**1** In triangle $ABC$, angle $BAC$ is $\frac{\pi}{6}$ radians, angle $ABC$ is $\frac{\pi}{3}+x$ radians. Given that $x$ is sufficiently small, show using the sine rule that

$$\frac{BC}{AC} \approx \frac{1}{\sqrt{3}\left(1-\frac{x^2}{2}\right)+x} \approx a+bx+cx^2 ,$$

where $a$, $b$ and $c$ are constants to be determined. **[4]**

**2** A sequence of real numbers $u_1$, $u_2$, $u_3$,… satisfies the recurrence relation

$$u_{n+1} = \left(\frac{n+2}{n}\right)u_n , \qquad n \geq 1.$$

**(i)** Show that $u_k = \frac{k(k+1)}{2}u_1$. **[1]**

**(ii)** Given that $u_1 = 2$, use the method of mathematical induction to show that

$$S_n = \frac{n}{3}(n+1)(n+2)$$

for all positive integers $n$, where $S_n$ denotes the sum of the first $n$ terms of the sequence $\{u_n\}$.

**[5]**

**3** **(i)** Solve the inequality $x^2 - \frac{2}{x} \geq \frac{3}{2}x, \quad x \in \mathbb{R}, \quad x \neq 0$. **[2]**

**(ii)** Without the use of graphic calculator, find the exact value of $a$ such that

$$\int_1^a \left| x^2 - \frac{2}{x} - \frac{3}{2}x \right| \mathrm{d}x = \frac{a^3}{3} - \frac{3a^2}{4} + \frac{1}{4} \quad \textit{where } a \in \mathbb{R},\ a > 2$$ **[4]**

**4** The complex number $z$ satisfies the relations $|iz+3| \leq 3$ and $\arg\left(z+\left(3+\frac{3}{i}\right)\right) \geq \frac{\pi}{4}$.

**(i)** Illustrate both of these relations on a single Argand diagram. **[3]**

**(ii)** Hence find in exact values, the range of possible values of

**(a)** $|z-3-3i|$

**(b)** $\arg(z-3-3i)$ **[3]**

**5** Let $\mathrm{f}(x) = \tan^{-1} x$

**(i)** Sketch the graph of $y = \mathrm{f}(x)$ for $-3 \le x \le 3$. **[1]**

**(ii)** Find the series expansion of $\mathrm{f}(x)$ in ascending powers of $x$, up to and including the term in $x^3$. **[3]**

Denote the answer to **(ii)** by $\mathrm{g}(x)$.

**(iii)** By substituting $x = \frac{1}{\sqrt{3}}$ into $\mathrm{g}(x)$, show that $\pi \approx \frac{16\sqrt{3}}{9}$. **[1]**

**(iv)** By substituting $x = \sqrt{3}$ into $\mathrm{g}(x)$, find another approximation for $\pi$. **[1]**

**(v)** Explain why the value of $x$ used for approximation in **(iii)** is better than that in **(iv)**. **[1]**

**6** A curve has parametric equations

$$x = at^2,\ y = at^3 \qquad \text{where } t \in \mathbb{R} \text{ and } a > 0.$$

Find, in terms of $a$,

**(i)** the equation of the tangent to the curve at the point $\left(\frac{25}{4}a, -\frac{125}{8}a\right)$. **[3]**

**(ii)** the coordinates of the point where this tangent meets the curve again. **[2]**

**(iii)** the exact coordinates of the point(s) on the curve at which the normal to the curve passes through the point $\left(\frac{21}{2}a, 0\right)$. **[3]**

**7** The functions f, g and $\mathrm{f}^{-1}\mathrm{h}$ are given by

$$\mathrm{f} : x \mapsto \frac{2x^2 + ax - 3}{x+1}, \qquad x \in \mathbb{R}, x > -1 \text{ where } a \text{ is a constant, } a \ne -1,$$

$$\mathrm{g} : x \mapsto (x-1)^2 - 0.25, \qquad x \in \mathbb{R}, 0 < x < 3.5,$$

$$\mathrm{f}^{-1}\mathrm{h} : x \mapsto e^x, \qquad x \in \mathbb{R}, x > 0.$$

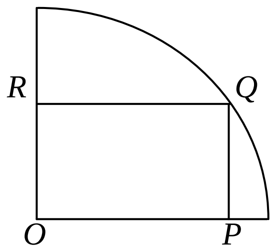
Find the range of values of $a$ for which f has stationary points. Hence, find the set of values of $a$ for which $\mathrm{f}^{-1}$ exists. **[4]**

Given that $a = -3$,

**(i)** find the exact range of fg. **[2]**

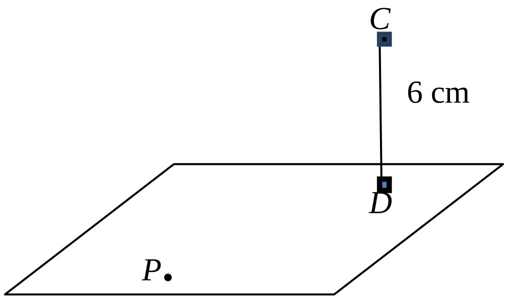
**(ii)** find h in a similar form. **[2]**

**8** **(a)**




The diagram above shows a rectangle *OPQR* inscribed on the quadrant of a circle of fixed radius *a* with *O* as the center of the circle as shown. If *OP* = *x*, find the area *A* of the rectangle *OPQR* in terms of *a* and *x*. Hence, show that *A* is maximum when the perimeter of the rectangle *OPQR* is 4 times the length of *OP*. **[5]**

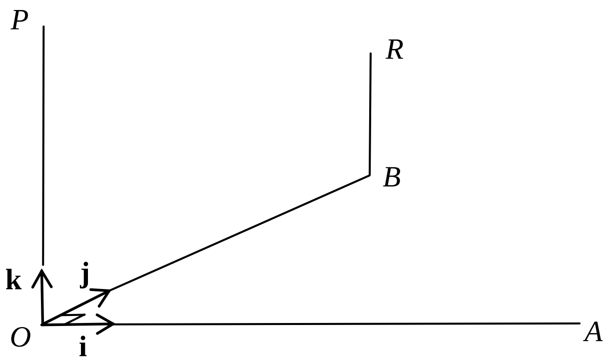
**(b)**




The line *CD* is perpendicular to a horizontal plane through the point D and *CD* is 6 cm. A variable point *P* moves along a straight line through *D* on the horizontal plane. Given that *P* is moving away from *D* at a speed of 2 cms$^{-1}$, find the rate of change of the angle *CPD* with respect to time when the distance of *P* from *D* is $6\sqrt{3}$ cm.

**[5]**

**9** In the diagram below, *OAB* is the horizontal base, where *OA* = 6 units, *OB* = 4 units, and $\angle AOB = 90°$. Poles *OP* and *BR* are placed vertically, where *OP* = 5 units and $BR = 2$ units. The unit vectors **i, j, k** are parallel to *OA*, *OB* and *OP* respectively. The point *O* is taken as the origin.




**(i)** Show that the equation of line *PR* is $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix}$, where $\lambda \in \mathbb{R}$ . **[2]**

**(ii)** A point $Q$ divides $AB$ such that $AQ{:}QB = 1{:}3$. Find $\overrightarrow{OQ}$. **[2]**

**(iii)** A line $l$ with equation $x = 0;\ \dfrac{1-y}{3} = z - 8$ intersects the line $PR$ at a point $X$. Find the position vector of $X$. **[3]**

**(iv)** Let $\mathbf{m}$ be a unit vector along $AB$. Find $|\ \overrightarrow{QX} \times \mathbf{m}\ |$. Give a geometrical meaning of $|\ \overrightarrow{QX} \times \mathbf{m}\ |$. Hence, or otherwise, find the area of triangle $AXB$. **[4]**

**10** A plane $\Pi_1$ is given by the equation $-x + 2y = -5$. A sphere with centre represented by the position vector $-4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ rests on the plane such that the plane touches the sphere only at one point $A$.

**(i)** Find the position vector of $A$. Hence, or otherwise, find the radius of the sphere exactly. **[4]**

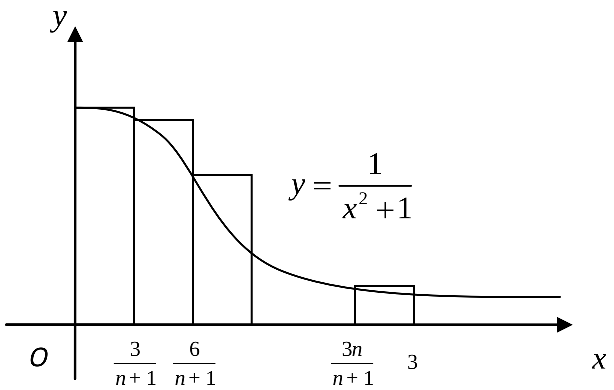
**(ii)** A line $l_1$ that passes through the point $a\mathbf{k}$ ( where $a > 0$) and the centre of the sphere makes an angle of $30^0$ with the plane $\Pi_1$. Find $a$ in exact form. **[4]**

**(iii)** Another plane $\Pi_2$ contains the line $\boldsymbol{r} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, $t$ Î ¡ and is also parallel to vector $-2\mathbf{i} + 2\ \mathbf{j} - \mathbf{k}$. Find the vector equation of $\Pi_2$ in scalar product form. Hence, show that $\Pi_1$ and $\Pi_2$ are perpendicular. **[3]**

**11 (a)** **(i)** The region $R$ is bounded by the curves $y = \dfrac{4}{x^2+1}$, $y = \ln(x+1)$ and the line $x = 1$ and the $y$-axis. Find the area of region $R$. **[2]**

**(ii)** Find the exact volume of the solid formed when $R$ is rotated 2p radians about the $y$-axis. **[4]**

**(b)** The diagram shows part of the graph of $y = \dfrac{1}{x^2+1}$.



**(i)** By considering $n+1$ rectangles of equal width from $x=0$ to $x=3$, show that for all non-negative integers $n$,

$$A < \sum_{r=0}^{n} \frac{3(n+1)}{9r^2+(n+1)^2},$$

where $A$ is the area bounded by the curve, the axes and the line $x = 3$. **[3]**

**(ii)** Deduce $\lim\limits_{n\to\infty} \sum_{r=0}^{n} \frac{3(n+1)}{9r^2+(n+1)^2}$ exactly. **[2]**

**12 (a)** By means of the substitution $u=\sqrt{x+1}$, find $\int \frac{2x}{\sqrt{x+1}}\,\mathrm{d}x$. **[4]**

**(b)** Find $\int \frac{\mathrm{d}x}{(1+x^2)\tan^{-1}x}$ where $x > 0$. **[2]**

**(c)** **(i)** Differentiate $e^{\sqrt{1-x^2}}$ with respect to $x$ for $|x|$ £ $1$. **[2]**

**(ii)** Hence, find the exact value of $\int_0^1 x\,e^{\sqrt{1-x^2}}\,\mathrm{d}x$. **[4]**

**End of Paper**