H2 Mathematics (9758)
Chapter 3 Functions
Assignment Solutions

1 2011/9740/II/3

The function f is defined by

$$\mathrm{f}: x \mapsto \ln(2x+1)+3, \quad x \in \mathbb{R},\ x > -\frac{1}{2}.$$

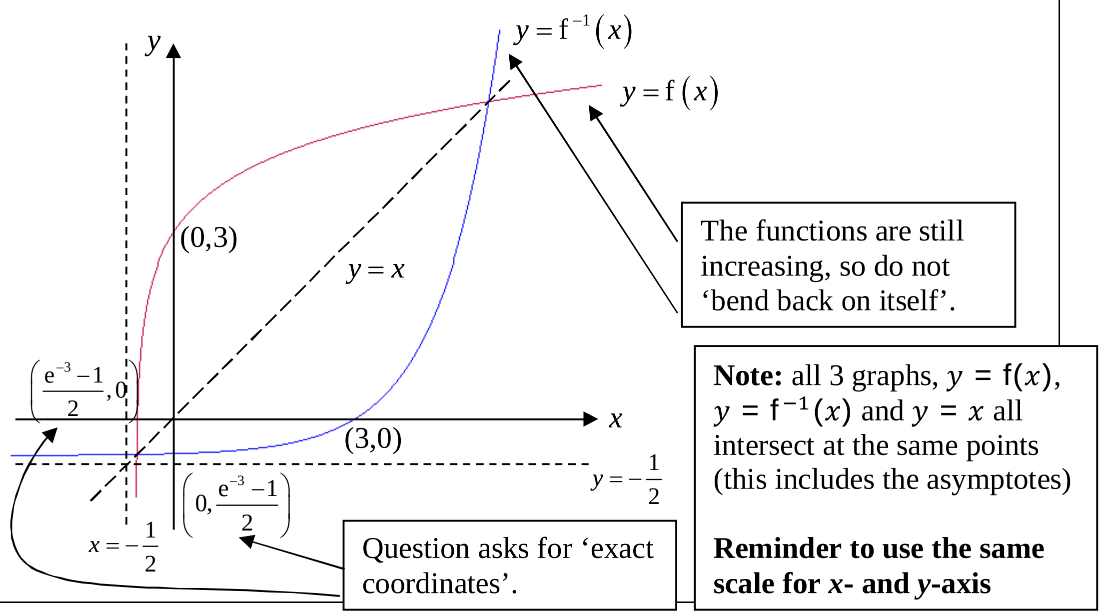
(i) Find $\mathrm{f}^{-1}(x)$ and write down the domain and range of f^{-1}. [4]

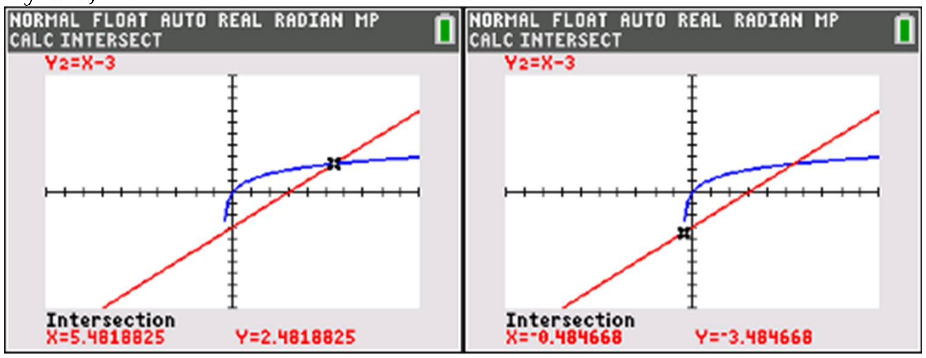
(ii) Sketch on the same diagram the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the *x*- and *y*-axes. [4]

(iii) Explain why the *x*-coordinates of the points of intersection of the curves in part **(ii)** satisfy the equation

$$\ln(2x+1)=x-3,$$

and find the values of these *x*-coordinates, correct to 4 significant figures. [3]

Q1	**Solution**
(i)	Let $y=\ln(2x+1)+3$. $2x+1=\mathrm{e}^{y-3}$ $x=\dfrac{\mathrm{e}^{y-3}-1}{2}$ $\therefore \mathrm{f}^{-1}(x)=\dfrac{\mathrm{e}^{x-3}-1}{2}$ $D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}=\mathbb{R}, \quad R_{\mathrm{f}^{-1}}=D_{\mathrm{f}}=\left(-\dfrac{1}{2},\infty\right)$
(ii)	

(iii)	The points of intersection also lies on the line $y = x$, thus the x-coordinates also satisfy the equation of $\mathrm{f}\left(x\right)=x \;\Rightarrow\; \ln\left(2x+1\right)+3=x \;\Rightarrow\; \ln\left(2x+1\right)=x-3$. By GC,  $x = -0.4847$ or $x = 5.482$ (4 s.f.)

There are 2 values of *x* so make sure the graphs in part **(ii)** intersect at 2 points. Note that GC does not show the intersection but you can still find the intersection.

2 2017/PJC Promo/Q7 (modified)

Functions f and g are defined by

$$\mathrm{f} : x \mapsto (x+1)^2 + 2 \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{1}{x-5} \quad \text{for } x \in \mathbb{R}, x \neq 5 .$$

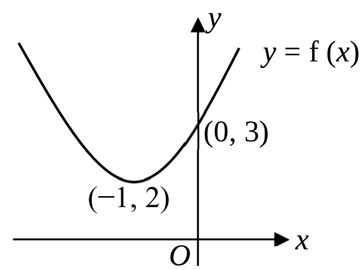
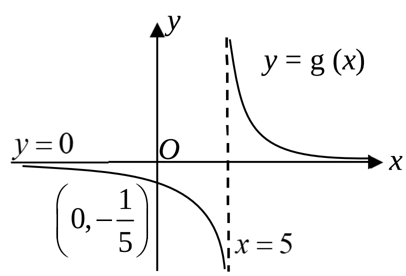
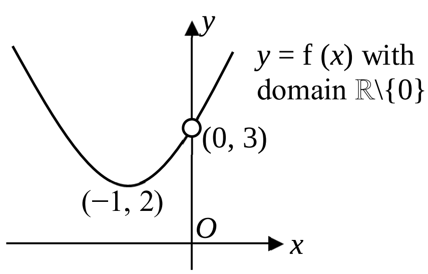
(i) Only one of the composite functions fg and gf exists. Give a definition (including the domain) of the composite that exists, and explain why the other composite does not exist. For the composite function that exist, find its range. [5]

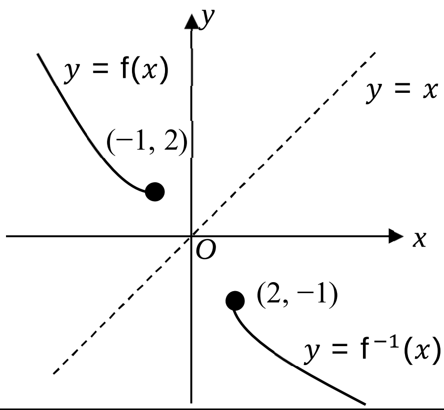
(ii) If the domain of f is restricted to $x \le k$, $k \in \mathbb{R}$, state the greatest value of k for which the function f^{-1} exists.

For the rest of the question, use the value of *k* found in part (ii).

(iii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$ for $x \le k$ and $y = \mathrm{f}^{-1}(x)$, showing clearly the relationship between the two graphs. [4]

(iv) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

Q2	Solution
(i)	$R_g = \mathbb{R}\backslash\{0\}$, $D_f = \mathbb{R}$ Since $R_g \subseteq D_f$, fg exists $\mathrm{fg}(x) = \mathrm{f}\left(\frac{1}{x-5}\right) = \left(\frac{1}{x-5}+1\right)^2 + 2$ $\mathrm{fg} : x \mapsto \left(\frac{1}{x-5}+1\right)^2 + 2, x \in \mathbb{R}, x \neq 5$ $D_{fg} = \mathbb{R}\backslash\{5\}$ $R_f = [2,\infty)$, $D_g = \mathbb{R}\backslash\{5\}$ Since $R_f \not\subseteq D_g$, gf does not exist.    $D_{fg} = D_g = \mathbb{R}\backslash\{5\} \xrightarrow{g} R_g = \mathbb{R}\backslash\{0\} \xrightarrow{f} R_{fg} = [2,\infty)$
(ii)	Greatest $k = -1$

(iii)	 **Reminder to use the same scale for *x*- and *y*-axis so that** $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are **symmetrical about the line** $y = x$
(iv)	Let $y = (x+1)^2 + 2 \quad , \ x \le -1$ $(x+1)^2 = y - 2$ $x = -1 \pm \sqrt{y-2}$ $x = -1 - \sqrt{y-2}$, since $x \le -1$ $\mathrm{f}^{-1}(x) = -1 - \sqrt{x-2}, x \in \mathbb{R}, x \ge 2$, $\mathrm{D}_{\mathrm{f}^{-1}} = [2, \infty)$

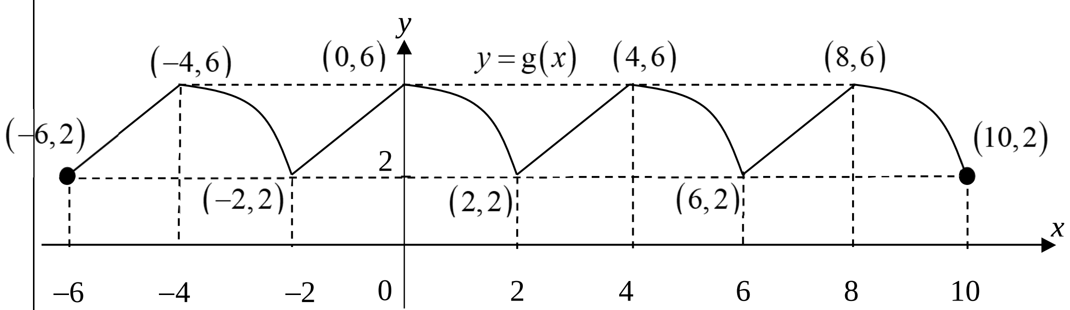
3 It is given that

$$g(x)=\begin{cases}6-x^2 & \text{for } 0<x\le 2,\\ 2x-2 & \text{for } 2<x\le 4,\end{cases}$$

and that $g(x)=g(x+4)$ for all real values of x.

(i) Sketch the graph of $y = g(x)$ for $-6\le x\le 10$. [4]

(ii) Evaluate $g(-3)+g(7)$. [3]

Q3	Solution
(i)	
(ii)	$g(-3)+g(7)=g(1)+g(3)$ $=(6-1^2)+(2(3)-2)$ $=9$