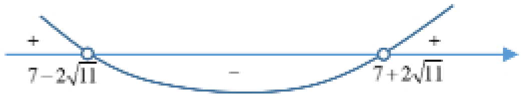
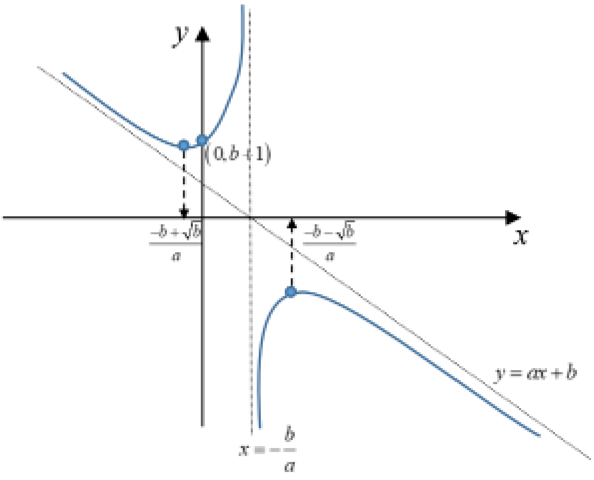
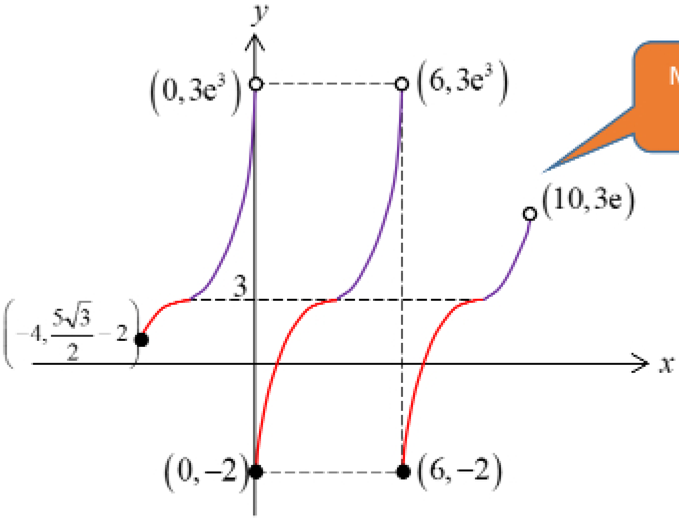
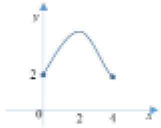
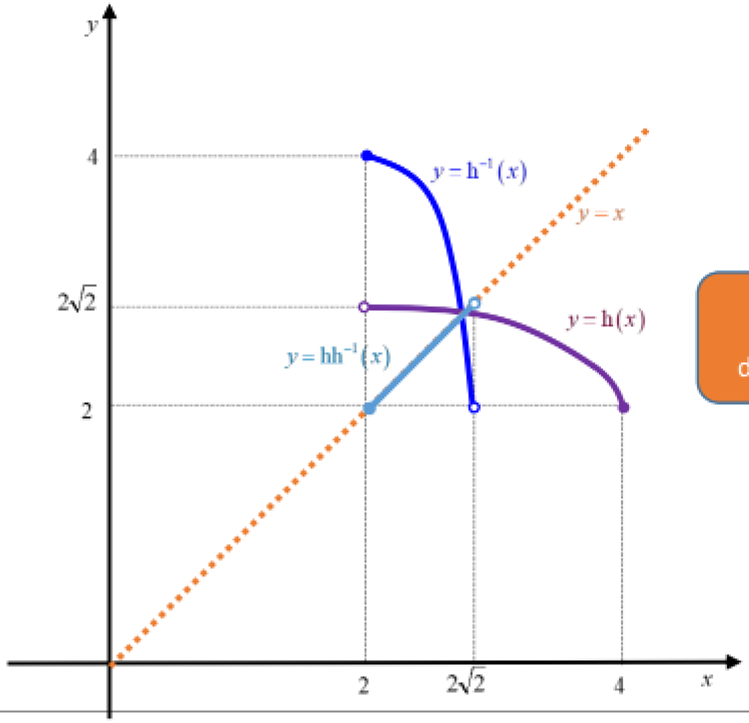
Qn	Solution
1(a)	$y=\dfrac{x^2+3x+1}{x-2}$ Algebraic Method. Do not use graph or differentiation. $y(x-2)=x^2+3x+1$ $xy-2y=x^2+3x+1$ $x^2+x(3-y)+(1+2y)=0$ For no real roots, $(3-y)^2-4(1)(1+2y)<0$ $y^2-14y+5<0$ $(y-7)^2+5-49<0$ $\left[(y-7)-\sqrt{44}\right]\left[(y-7)-\sqrt{44}\right]<0$  $7-2\sqrt{11}<y<7+2\sqrt{11}$ $\left\{y\in\mathbb{R}:7-2\sqrt{11}<y<7+2\sqrt{11}\right\}$ or $\left(7-2\sqrt{11},7+2\sqrt{11}\right)$
2	$\dfrac{xy-y^2}{(x+1)^2}=x$ $xy-y^2=x(x+1)^2$ Differentiate w.r.t. x: $y+x\dfrac{dy}{dx}-2y\dfrac{dy}{dx}=(x+1)^2+2x(x+1)$ Apply implicit differentiation $\dfrac{dy}{dx}(x-2y)=3x^2+4x+1-y$ $\dfrac{dy}{dx}=\dfrac{3x^2+4x+1-y}{x-2y}$ **Method 1** When tangent is parallel to y-axis, $x-2y=0$ Tangent parallel to y-axis, gradient is undefined i.e. $x=2y$ and we substitute into equation of C obtaining $\dfrac{x\left(\frac{x}{2}\right)-\left(\frac{x}{2}\right)^2}{(x+1)^2}=x$ $x^2=4x^3+8x^2+4x$

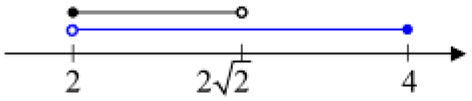
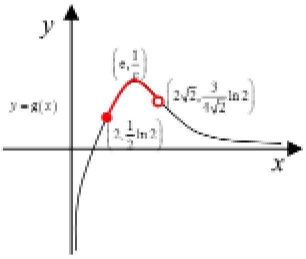
Qn	**Solution**
	$x(4x^2+7x+4)=0$ $x=0$ or $4x^2+7x+4=0$ $\because D<0, \quad 4x^2+7x+4>0$ $\therefore x=0$ **<u>Method 2</u>** When tangent is parallel to y-axis, $x-2y=0$ i.e. $x=2y$ and we substitute into equation of C obtaining $\dfrac{2y^2-y^2}{(2y+1)^2}=2y$ $8y^3+7y^2+2y=0$ $y\left(8y^2+7y+2\right)=0$ $y=0$ or $8y^2+7y+2=0$ $\because D<0, \quad 8y^2+7y+2>0$ When $y=0$, $\therefore x=0$
3(i)	**Stationary points:** $\dfrac{\mathrm{d}y}{\mathrm{d}x}=a-\dfrac{ab}{(ax+b)^2}$ When $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0, a-\dfrac{ab}{(ax+b)^2}=0$ $(ax+b)^2=b$ $x=\dfrac{\pm\sqrt{b}-b}{a}$
(ii)	**Method 1** Since $0<b<1$, $0<b^2<b$ (i.e. multiply throughout by b) $0<b^2<b$ $\lvert b\rvert<\sqrt{b}$ Note: $\sqrt{b^2}=\lvert b\rvert$ Since $b>0$ $b<\sqrt{b}$ $\sqrt{b}-b>0$ **Method 2** $\sqrt{b}-b$ $=\sqrt{b}\left(1-\sqrt{b}\right)$ Since $b>0$, $\sqrt{b}>0$

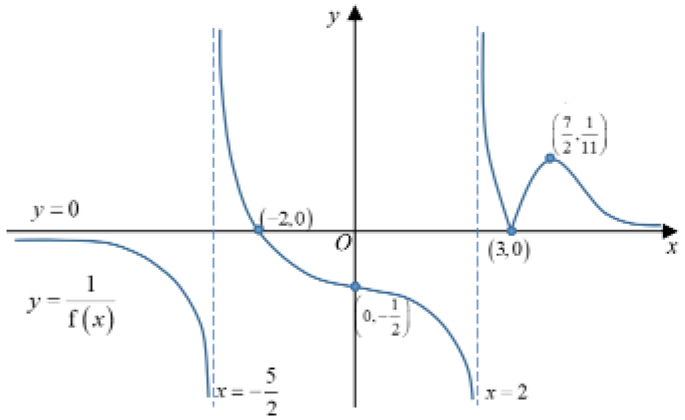
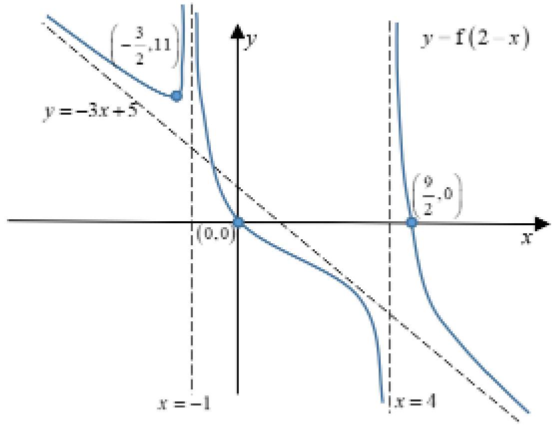
Qn	**Solution**
	$0<\sqrt{b}<1$ $-\sqrt{b}>-1$ $1-\sqrt{b}>0$ $\therefore \sqrt{b}-b=\sqrt{b}\left(1-\sqrt{b}\right)>0$
(iii)	$a<0<b<1$ **Asymptotes:** $y=ax+b$ (negative gradient) Oblique asymptote $x=-\frac{b}{a}>0$ Vertica asymptote Note that the two asymptotes intersect at $\left(-\frac{b}{a},0\right)$ **Intercepts:** When $x=0$, $y=b+1>0$ When $y=0$, $(ax+b)^2=-b$ $x=\frac{\pm\sqrt{-b}-b}{a}$ Since $b>0$, $\sqrt{-b}$ is undefined Therefore curve does not cut the x-axis. **Stationary points:** $x=\frac{-\sqrt{b}-b}{a}>0$ $x=\frac{\sqrt{b}-b}{a}<0$ Use $\sqrt{b}-b>0$ from (ii) 

Qn	Solution
4(i)	$a_{n+1}=a_n+ka_{n-1}$ $a_2=a_1+ka_0$ $11=7+k(2)$ $k=2$
(ii)	$a_n=A\left(2^n\right)+B(-1)^n+C$ $2=A+B+C$ $7=2A-B+C$ $11=4A+B+C$ Solve System of Equations $A=3,\quad B=-1,\quad C=0$
(iii)	**Method 1** $a_r=3\left(2^r\right)-(-1)^r$ $\sum_{r=1}^{n}a_r$ $=\sum_{r=1}^{n}\left[3\left(2^r\right)-(-1)^r\right]$ $=3\sum_{r=1}^{n}\left[\left(2^r\right)\right]-\sum_{r=1}^{n}(-1)^r$ $=3\left[\frac{2\left(2^n-1\right)}{2-1}\right]-\left[(-1)^1+(-1)^2+\ldots+(-1)^n\right]$ $=6\left(2^n-1\right)-\frac{(-1)\left[(-1)^n-1\right]}{-1-1}$ $=6\left(2^n-1\right)-\frac{1}{2}\left[(-1)^n-1\right]$ $=6\left(2^n\right)-\frac{1}{2}(-1)^n-\frac{11}{2}$ **Method 2** When n is odd, $\sum_{r=1}^{n}a_r$ $=\sum_{r=1}^{n}\left[3\left(2^r\right)-(-1)^r\right]$ $=3\left[\frac{2\left(2^n-1\right)}{2-1}\right]-(-1)$ $=6\left(2^n-1\right)+1$ or $=6\left(2^n\right)-5$ When n is even,

Qn	**Solution**
	$\sum_{r=1}^{n} a_r$ $= \sum_{r=1}^{n}\left[3\left(2^r\right)-1\left(-1\right)^r\right]$ $= 3\left[\frac{2\left(2^n-1\right)}{2-1}\right]$ $= 6\left(2^n-1\right)$
5(i)	 Must indicate clearly all the end points $\mathrm{f}(10) = \mathrm{f}(4) = 3e^{\lvert 3-4\rvert} = 3e$ $\mathrm{f}(-4) = \mathrm{f}(2) = 5\cos\left(\frac{\pi}{3}-\frac{\pi}{2}\right)-2$ $= 5\cos\left(-\frac{\pi}{6}\right)-2$ $= \frac{5\sqrt{3}}{2}-2$
(ii)	Let $y = 5\cos\left(\frac{\pi}{6}x-\frac{\pi}{2}\right)-2$ Since $0 \le x < 3$, $-\frac{\pi}{2} \le \frac{\pi}{6}x-\frac{\pi}{2} < 0$ $\frac{\pi\left(x-3\right)}{6} = -\cos^{-1}\left(\frac{y+2}{5}\right)$

Qn	Solution
	$x-3=-\frac{6}{\pi}\cos^{-1}\left(\frac{y+2}{5}\right)$ $x=3-\frac{6}{\pi}\cos^{-1}\left(\frac{y+2}{5}\right)$ Since $3\le x<6$. $y=3\mathrm{e}^{\lvert 3-x\rvert}=3\mathrm{e}^{x-3}$ $\lvert 3-x\rvert=-(3-x)\ \because 3\le x<6$ $y=3\mathrm{e}^{x-3}$ $x=3+\ln\frac{y}{3}$ Don't forget to express in terms of *x* $\mathrm{f}^{-1}(x)=\begin{cases}3-\frac{6}{\pi}\cos^{-1}\left(\frac{x+2}{5}\right) & \text{, where } -2\le x<3,\\ 3+\ln\frac{x}{3} & \text{, where } 3\le x<3\mathrm{e}^{3}.\end{cases}$
6(i)	$y=x^{\frac{1}{2}}+(4-x)^{\frac{1}{2}}$ By observation, $q=2$ Any *q* smaller than 2, f is not one-to-one 
(ii)	 Take note: domain of hh^{-1} = domain of h^{-1}

Qn	Solution
(iii)	$\mathrm{h}^{-1}\mathrm{h}$ and $\mathrm{h}\,\mathrm{h}^{-1}$ have the same rule but different domain. $\mathrm{h}^{-1}\mathrm{h}(x)=x$, $\mathrm{D}_{\mathrm{h}^{-1}\mathrm{h}}=\mathrm{D}_{\mathrm{h}}=(2,4]$ $\mathrm{hh}^{-1}(x)=x$, $\mathrm{D}_{\mathrm{h}\,\mathrm{h}^{-1}}=\mathrm{D}_{\mathrm{h}^{-1}}=\mathrm{R}_{\mathrm{h}}=\left[2,2\sqrt{2}\right)$  $\left\{x\in\mathbb{R}:2<x<2\sqrt{2}\right\}$ or $\left(2,2\sqrt{2}\right)$ Take note of round bracket
(iv)	$\mathrm{g}(x)=\dfrac{\ln x}{x}$ $\mathrm{g}'(x)=\dfrac{1-\ln x}{x^2}=0$ Stationary point at $\left(\mathrm{e},\dfrac{1}{\mathrm{e}}\right)$  $\mathrm{R}_{\mathrm{h}}=\left[2,2\sqrt{2}\right)$ $\mathrm{D}_{g}=\mathbb{R}^{+}$ $\mathrm{R}_{\mathrm{h}}\subseteq\mathrm{D}_{g}$ $\therefore$ gh exists. (shown) Restrict $\mathrm{D}_{g}=\left[2,2\sqrt{2}\right)$ $\mathrm{g}(2)=\dfrac{1}{2}\ln 2$ and $\mathrm{g}\left(2\sqrt{2}\right)=\dfrac{3}{4\sqrt{2}}\ln 2$ $\therefore \mathrm{R}_{\mathrm{gh}}=\left[\dfrac{1}{2}\ln 2,\dfrac{1}{\mathrm{e}}\right]$

Qn	Solution
7(a) (i)	
(a) (ii)	 (1) Translate by 2 units in the negative *x*-direction (2) Reflection in the *y*-axis
(b)	$y=\frac{1}{x^2+4x+3}=\frac{1}{(x+2)^2-1}$ $y=\frac{3x^2-4}{x^2-1}=3-\frac{1}{x^2-1}$ $y=\frac{1}{(x+2)^2-1}\xrightarrow[\text{with } x-2]{\text{Replace } x} y=\frac{1}{x^2-1}$ $\xrightarrow[\text{with } -y]{\text{Replace } y} -y=\frac{1}{x^2-1}\Rightarrow y=-\frac{1}{x^2-1}$ $\xrightarrow[\text{with } y-3]{\text{Replace } y} y-3=-\frac{1}{x^2-1}\Rightarrow y=3-\frac{1}{x^2-1}$ **Method 1** Hence 1. Translate 2 units in positive x-direction. 2. Reflect in x-axis. (or Scale parallel to y-axis with factor -1.) 3. Translate 3 units in positive y-direction.

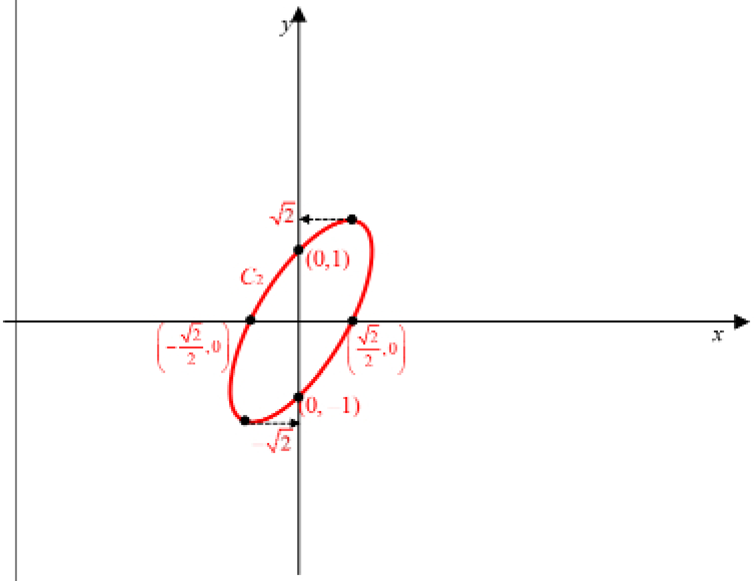
Qn	Solution
	$y=\frac{1}{(x+2)^2-1} \xrightarrow[\substack{\text{Replace } x\\ \text{with } x-2}]{} y=\frac{1}{x^2-1}$ $\xrightarrow[\substack{\text{Replace } y\\ \text{with } y+3}]{} y+3=\frac{1}{x^2-1} \Rightarrow y=-3+\frac{1}{x^2-1}$ $\xrightarrow[\substack{\text{Replace } y\\ \text{with } -y}]{} -y=-3+\frac{1}{x^2-1} \Rightarrow y=3-\frac{1}{x^2-1}$ **Method 2** Hence 1. Translate 2 units in positive x-direction. 2. Translate 3 units in negative y-direction. 3. Reflect in x-axis. (or Scale parallel to y-axis with factor -1.)

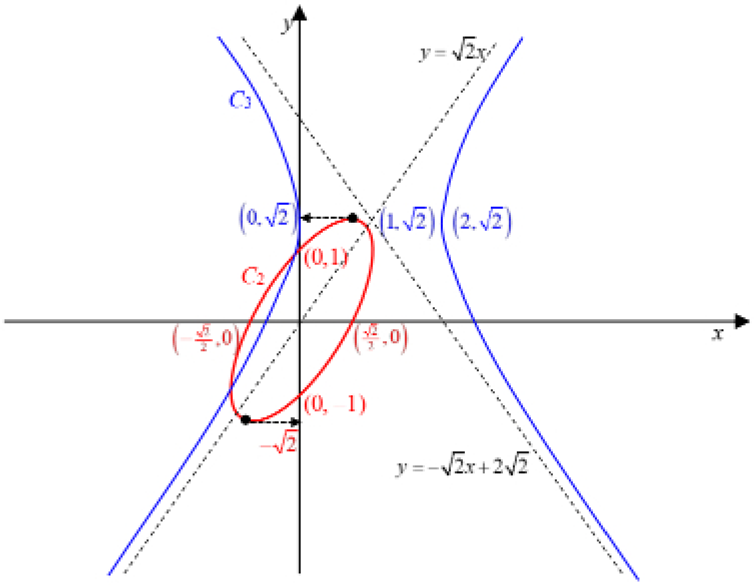
8(i)	$\frac{6}{(r-2)(r)(r+1)}=\frac{A}{r-2}+\frac{B}{r}+\frac{C}{r+1}$
	$6=A(r)(r+1)+B(r-2)(r+1)+C(r-2)(r)$
	Sub $r=2, A=1$
	Sub $r=0, B=-3$
	Sub $r=1, C=2$
	Hence, $\frac{6}{(r-2)(r)(r+1)}=\frac{1}{r-2}-\frac{3}{r}+\frac{2}{r+1}$

(ii)	$\sum_{r=3}^{N}\frac{1}{(r-2)(r)(r+1)}=\frac{1}{6}\sum_{r=3}^{N}\frac{6}{(r-2)(r)(r+1)}$ $=\frac{1}{6}\sum_{r=3}^{N}\left(\frac{1}{r-2}-\frac{3}{r}+\frac{2}{r+1}\right)$

Qn	Solution
	$= \frac{1}{6}\left(\begin{array}{ccc} \frac{1}{1} & -\frac{3}{3} & +\cancel{\frac{2}{4}} \\ +\frac{1}{2} & -\cancel{\frac{3}{4}} & +\cancel{\frac{2}{5}} \\ +\frac{1}{3} & -\cancel{\frac{3}{5}} & +\cancel{\frac{2}{6}} \\ +\cancel{\frac{1}{4}} & -\cancel{\frac{3}{6}} & +\cancel{\frac{2}{7}} \\ & \vdots & \\ +\cancel{\frac{1}{N-5}} & -\cancel{\frac{3}{N-3}} & +\cancel{\frac{2}{N-2}} \\ +\cancel{\frac{1}{N-4}} & -\cancel{\frac{3}{N-2}} & +\frac{2}{N-1} \\ +\cancel{\frac{1}{N-3}} & -\frac{3}{N-1} & +\frac{2}{N} \\ +\cancel{\frac{1}{N-2}} & -\frac{3}{N} & +\frac{2}{N+1} \end{array}\right)$ $=\frac{1}{6}\left(\frac{5}{6}-\frac{1}{N-1}-\frac{1}{N}+\frac{2}{N+1}\right)$ $=\frac{5}{36}-\frac{1}{6(N-1)}-\frac{1}{6N}+\frac{1}{3(N+1)}$
(iii)	As $N \to \infty$, $\left(-\frac{1}{6N-6}-\frac{1}{6N}+\frac{2}{6N+6}\right) \to 0$, $\therefore \sum_{r=3}^{N} \frac{1}{(r-2)(r)(r+1)} \to \frac{5}{36}$ the series is convergent. $\lim_{N\to\infty}\left(\sum_{r=3}^{N}\frac{1}{(r-2)(r)(r+1)}\right)=\frac{5}{36}$
(iv)	$\sum_{r=10}^{r=2N} \frac{1}{(r-3)(r-1)(r)} = \sum_{i+1=10}^{i+1=2N} \frac{1}{(i+1-3)(i+1-1)(i+1)}$

Qn	Solution
	$=\sum_{i=9}^{i=2N-1}\frac{1}{(i-2)(i)(i+1)}$ $=\sum_{i=9}^{i=2N-1}\frac{1}{(i-2)(i)(i+1)}$ $=\sum_{i=3}^{2N-1}\frac{1}{(i-2)(i)(i+1)}-\sum_{i=3}^{8}\frac{1}{(i-2)(i)(i+1)}$ $=\left(\begin{array}{r}\frac{5}{36}-\frac{1}{6(2N-1-1)}-\frac{1}{6(2N-1)}+\frac{2}{6(2N-1+1)}\\-\frac{5}{36}+\frac{1}{6(8-1)}+\frac{1}{6(8)}-\frac{2}{6(8+1)}\end{array}\right)$ $=\frac{5}{36}-\frac{1}{6(2N-1-1)}-\frac{1}{6(2N-1)}+\frac{2}{6(2N-1+1)}-\frac{397}{3024}$ $=\frac{23}{3024}-\frac{1}{12(N-1)}-\frac{1}{6(2N-1)}+\frac{1}{6N}$
9(i)	$\sin t+\cos t=\sqrt{2}\sin\left(t+\frac{\pi}{4}\right)$ for $0\le t<2\pi,\ -\sqrt{2}\le\sqrt{2}\sin\left(t+\frac{\pi}{4}\right)\le\sqrt{2}$ $\therefore\left\{y\in\mathbb{R}:-\sqrt{2}\le y\le\sqrt{2}\right\}$
(ii)	when $x=0,\ \cos t=0\Rightarrow t=\frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\therefore\ y=\sin\frac{\pi}{2}+0=1$ or $y=\sin\frac{3\pi}{2}+0=-1$ y-intercepts : $(0,\pm1)$ when $y=0,\ \sin t+\cos t=0\Rightarrow\tan t=-1\Rightarrow t=\frac{3\pi}{4}$ or $\frac{7\pi}{4}$ $\therefore\ x=\cos\frac{3\pi}{4}=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$ or $x=\cos\frac{7\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ x-intercepts : $\left(\pm\frac{\sqrt{2}}{2},0\right)$

Qn	Solution
(iii)	$y=\sin t+x$ $\sin t=y-x$ $\cos t=x$ $\sin^2 t+\cos^2 t=1$ $(y-x)^2+x^2=1$ $y^2-2xy+2x^2=1$ Cartesian Equation of C_1 : $y^2-2xy+2x^2=1$
(iv)	
(v)	To find centre of hyperbola, equate the two asymptotes: $\begin{cases} y=\sqrt{2}x \\ y=-\sqrt{2}x+2\sqrt{2} \end{cases}$ $\Rightarrow \sqrt{2}x=-\sqrt{2}x+2\sqrt{2}$ $\begin{cases} x=1 \\ y=\sqrt{2} \end{cases}$ $\therefore$ Centre $\left(1,\sqrt{2}\right)$ $h=1,\quad k=\sqrt{2}$

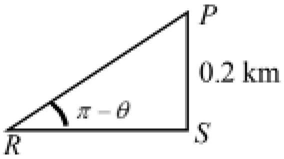
Qn	Solution
	Since vertex $\left(0,\sqrt{2}\right)$ is 1 unit away from centre $\left(1,\sqrt{2}\right)$, we must have $a=1$. Gradient of asymptote: $\dfrac{b}{a}=\sqrt{2}\Rightarrow b=\sqrt{2}a$ $\therefore b=\sqrt{2}$
(vi)	The **number of distinct *x*-coordinates** values of the points of intersections between C_3 and C_2 **could be used to solve for the distinct solutions for** t where $0\le t<2\pi$. $\left[b\left(\cos t-h\right)\right]^2-\left[a\left(\sin t+\cos t-k\right)\right]^2=\left(ab\right)^2$ $\dfrac{\left(\cos t-h\right)^2}{a^2}-\dfrac{\left(\sin t+\cos t-k\right)^2}{b^2}=1$ 
10 (i)	$S_m=1\,300\,000-1\,300\,000\left(0.9\right)^m$ $a_m=\left[1\,300\,000-1\,300\,000\left(0.9\right)^m\right]$ $\quad-\left[1\,300\,000-1\,300\,000\left(0.9\right)^{m-1}\right]$ $\quad=130\,000\left(0.9\right)^{m-1}$ $\dfrac{a_{m+1}}{a_m}=\dfrac{130000\left(0.9\right)^m}{130000\left(0.9\right)^{m-1}}$ $\quad=0.9$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td></td><td>Since $\frac{a_{m+1}}{a_m}$ is a constant, $\{a_m\}$ is a GP with common ratio 0.9.</td></tr>
<tr><td>(ii)</td><td>Sum to infinity $= \frac{130\,000}{1-0.9} = 1\,300\,000$
<u>Alternative</u>
As $m \to \infty$, $0.9^m \to 0$.
So, $s_m \to 1\,300\,000$</td></tr>
<tr><td>(iii)</td><td>Number of nurses $= 60 + (n-1)(8) = 8n + 52$

No. of citizens vaccinated by Butua in the nth week
$b_n = 24 \times 5 \times (8n + 52)$
$= 960n + 6240$
No. of citizens vaccinated by Butua in the 20th week
$b_{20} = 960(20) + 6240$
$= 25440$
Total number of citizens Butua vaccinated by the 20th week
$= b_1 + b_2 + b_3 + \cdots + b_{20}$
$= \frac{20}{2}[b_1 + b_{20}]$
$= 10[7200 + 25440]$
$= 326400$</td></tr>
<tr><td>(iv)</td><td>Solving $a_n < b_n$,
$\frac{1\,300\,000}{9}(0.9)^n < 960n + 6240$
$\frac{1\,300\,000}{9}(0.9)^n - 960n - 6240 < 0$
Let $y = \frac{1\,300\,000}{9}(0.9)^n - 960n - 6240$

<table>
<tr><th>n</th><th>y</th></tr>
<tr><td>17</td><td>$1529.3 > 0$</td></tr>
<tr><td>18</td><td>$-1839.7 < 0$</td></tr>
<tr><td>19</td><td>$-4967.7 < 0$</td></tr>
</table>
$n = 18$

Alternatively,
<table>
<tr><th>n</th><th>a_n</th><th>b_n</th></tr>
<tr><td>17</td><td>24089</td><td>22560</td></tr>
<tr><td>18</td><td>21680</td><td>23520</td></tr>
</table>
</td></tr>
</table>

Qn	Solution
11 (a) (i)	 $\sin(\pi-\theta)=\frac{0.2}{PR}\Rightarrow PR=\frac{0.2}{\sin\theta}$ **(1)** $\text{Time}_{PR}=\frac{\text{Distance}}{\text{Speed}}=\frac{0.2}{\sin\theta(2.4)}=\frac{1}{12\sin\theta}$ **(3)** $\tan(\pi-\theta)=\frac{0.2}{RS}\Rightarrow RS=\frac{0.2}{-\tan\theta}$ **(2)** $QR=8-RS=8+\frac{0.2}{\tan\theta}$ $\text{Time}_{QR}=\frac{\text{Distance}}{\text{Speed}}=\frac{8+\frac{0.2}{\tan\theta}}{4}=2+\frac{1}{20\tan\theta}$ **(4)** Total time taken to travel from P to Q in hours, $T=\frac{1}{12\sin\theta}+2+\frac{1}{20\tan\theta}$ $=\frac{1}{12}\operatorname{cosec}\theta+\frac{1}{20}\cot\theta+2$ $\alpha=\frac{1}{12},\quad\beta=\frac{1}{20},\quad\gamma=2$
(ii)	$\frac{\mathrm{d}T}{\mathrm{d}\theta}=-\frac{1}{12}\operatorname{cosec}\theta\cot\theta-\frac{1}{20}\operatorname{cosec}^2\theta$ $\frac{\mathrm{d}T}{\mathrm{d}\theta}=-\frac{1}{12}\operatorname{cosec}^2\theta\left[\cos\theta+\frac{3}{5}\right]$ Let $\frac{\mathrm{d}T}{\mathrm{d}\theta}=0$, i.e. $-\frac{1}{12}\operatorname{cosec}\theta\cot\theta-\frac{1}{20}\operatorname{cosec}^2\theta=0$ $-\frac{1}{12}\operatorname{cosec}^2\theta\left[\cos\theta+\frac{3}{5}\right]=0$ Since $\operatorname{cosec}^2\theta>0$ $\Rightarrow\cos\theta=-\frac{3}{5}$ $\Rightarrow\theta=\pi-\cos^{-1}\left(\frac{3}{5}\right)=2.2143\text{ rad}$

Qn **Solution**

Using **First Order Derivative Test**:

θ	$\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]^-$ 2.21^-	$\cos^{-1}\left(-\frac{3}{5}\right)$ 2.21	$\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]^+$ 2.21^+
$\frac{\mathrm{d}T}{\mathrm{d}\theta}$	−ve	0	+ve

Using **Second Order Derivative Test**:

$$\frac{\mathrm{d}^2T}{\mathrm{d}\theta^2}$$

$$=\frac{\mathrm{d}}{\mathrm{d}\theta}\left[-\frac{1}{12}\operatorname{cosec}^2\theta\left(\cos\theta+\frac{3}{5}\right)\right]$$

$$=-\frac{1}{12}\operatorname{cosec}^2\theta(-\sin\theta)+\left(\cos\theta+\frac{3}{5}\right)\left[-\frac{1}{6}\operatorname{cosec}\theta\right]\left[-\operatorname{cosec}\theta\cot\theta\right]$$

$$=\frac{1}{12}\operatorname{cosec}^2\theta\sin\theta+\frac{1}{6}\left(\cos\theta+\frac{3}{5}\right)\operatorname{cosec}^2\theta\cot\theta$$

$$\cos\theta=-\frac{3}{5}\Leftrightarrow\sin\theta=\frac{4}{5}$$

$$\left.\frac{\mathrm{d}^2T}{\mathrm{d}\theta^2}\right|_{\theta=\cos^{-1}\left(-\frac{3}{5}\right)}=\frac{1}{12}\left(\frac{5}{4}\right)+0=\frac{5}{48}=0.104167=0.104\ (3\text{ s.f.})>0$$

$$T=\frac{1}{12\sin\theta}+2+\frac{1}{20\tan\theta}=2.07\text{ hour}$$

Must convert to hours & minutes

$= 2$ hour 4 min

Hence earliest arrival time is 10.24 am

(a) (ii)

Assume that

- The **speed of paddling & walking remain constant** despite the worker feeling tired after some time.
- The **current** in the canal is negligible and hence will not have **any effect on the speed of the paddling**.

(b) (i)

Using Similar Triangles,

$$\frac{w}{h}=\frac{A}{B}\Rightarrow w=\frac{Ah}{B}$$

$$V=\frac{1}{2}\times\text{base}\times\text{height}\times\text{length}$$

$$=\frac{1}{2}\times\frac{Ah}{B}\times h\times 400$$

$$=200\frac{A}{B}h^2$$

(b) (ii)

$$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{A}{B}400h$$

Given $\frac{\mathrm{d}V}{\mathrm{d}t}=10$

Qn	Solution
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ $\quad=\frac{B}{400Ah}\times 10$ When $t=30\text{ min}=1800\text{ sec}$, Remember to change minutes to seconds $V=1800\times 10=\frac{A}{B}\times 200\times h^2$ $h^2=\frac{90B}{A}$ $h=\sqrt{\frac{90B}{A}}$, since $h>0$ $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{10B}{400A\sqrt{\frac{90B}{A}}}=\frac{1}{40\sqrt{90}}\sqrt{\frac{B}{A}}$ $=\frac{1}{120\sqrt{10}}\sqrt{\frac{B}{A}}$ $\therefore k=10$