) \Rightarrow B$ is not independent of A' $\therefore$ A and B are not independent.

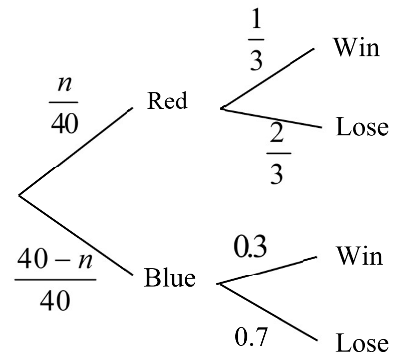
30	(a)(i) By considering $P(A \cup B) \leq 1$ $P(A \cap B') + P(A \cap B) + P(B \cap A') \leq 1$ Let $P(A \cap B) = c$ Then we have $a - c + c + b - c \leq 1$ $c \geq a + b - 1$ Note: Can also use $P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq 1 \Rightarrow a + b - c \leq 1 \Rightarrow c \geq a + b - 1$ Thus, minimum possible value of $P(A \cap B)$ is $a + b - 1$.
	(a)(ii) when $P(A) = 0.6$ and $P(B) = 0.2$, [note that $P(A) + P(B) < 1$]. $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = 0.8 - P(A \cup B)$ minimum possible value of $P(A \cap B)$ is 0, i.e., A and B are disjoint.

	[i.e. when $P(A\cup B)=P(A)+P(B)$].
	(b) $P(A'\cap B')=1-P(A\cup B)$ $=1-\left[P(A)+P(B)-P(A\cap B)\right]$ $=1-P(A)-P(B)+P(A)P(B)$ (since A, B are independent) $=P(A')-P(B)\left[1-P(A)\right]$ $=P(A')-P(B)\left[P(A')\right]$ $=P(A')\left[1-P(B)\right]$ $=P(A')P(B')$ Thus A' and B' are independent as well.
31(i)	
(ii)	P(player wins $12 or more when the game ends) = P(BRR) + P(BBR) +P(R) $=\frac{5}{10}\left(\frac{2}{10}\right)\left(\frac{2}{10}\right)+\frac{5}{10}\left(\frac{5}{10}\right)\left(\frac{2}{10}\right)+\left(\frac{2}{10}\right)=\frac{27}{100}$

(iii)	P (second spin lands on a blue sector | the player wins $12 or more when the game ends) $=\dfrac{\left(\frac{5}{10}\right)\left(\frac{5}{10}\right)\left(\frac{2}{10}\right)}{0.27}=\dfrac{5}{27}$
(iv)	P (no spins result on a red sector and wins nothing when game ends) $=\frac{3}{10}+\left(\frac{5}{10}\right)\left(\frac{3}{10}\right)+\left(\frac{5}{10}\right)^2\left(\frac{3}{10}\right)+\left(\frac{5}{10}\right)^3\left(\frac{3}{10}\right)+\ldots=\dfrac{\frac{3}{10}}{1-\frac{5}{10}}=\frac{3}{5}$

32 (i)	Req Prob $=\mathrm{P}(A\cup B)-\mathrm{P}(A\cap B)=\mathrm{P}(A)+\mathrm{P}(B)-2\mathrm{P}(A\cap B)=\frac{11}{20}+\frac{2}{5}-\frac{1}{2}=\frac{9}{20}$
(ii)	$\mathrm{P}(B)-\mathrm{P}(A\cap B)=\frac{2}{5}-\frac{1}{4}=\frac{3}{20}$
(iii	
	$\mathrm{P}(A\cap C)=P(A)\times P(C)=\frac{11}{20}\times\frac{1}{2}=\frac{11}{40}$ P(C)= $\mathrm{P}(A\cap C)+b+a$ from (ii) b+c = $\frac{3}{20}$ $b+a=\frac{1}{2}-\frac{11}{40}$ $a+b=\frac{9}{40}$ Maximum a occurs when b=0 , so a= 9/40 Minimum a occurs when b is maximum (b= 3/20 when c=0), So minimum a = $\frac{9}{40}-\frac{3}{20}=\frac{3}{40}$ Hence $\frac{3}{40}\le \mathrm{P}(A'\cap B'\cap C)\le\frac{9}{40}$

33(i)



P(a player wins the game) $= \frac{15}{40}\times\frac{1}{3}+\frac{25}{40}\times\frac{3}{10}=\frac{5}{16}$

P(exactly 2 of 3 players win) $= {}^{3}C_{2}\left(\frac{5}{16}\right)^{2}\left(\frac{11}{16}\right)=\frac{825}{4096}$

(ii)

P(a player wins the game)

$$=\frac{n}{40}\times\frac{1}{3}+\frac{40-n}{40}\times\frac{3}{10}=\frac{10n}{1200}+\frac{360-9n}{1200}=\frac{360+n}{1200}$$

$\mathrm{f}(n)$

= P(player draws blue | player wins)

$$=\frac{\mathrm{P}(\text{player draws blue and wins})}{\mathrm{P}(\text{player wins})}$$

$$=\frac{\frac{40-n}{40}\times\frac{3}{10}}{\frac{360+n}{1200}}=\frac{360-9n}{360+n}=\frac{-9(360+n)+3600}{360+n}=-9+\frac{3600}{360+n}$$

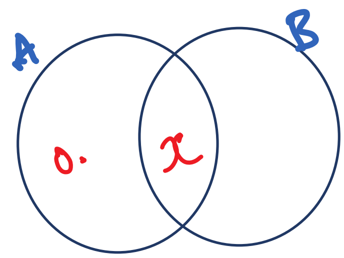
As n increases, $\frac{3600}{360+n}$ decreases, hence f(n) decreases.

Hence f is decreasing for all n, $0\le n\le 40$.

This means that as the number of red counters increase (i.e. n increases), the probability that a winning player drew a blue counter decreases (i.e. f(n) decreases).

34. Suggested solution

(i)



Let $P(A\cap B)=x$.

$$P(B|A)=\frac{7}{23}$$

$$\Rightarrow \frac{P(A\cap B)}{P(A)}=\frac{7}{23}\Rightarrow \frac{x}{0.32+x}=\frac{7}{23}$$

$$\Rightarrow 23x=7x+2.24\Rightarrow x=0.14$$

(ii)

$$P(A|B)=\frac{7}{19}$$

$$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{7}{19}$$

$$\Rightarrow \frac{0.14}{P(B)}=\frac{7}{19}$$

$$\Rightarrow P(B)=0.38$$

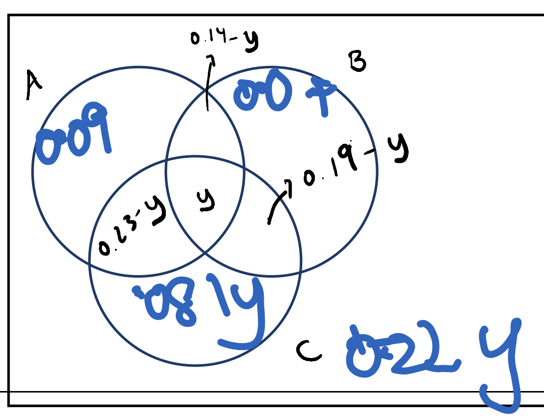
$$P(A\cup B)=P(B)+P(A\cap B')$$
$$=0.38+0.32=0.7$$

(iii)

Since A, C are independent, $P(A\cap C)=0.5(0.46)=0.23$

Since B, C are independent, $P(B\cap C)=0.5(0.38)=0.19$

Let $P(A\cap B\cap C)=y$



The values of *y* can range from 0 to 0.14.

When $y=0$,

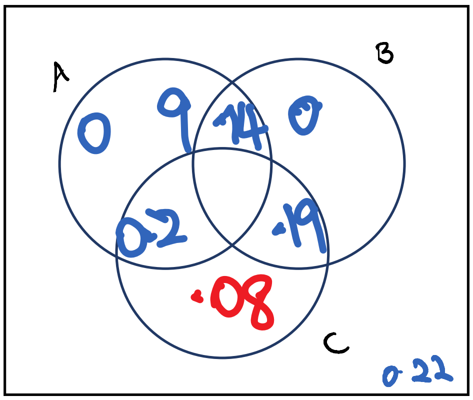
$P\left(A'\cap B'\cap C\right)=0.5-(0.19+0.23)=0.08$ **[Min value]**

When $y=0.14$,

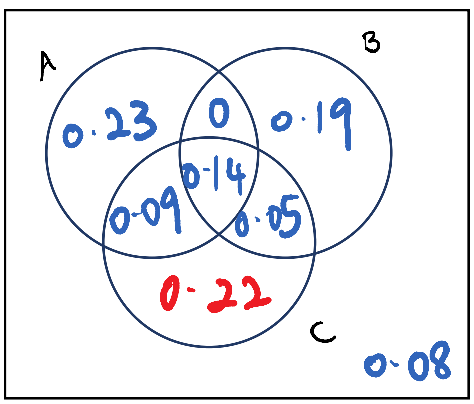
$P\left(A'\cap B'\cap C\right)=0.5-(0.19+0.23-0.14)=0.22$ **[Max value]**

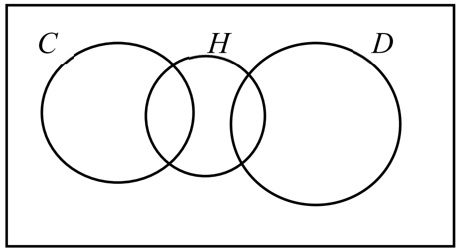
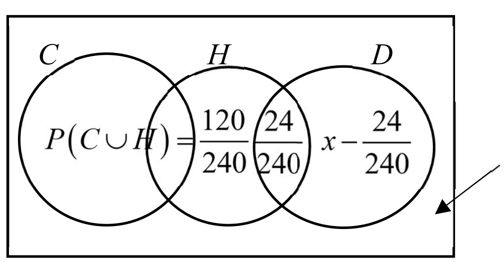
For a complete solution, we would still need to verify if the remaining regions are non-negative for this range of *y*.

For y=0



For *y=0.14*



35.	Suggested Answers
(i)	
(ii)	$\mathrm{P}(H)=\frac{80}{240}=\frac{1}{3}$; $\quad \mathrm{P}(C\cup H)=\frac{120}{240}=\frac{1}{2}$ $\mathrm{P}(C'\cap H')=1-\mathrm{P}(C\cup H)=1-\frac{120}{240}=\frac{1}{2}$
(iii)	$\mathrm{P}(C\cup H)=\mathrm{P}(C)+\mathrm{P}(H)-\mathrm{P}(C\cap H)$ $=\mathrm{P}(C)+\mathrm{P}(H)-\mathrm{P}(C)\mathrm{P}(H)$ since C and H are independent $\frac{1}{2}=\mathrm{P}(C)+\frac{1}{3}-\frac{1}{3}\mathrm{P}(C)$ $\frac{2}{3}\mathrm{P}(C)=\frac{1}{6}$ $\mathrm{P}(C)=\frac{1}{4}$ Alternative method: From (ii), $\mathrm{P}(C'\cap H')=\frac{1}{2}$ Since C and H are independent, C' and H' are also independent $\mathrm{P}(C')\mathrm{P}(H')=\frac{1}{2}$ $\mathrm{P}(C')\left(1-\frac{80}{240}\right)=\frac{1}{2}$ $\mathrm{P}(C')=\frac{3}{4}$ $\mathrm{P}(C)=\frac{1}{4}$
(iv)	 $\frac{120}{240}-\left(x-\frac{24}{240}\right)=\frac{144}{240}-x$

Let $\mathrm{P}(D) = x$

$$x - \frac{24}{240} \geq 0 \quad \Rightarrow \quad x \geq \frac{24}{240} = \frac{1}{10}$$

$$\frac{144}{240} - x \geq 0 \quad \Rightarrow \quad x \leq \frac{144}{240} = \frac{3}{5}$$

Hence greatest value of $\mathrm{P}(D) = \frac{3}{5}$

Alternative method:
From (ii) and Venn diagram, greatest x happens when all those who do not like cats or hamsters like dogs.
Hence largest $\mathrm{P}(D) = \mathrm{P}(C' \cap H') + P(D \cap H)$

$$= \frac{1}{2} + \frac{24}{120} = \frac{3}{5}$$

(v)

$$\mathrm{P}(D \mid H') = \frac{1}{8}$$

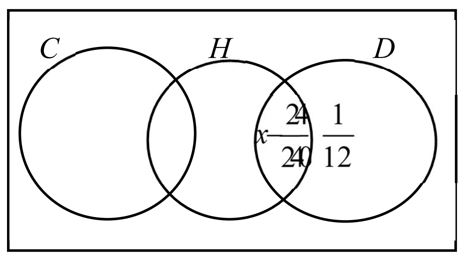
$$\frac{\mathrm{P}(D \cap H')}{\mathrm{P}(H')} = \frac{1}{8}$$

$$\mathrm{P}(D \cap H') = \frac{1}{8} \times \frac{2}{3} = \frac{1}{12}$$

$$\mathrm{P}(D) = \mathrm{P}(D \cap H) + \mathrm{P}(D \cap H')$$

$$= \frac{24}{240} + \frac{1}{12}$$

$$= \frac{11}{60}$$



36.	**Suggested Answers**
(i)	Let G_A and G_B represent the event that the ball drawn is green from Box A and Box B respectively. Let W_A and W_B represent the event that the ball drawn is white from Box A and Box B respectively.

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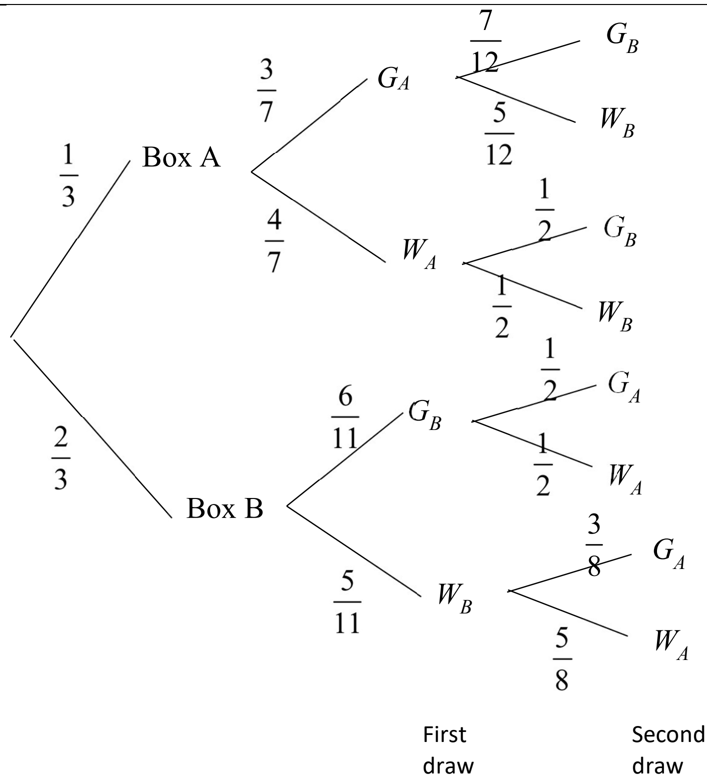

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<td>(ii)</td>
<td>
P(wins the game) = P(second ball is white)

$= \text{P}(G_A, W_B) + \text{P}(W_A, W_B) + \text{P}(G_B, W_A) + \text{P}(W_B, W_A)$

$= \frac{1}{3}\left[\left(\frac{3}{7}\right)\left(\frac{5}{12}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)\right] + \frac{2}{3}\left[\left(\frac{6}{11}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{11}\right)\left(\frac{5}{8}\right)\right]$

$= \frac{81}{154}$

P(first ball from Box A| Player wins the game)

$= \frac{\text{P(first ball from } A \cap \text{player wins the game)}}{\text{P(player wins the game)}}$

$= \frac{\text{P}\begin{pmatrix}\text{score of die is less than 3 followed by} \\ \text{drawing a ball from A} \\ \text{and the second ball drawn is white}\end{pmatrix}}{\text{P}(\text{player wins the game})}$

$= \frac{\frac{1}{3}\left[\left(\frac{3}{7}\right)\left(\frac{5}{12}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)\right]}{\frac{81}{154}} = \frac{143}{486}$
</td>
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37.	**Suggested Answers**
(a)(i)	$\text{P}(A \cap B)$ $= \text{P(fall, rise, rise)} + \text{P(fall, fall, rise)}$ $= (0.4 \times 0.15 \times 0.6) + (0.4 \times 0.85 \times 0.15)$ $= 0.087$
(ii)	$\text{P}(B)$ $= \text{P}(A \cap B) + \text{P}(A' \cap B)$ $= 0.087 + \text{P(rise, rise, rise)} + \text{P(rise, fall, rise)}$ $= 0.087 + (0.6 \times 0.6 \times 0.6) + (0.6 \times 0.4 \times 0.15)$ $= 0.339$
(iii)	$\text{P}(B \mid A)$ $= \dfrac{\text{P}(B \cap A)}{\text{P}(A)}$ $= \dfrac{0.087}{0.4}$ $= 0.2175$ Since $\text{P}(B \mid A) = 0.2175 \neq 0.339 = \text{P}(B)$, A and B are not independent. Let W be the number of Tuesdays in which the unit price of X rises, out of 12 Tuesdays. $W \sim \text{B}(12, 0.6)$ $\text{P}(W = 5) = 0.101$ (3 s.f.)
(b)	$\text{P}(A \cap B)$ $= \text{P(fall, rise, rise)} + \text{P(fall, fall, rise)}$ $= (0.4 \times 0.15 \times 0.6) + (0.4 \times 0.85 \times 0.15)$ $= 0.087$ $\text{P}(B)$ $= \text{P}(A \cap B) + \text{P}(A' \cap B)$ $= 0.087 + \text{P(rise, rise, rise)} + \text{P(rise, fall, rise)}$ $= 0.087 + (0.6 \times 0.6 \times 0.6) + (0.6 \times 0.4 \times 0.15)$ $= 0.339$
(c)	$\text{P}(B \mid A)$ $= \dfrac{\text{P}(B \cap A)}{\text{P}(A)}$ $= \dfrac{0.087}{0.4}$ $= 0.2175$