1 Without using a graphic calculator, solve the inequality $\frac{x+2}{2x-1} < 2x+1$. [3]

Hence find the exact solution of the inequality $\frac{2x^2+1}{2-x^2} < \frac{2+x^2}{x^2}$. [3]

2 It is given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+5}{y^2}$ and when $x=0$, $y=5$. Find the Maclaurin's series expansion for y, up to and including the term in x^3, leaving the coefficients in exact form. [6]

3 The complex number z satisfies

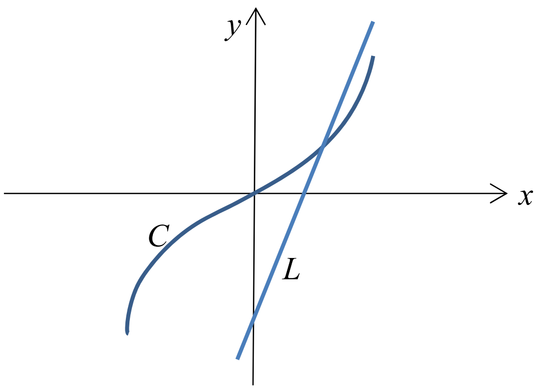
$$|z-4-3\mathrm{i}| \le 2 \quad \text{and} \quad \pi < \arg\left[(z-4)^2\right] < 2\pi$$

(i) Sketch clearly the locus of z on an Argand diagram. [3]

(ii) Find the range of values of $|z-8|$. [2]

(iii) Find maximum value of $\arg(z-8)$. [3]

4 The diagram shows the curve C with equation $y = 2\sin^{-1}x$ and the line L with equation $y = \frac{8\pi}{3}x - \pi$. C and L intersect at the point where $x = \frac{1}{2}$.



The region S is defined by $y \le 2\sin^{-1}x$, $y \ge \frac{8\pi}{3}x - \pi$ and $x \ge 0$.

Find the exact volume of the solid obtained when S is rotated through 2π radians about the y-axis. [6]

5 In the triangle ABC, angle $BAC = \theta$ radians, angle $ACB = \dfrac{\pi}{6}$ radians and $AC = \sqrt{3}$.

Given that θ is sufficiently small, show that

$$AB \approx \frac{2\sqrt{3}}{2 + 2\sqrt{3}\theta - \theta^2} \approx a + b\theta + c\theta^2,$$

where a, b and c are constants to be determined in exact form. [7]

6 **Do not use a graphic calculator in answering this question.**

It is given that $\sin x > \dfrac{2x}{\pi}$ for $0 < x < \dfrac{\pi}{2}$.

(i) Explain why $\displaystyle\int_0^{\frac{\pi}{2}} \mathrm{e}^{-\sin x}\,\mathrm{d}x < \int_0^{\frac{\pi}{2}} \mathrm{e}^{-\frac{2x}{\pi}}\,\mathrm{d}x$. [2]

(ii) By making the substitution $u = \pi - x$, show that

$$\int_{\frac{\pi}{2}}^{\pi} \mathrm{e}^{-\sin x}\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \mathrm{e}^{-\sin u}\,\mathrm{d}u .$$ [2]

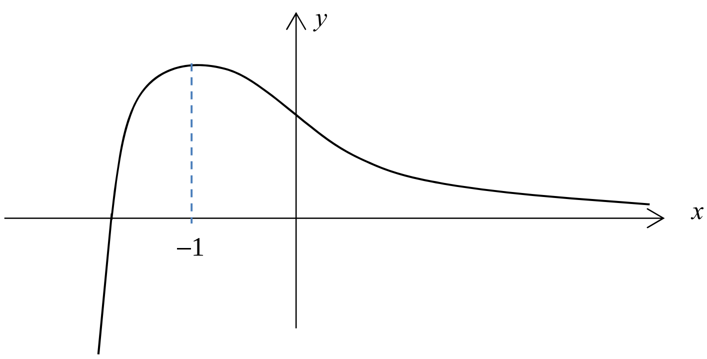
(iii) Hence show that $\displaystyle\int_0^{\pi} \mathrm{e}^{-\sin x}\,\mathrm{d}x < \frac{\pi}{\mathrm{e}}(\mathrm{e}-1)$. [3]

7 A curve C_1 has the equation $p^2x^2 - y^2 = p^2$ where $p > 1$.

(i) Sketch C_1, stating the coordinates of any points of intersection with the axes, the coordinates of any stationary points and the equations of any asymptotes in terms of p. [3]

(ii) C_1 undergoes a single transformation to become C_2. Given that C_2 has a line of symmetry $x = 2$ and the point $(4, 3)$ lies on C_2, find p. [2]

(iii) The graph of $y = \mathrm{f}(x)$ is given below. It has a maximum point at $x = -1$ and a horizontal asymptote $y = 0$.



Sketch the graph of $y = \mathrm{f}'(x)$ on the same diagram as C_1.

Hence, state the number of roots of the equation $p^2x^2 - \left[\mathrm{f}'(x)\right]^2 = p^2$. [4]

8 **(i)** Prove by the method of mathematical induction that

$$\sum_{r=2}^{n} \frac{2}{(r+3)(r+5)} = \frac{11}{30} - \frac{2n+9}{(n+4)(n+5)}.$$ [5]

(ii) Hence find $\displaystyle\sum_{r=4}^{n+4} \frac{2}{r(r+2)}$. [3]

(iii) Deduce that $\displaystyle\sum_{r=4}^{n+4} \frac{1}{(r+1)^2} < \frac{9}{40}$. [2]

9 Show that for $x > 1$,

(i) $$\frac{d}{dx}\left(\frac{1}{\sqrt{x^2-1}}\right) = -\frac{x}{\left(x^2-1\right)^{\frac{3}{2}}}.$$ [1]

(ii) $$\frac{d}{dx}\left(\sin^{-1}\frac{1}{x}\right) = -\frac{1}{x\sqrt{x^2-1}}$$ [2]

A curve C has parametric equations

$$x = \frac{1}{\sqrt{t^2-1}}, \quad y = \ln t \qquad \text{where } t > 1.$$

R is the finite region bounded by the curve C, the x-axis and the lines $x = \frac{1}{\sqrt{3}}$ and $x = \sqrt{3}$. Show that the area of R is given by $\displaystyle\int_{\frac{2}{\sqrt{3}}}^{2} \frac{t}{\left(t^2-1\right)^{\frac{3}{2}}}(\ln t)\ dt$.

By using the results in **(i)** & **(ii)**, find the exact area of R. [6]

10 The function f is defined by

$$f : x \mapsto \frac{1}{2-x^2}, \ x \in \mathbb{R}, \ x \le 0, \ x \ne -\sqrt{2}$$

(i) Define the inverse function f^{-1} in a similar form. [3]

(ii) Sketch the graphs of f and f^{-1} on the same diagram, giving the exact equation of any asymptote(s) and showing clearly the relationship between the two graphs. Hence find the set of values of x, in exact form, for which $f(x) \le f^{-1}(x)$. [6]

Another function g is defined by

$$g : x \mapsto 1 - e^{\lambda - x}, \ x \in \mathbb{R}, x \ge 0 \ \text{where } \lambda \text{ is a constant.}$$

(iii) Given that the composite function $f^{-1}g$ exists, find the greatest value of λ. [2]

With this value of λ, find the range of $f^{-1}g$. [1]

11

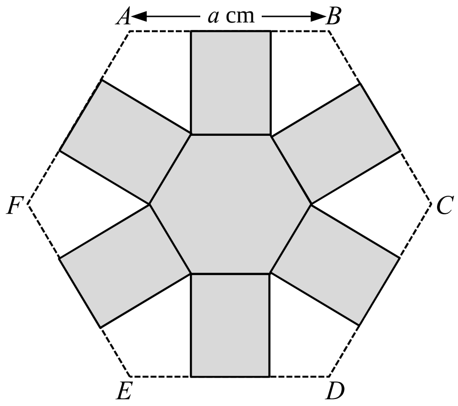


Fig. 1

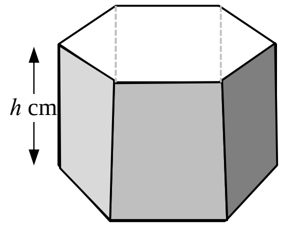


Fig. 2

A piece of vanguard sheet, *ABCDEF*, is in the form of a regular hexagon of side a cm. A kite shape is cut out from each corner to form the shaded shape, as shown in Fig. 1. It is then folded to form the open hexagonal box of height h cm, as shown in Fig. 2.

(i) Show that the volume V of the box in cm^3 is given by $V = 2\sqrt{3}\,h\left(\frac{\sqrt{3}}{2}a - h\right)^2$. [4]

(ii) Use differentiation to find, in terms of a, the value of h which would result in the volume of the box being maximum. [4]

12 The line l passes through the points $P(0,0,1)$ and $Q(0,6,2)$. The plane p_1 is perpendicular to the vector $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and contains the point Q.

(i) Find the acute angle between line l and plane p_1. [3]

(ii) The point R lies on the x-y plane such that its distance from the mid-point M of the line segment PQ is 2 units. If MR is perpendicular to the line l, find the coordinates of R. [5]

The plane p_2 is given by the equation $x + 5y - 10z - 10 = 0$.

(iii) The plane p_3 passes through the point $(1,1,1)$ and contains all the common points of p_1 and p_2. Find a vector equation of p_3, giving your answer in the scalar product form. [4]

END OF PAPER