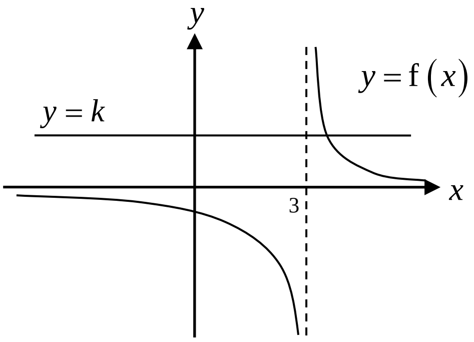
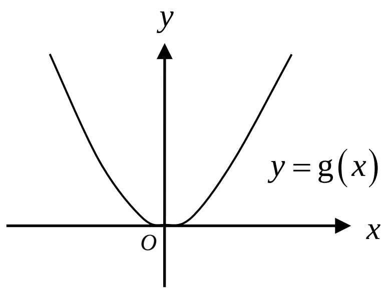
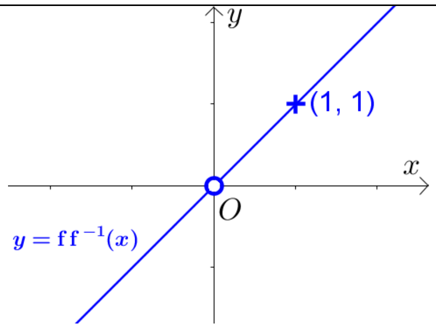
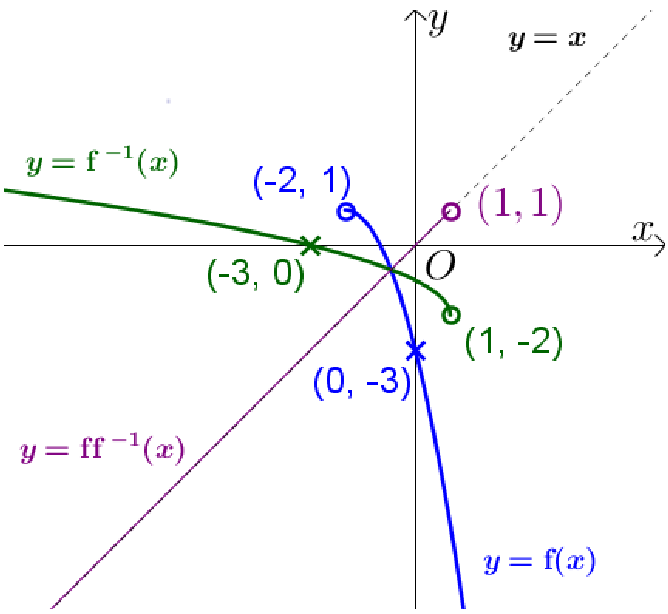
National Junior College
2016 – 2017 H2 Mathematics
Functions **Assignment Solutions**

1(i)	  $R_{\mathrm{g}} = [0, \infty) \not\subseteq \mathbb{R} \setminus \{3\} = D_{\mathrm{f}} \quad \Rightarrow$ fg does not exist. $R_{\mathrm{f}} = \mathbb{R} \setminus \{0\} \subseteq \mathbb{R} = D_{\mathrm{g}} \quad \Rightarrow$ gf exists. $\therefore \mathrm{gf} : x \mapsto \dfrac{1}{(x-3)^2}, \quad x \in \mathbb{R},\ x \neq 3.$	Take note of the correct use of set notations. Domain is required whenever a function is defined.
1(ii)	$\mathbb{R} \setminus \{3\} \to \mathbb{R} \setminus \{0\} \to (0, \infty)$ Therefore we have $\mathrm{R}_{\mathrm{gf}} = (0, \infty)$.	The mapping method is shown here. An alternative approach is through a sketch of the graph of the function gf over its domain.
1(iii)	Every horizontal line $y = k$, $k \in \mathbb{R} \setminus \{0\}$ cuts the graph of $y = \mathrm{f}(x)$ exactly once. Hence f is one-one, and so f^{-1} exists. $y = \dfrac{1}{x-3} \Rightarrow x - 3 = \dfrac{1}{y} \Rightarrow x = 3 + \dfrac{1}{y}$ $\therefore \mathrm{f}^{-1} : x \mapsto 3 + \dfrac{1}{x}, \quad x \in \mathbb{R} \setminus \{0\}.$	The inverse function has to be defined in a similar form as stated in the question. $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = \mathbb{R} \setminus \{0\}$
1(iv)		The origin is excluded because $\mathrm{D}_{\mathrm{ff}^{-1}} = \mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = \mathbb{R} \setminus \{0\}$.

2(i)	$R_f = (-\infty, \lambda) \subseteq (-\infty, 1) = D_g$. Therefore largest value of $\lambda = 1$.	
2(ii)	 	Need to show clearly the symmetrical relationship between the function and its inverse about the line $y = x$. You are to use the same scale on both axes to illustrate this. The coordinates of the end points of each curve must be clearly stated.
2(iii)	From the graph, the solution to $f(x) = f^{-1}(x)$ is the same as that of $f(x) = x$. $\Rightarrow \quad 1-(x+2)^2 = x$ $\Rightarrow \quad x^2+5x+3=0$ $\Rightarrow \quad x = \dfrac{-5\pm\sqrt{13}}{2}$ $\Rightarrow \quad x = \dfrac{-5+\sqrt{13}}{2}$ (reject $\dfrac{-5-\sqrt{13}}{2}$ $\because x > -2$).	From the graphs in 2(ii), solving $f(x) = f^{-1}(x)$ is equivalent to solving $f(x) = x$. Hence it is not necessary to find the inverse function to solve the given equation.

3(i)	$f(\lambda) = \lambda^2 - 6\lambda^2 = -5\lambda^2$. $f^2(\lambda) = f\left(-5\lambda^2\right) = 25\lambda^4 + 30\lambda^3$.	Note that f^2 is a composite function and not the square of the function.
3(ii)	$f(x) = (x-3\lambda)^2 - 9\lambda^2$. Therefore $k = 3\lambda$. When $x \le 3\lambda$, $y = (x-3\lambda)^2 - 9\lambda^2 \quad \Rightarrow \quad (x-3\lambda)^2 = y + 9\lambda^2$ $\Rightarrow \quad x = 3\lambda \pm \sqrt{y+9\lambda^2}$ $\Rightarrow \quad x = 3\lambda - \sqrt{y+9\lambda^2}$. $\therefore f^{-1}(x) = 3\lambda - \sqrt{x+9\lambda^2}$.	