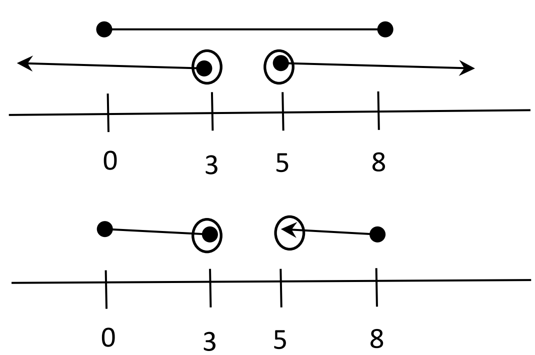
2021 KSS AM Prelim P1 (Solutions)

1	$x^3+2x\,\overline{)\,x^4+1}$ with quotient x $-(x^4+2x^2)$ $-2x^2+1$ $\dfrac{x^4+1}{x^3+2x}=x+\dfrac{1-2x^2}{x(x^2+2)}$ $Let\ \dfrac{1-2x^2}{x(x^2+2)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+2}$ $when\ x=0$ $A=\dfrac{1}{2}$ $By\ comparing,$ $\dfrac{1}{2}(x^2+2)+x(Bx+C)=1-2x^2$ $B=-\dfrac{5}{2}$ $C=0$ $\therefore \dfrac{x^4+1}{x^3+2x}=x+\dfrac{1}{2x}-\dfrac{5x}{2(x^2+2)}$	
2	$-15<x^2-8x \quad and \quad x^2-8x\le 0$ $0<x^2-8x+15$ $0<(x-5)(x-3) \quad\quad x(x-8)\le 0$ $(sketch)$ $\therefore x<3 \quad or\ x>5 \quad\quad \therefore 0\le x\le 8$  The solution set is $0\le x<3 \quad and \quad 5<x\le 8$, hence x = 5 does not satisfy the inequality.	

3i	$f(x)=2(x+1)(x^2-2x+3)=2x^3-2x^2+2x+6$	
3ii	$f(x)=0$ $x^2-2x+3=0$ $b^2-4ac=(-2)^2-4(1)(3)=-8<0$ Since there are no real roots for the quadratic factor, f(x) has only 1 real root, x = -1.	
3iii	$f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^3-2\left(\frac{3}{2}\right)^2+2\left(\frac{3}{2}\right)+6=\frac{45}{4}\quad or\quad 11.25$	
4ai	$\text{center}=(-2,3)$ $\text{radius}=\sqrt{2^2+3^2+131}=12\text{units}$	
4aii	$\text{Distance of } A \text{ to center}=\sqrt{(10-(-2))^2+(7-3)^2}=\sqrt{60}=12.64units$ [M1] Since distance of A to center = 12.6 units (3s.f.) > 12 = radius, The person will not be able to make use of free WiFi. $sub\ x=1-,\ y=7$ $LHS=10^2+7+4(10)-6(7)=147\neq 131=RHS$ Since 147>131, i.e. standing outside the circumference of the coverage, hence the person will not be able to make use of free WiFi.	
4bi	$Midpo\text{int}_{AC}=\left(\frac{1-6}{2},\frac{5+2}{2}\right)=\left(\frac{-5}{2},\frac{7}{2}\right)$ $m_{AC}=\frac{3}{7}$ $m_{BD}=-\frac{7}{3}$ $l_{BD}:\ y-\frac{7}{2}=-\frac{7}{3}(x+\frac{5}{2})$ $y=-\frac{7}{3}x-\frac{7}{3}$	
4bii	$B(-1,0)$ $\text{Let } D \text{ be } (x,y)$ $\left(\frac{x-1}{2},\frac{y+0}{2}\right)=\left(\frac{-5}{2},\frac{7}{2}\right)$ $x=-4\ ;\ y=7$ $\therefore D(-4,7)$	

4biii	$m_{AB}=\frac{5}{2}$ and $m_{AD}=-\frac{2}{5}$ Since $m_{AB}\times m_{AD}=\frac{5}{2}\times\left(-\frac{2}{5}\right)=-1$ and *ABCD* is given as a rhombus, i.e. *AB* = *AD* ∴ *ABCD* is a square.	
5i	$y=\ln(8+3x)-\ln(3x-4)$ $\frac{dy}{dx}=\frac{3}{8+3x}-\frac{3}{3x-4}=-\frac{36}{(8+3x)(3x-4)}\neq 0$ Since $\frac{dy}{dx}\neq 0$, there is no turning point for all values of *x*. (shown)	
5ii	$-\frac{36}{(8+3x)(3x-4)}<0$ $(8+3x)(3x-4)>0$ $x<-\frac{8}{3}$ *or* $x>\frac{4}{3}$	
6	$\frac{dy}{dx}=\cos x+\sqrt{3}\sin x$ $\frac{dy}{dx}=0$ $\tan x=-\frac{1}{\sqrt{3}}$ $basic\measuredangle=\frac{\pi}{6}$ $x=\frac{5\pi}{6}\quad,\quad\frac{11\pi}{6}$ $y=2\quad,\quad-2$ $\therefore\left(\frac{5\pi}{6},2\right),\left(\frac{11\pi}{6},-2\right)$ $\frac{d^2y}{dx^2}=-\sin x+\sqrt{3}\cos x$ At $\left(\frac{5\pi}{6},2\right)$, $\frac{d^2y}{dx^2}=-2<0$ ∴ maximum point At $\left(\frac{11\pi}{6},-2\right)$, $\frac{d^2y}{dx^2}=2>0$ ∴ minimum point	

7i	$B(4.5\,,0)$ $x^2-8x=2x-9$ $(x-9)(x-1)=0$ $x=1 \quad or \quad x=9$ $A(1,-7)$ and $C(9,9)$	
7ii	Area below x-axis $=-\int_0^1 x^2-8x\,dx+\frac{1}{2}\times 3.5\times 7 \quad =-\left[\frac{x^3}{3}-4x^2\right]_0^1+\frac{49}{4}=\frac{191}{12}units^2$ Area above x-axis $=\frac{1}{2}\times 9\times 4.5-\int_8^9 x^2-8x\,dx=\frac{81}{4}-\left[\frac{x^3}{3}-4x^2\right]_8^9=\frac{191}{12}units^2$ $Total\ area=\frac{191}{12}+\frac{191}{12}=\frac{191}{6} \quad or \quad 31.8units^2\ (3sf)$	
8i	$\frac{dv}{dt}=-\frac{1}{5}ke^{-\frac{1}{5}t}$ $-2=-\frac{1}{5}ke^0$ $k=10 \quad (shown)$	
8ii	$v=10e^{-\frac{1}{5}t}-2$ Given $v=0$, $\frac{1}{5}=e^{-\frac{1}{5}t}$ $t=5\ln 5$ $s=\int_0^{5\ln 5}10e^{-\frac{1}{5}t}-2dt=\left[-50e^{-\frac{1}{5}t}-2t\right]_0^{5\ln 5}=23.9m \quad (3s.f.)$	
8iii	When $t=0$, $s=0 \Rightarrow c=50$ $s=-50e^{-\frac{1}{5}t}-2t+50$ When $t=24s$, $s=-50e^{-\frac{1}{5}(24)}-2(24)+50=1.589$ When $t=25s$, $s=-50e^{-\frac{1}{5}(25)}-2(25)+50=-0.3369$ Since the displacement of the particle **changes from a positive value.** (when t = 24) **to a negative value** (when t = 25), the particle passes through O at some instant during the twenty-fifth sec. [A1]	

9a	$LHS: \frac{2\sin A\cos A+2\cos^2 A}{2\sin A\cos A-2\sin^2 A}=\frac{\cos A(\sin A+\cos A)}{\sin A(\cos A-\sin A)}=\frac{1}{\tan A}\times\frac{\frac{\sin A}{\cos A}+1}{1-\frac{\sin A}{\cos A}}$ $=\frac{1}{\tan A}\times\frac{\tan A+\tan 45^\circ}{1-\tan A\tan 45^\circ}$ $=\frac{\tan(45^\circ+A)}{\tan A} \quad (proven)$	
9bi	$(\sin\theta-\cos\theta)^2=\frac{9}{16}$ $1-2\sin\theta\cos\theta=\frac{9}{16}$ $\sin\theta\cos\theta=\frac{7}{32} \quad (shown)$	
9bii	$7\cot\theta+7\tan\theta=\frac{7\cos^2\theta+7\sin^2\theta}{\sin\theta\cos\theta}=\frac{7}{\frac{7}{32}}=32$	
10i	h = 2(1 – cos kt). Since the diameter of the carousel = max value of h, h = 2 (1 – (-1)) = 4 m	
10ii	Period = 20 s $\frac{2\pi}{k}=20$ $k=\frac{\pi}{10} rad/s \quad (shown)$	
10iii	$1=2(1-\cos\frac{\pi}{10}t)$ $\frac{1}{2}=\cos\frac{\pi}{10}t$ $basic\measuredangle=\frac{\pi}{3}$ $\frac{\pi}{10}t=\frac{\pi}{3} \quad ,\frac{5\pi}{3}$ $t=\frac{10}{3} \quad ,\frac{50}{3}$ $Not\ able\ to$ view $:\frac{50}{3}-\frac{10}{3}=\frac{40}{3}s \quad or \quad 13.3\,s\ (3s.f.)$	

11a	$a=-4$; $M(-2,0)$; $b=2$; $c=0$	
11bi	$x^2+2x=x^2-4x+4$ $x=\frac{2}{3}$ $y=-(x-\frac{2}{3})^2+k$ $0=-(x-\frac{2}{3})^2+k$ $k=\frac{16}{9}$ $N(\frac{2}{3},\frac{16}{9})$	
11bii	$\Rightarrow y=-(x-\frac{2}{3})^2+\frac{16}{9}$ $m=-1$; $n=\frac{4}{3}$; $r=\frac{4}{3}$	
12	$2^{5r}3^{5r}2^{-\frac{r}{2}}3^{-\frac{r}{2}}=2^{\frac{1}{2r}}3^{\frac{1}{2r}}$ $5r-\frac{r}{2}=\frac{1}{2r}$ $r^2=\frac{1}{9}$ $r=\frac{1}{3}$ *or* $-\frac{1}{3}$ (reject , $r>0$)	