2023 4E Additional Mathematics WA2 Marking Scheme

1	Given that $\mathrm{f}(x)=\dfrac{3(3-4x)^2}{2\sqrt{3-4x}}$, find $\mathrm{f}'(x)$.	[3]
	$\mathrm{f}(x) = \dfrac{3(3-4x)^{\frac{3}{2}}}{2}$ $\mathrm{f}'(x) = \dfrac{3}{2}\left(\dfrac{3}{2}\right)(3-4x)^{\frac{1}{2}}(-4)$ [M2] $= -9(3-4x)^{\frac{1}{2}}$ [A1]	
2	The total surface area of a spherical ball of ice is decreasing at a rate of $2\,\mathrm{cm}^2/\mathrm{s}$. Find the rate of change of the volume when its radius is 0.5 cm. [Volume of sphere = $\dfrac{4}{3}\pi r^3$; total surface area of sphere = $4\pi r^2$]	[5]
	$A = 4\pi r^2$ $\dfrac{\mathrm{d}A}{\mathrm{d}r} = 8\pi r$ $\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}r}\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$ $-2 = 8\pi(0.5)\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$ [M1 – connected rates of change] $\dfrac{dr}{dt} = \dfrac{1}{8\pi(0.5)}\times -2$ $= -\dfrac{1}{2\pi}$ cm/s [M1 - $\dfrac{\mathrm{d}r}{\mathrm{d}t}$] $V = \dfrac{4}{3}\pi r^3$ $\dfrac{dV}{dr} = 4\pi r^2$ [M1 – $\dfrac{\mathrm{d}A}{\mathrm{d}r} = 8\pi r$ and $\dfrac{dV}{dr} = 4\pi r^2$] $\dfrac{dV}{dt} = \dfrac{dV}{dr}\times\dfrac{dr}{dt}$ $= 4\pi(0.5)^2 \times -\dfrac{1}{2\pi}$ [M1 – connected rates of change] $= -\dfrac{1}{2}cm^3/s$ The rate of change of volume at r = 0.5 cm is $-\dfrac{1}{2}cm^3/s$. [A1 – with units and statements]	

3 It is given that $y=(3x+1)\sqrt{2x-1}$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $\frac{mx+n}{\sqrt{2x-1}}$ where m and n are integers. [3]

$y=(3x+1)\sqrt{2x-1}$

$\frac{\mathrm{d}y}{\mathrm{d}x}=(3x+1)\left(\frac{1}{2}\right)(2x-1)^{-\frac{1}{2}}(2)+(2x-1)^{\frac{1}{2}}(3)$ [M1 – product law]

$=(3x+1)(2x-1)^{-\frac{1}{2}}+3(2x-1)^{\frac{1}{2}}$

$=(2x-1)^{-\frac{1}{2}}\left[(3x+1)+3(2x-1)\right]$ [M1 – factorisation of $\left(x^2+1\right)^{-\frac{1}{2}}$]

$=(2x-1)^{-\frac{1}{2}}(3x+1+6x-3)$

$=(2x-1)^{-\frac{1}{2}}(9x-2)$

$=\frac{9x-2}{\sqrt{2x-1}}$ [A1]

(ii) Show that y is an increasing for $x>\frac{1}{2}$. [2]

For the numerator, when $x>\frac{1}{2}$,

$9x>\frac{9}{2}$

$9x-2>\frac{7}{2}$

For the denominator, when $x>\frac{1}{2}$,

$2x>1$

$2x-1>0$

$\sqrt{2x-1}>0$ [M1 – for both numerator and denominator]

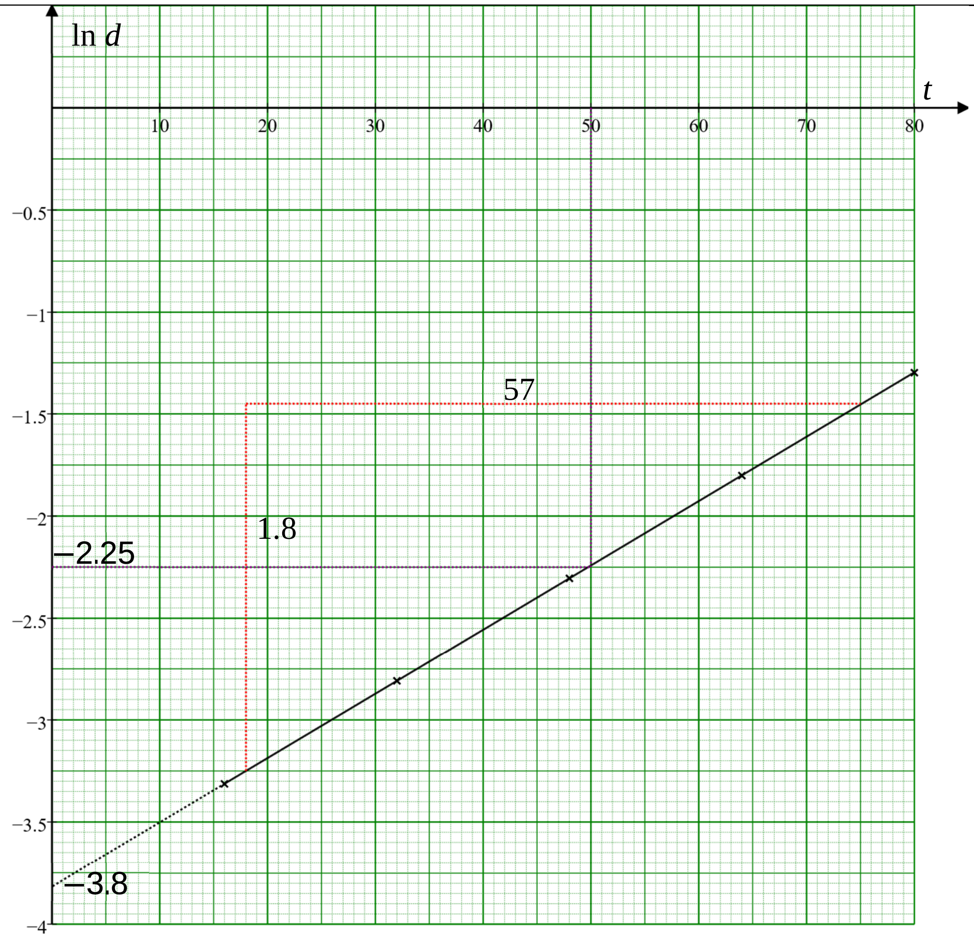
Since $9x-2>0$ and $\sqrt{2x-1}>0$, $\frac{\mathrm{d}y}{\mathrm{d}x}>0$ and y is an increasing function. [A1]

4	A curve has the equation $y = \frac{2-x}{3x-4}$, $x \neq \frac{4}{3}$.	
	(i)	Find an expression for $\frac{dy}{dx}$. [2] $\frac{dy}{dx} = \frac{(3x-4)(-1)-(2-x)(3)}{(3x-4)^2}$ [M1 – quotient rule] $= \frac{-3x+4-6+3x}{(3x-4)^2}$ $= \frac{-2}{(3x-4)^2}$ [A1]
	(ii)	Find the coordinates of the points on the curve where the normal is parallel to the line $2y-16x=3$. [3] $y = 8x + \frac{3}{2}$ Gradient of normal $= 8$ Gradient of tangent $= -\frac{1}{8}$ $\frac{dy}{dx} = -\frac{1}{8}$ $\frac{-2}{(3x-4)^2} = -\frac{1}{8}$ [M1 – equate gradient of tangent to $\frac{dy}{dx}$] [M1] $(3x-4)^2 = 16$ $3x-4 = \pm 4$ $x = \frac{8}{3}$ or $x = 0$ [M1] When $x = \frac{8}{3}$, $y = \frac{2-\left(\frac{8}{3}\right)}{3\left(\frac{8}{3}\right)-4} = -\frac{1}{6}$ When $x = 0$, $y = \frac{2-0}{3(0)-4} = -\frac{1}{2}$

		The coordinates are $\left(\frac{8}{3},-\frac{1}{6}\right)$ and $\left(0,-\frac{1}{2}\right)$. [A1 for both]	
5	$y=cx^3+dx$ $\frac{y}{x}=cx^2+d$ $c=\frac{8-5}{7-4}$ $=1$ $5=1(4)+d$ $d=1$		[M1] for dividing by x on both sides. [M1] for finding gradient. [A1] [A1]

6(i)	$d = Ae^{kt}$ $\ln d = \ln A + \ln e^{kt}$ $\ln d = \ln A + kt \ln e$ $\ln d = kt + \ln A$	[B1] for table of values for $\ln d$

t	16	32	48	64	80
d	0.0364	0.0603	0.0998	0.165	0.273
$\ln d$	−3.313	−2.808	−2.305	−1.802	−1.298



[B1] for plot of points and labelling of axes.

[B1] for straight line passing through the points

6(ii)	From the graph, the vertical intercept, $\ln A = -3.8$ $A = 0.0224$ From the graph, the gradient, $b = \dfrac{-3.25-(-1.45)}{18-75}$ $= \dfrac{3}{95}$ $= 0.0316$ (3 s.f)	[B1] A accept 0.0213 to 0.0235 [M1] for finding 2 convenient points to find gradient. [A1] for b between 0.0310 to 0.0320
6(iii)	Method 1: When $t = 50$, $d = e^{-3.8} e^{\frac{3}{95} \times 50}$ $= 0.108$ (3 s.f)	[M1] [A1]
	Method 2: From the graph, $\ln d = -2.25$ $d = 0.105$ (3 s.f)	[M1] for reading from graph [A1] (accept 0.100-0.111)