St Margaret School (Sec)

Preliminary Examinations 2024

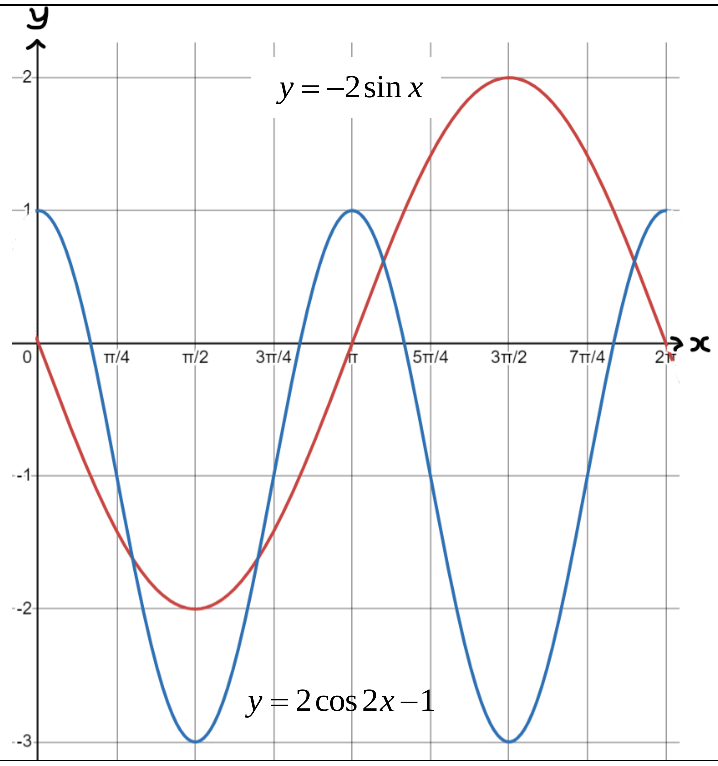
Sec 4E5N Additional Mathematics Paper 1

Question		Answer
1	**(a)**	$-2\left[x^2+6x-\frac{1}{2}\right]$ $=-2[(x+3)^2-3^2-\frac{1}{2}]$ $=-2(x+3)^2+19$ Max y value is 19 Corresponding x value is -3.
	(b)	$y=-2x^2-12x+1---(1)$ $y=-2x+1-------(2)$ Sub (1) into (2) $-2x+1=-2x^2-12x+1$ $2x^2+10x=0$ $2x(x+5)=0$ $x=0 \; or \; x=-5$ $y=1 \; or \; y=11$ $AB=\sqrt{(0+5)^2+(1-11)^2}$ $=\sqrt{125}$ $\therefore k=125$
2	**(a)**	Refer to last page
	(b)	$\lg T = x\lg B+\lg A$ $\lg A=1.78 \; (\pm 0.01)$ $A=60.3 \; (\pm 2)$ $Gradient=\frac{0.63-1.55}{25-5}$ $=-0.046$ $\lg B=-0.046$ $B=0.899 \; (\pm 0.01)$
	(c)	$At \; x=13\text{min}, \; \lg T=1.18$ $T=15.1°C<16°C \; (FreezingPt)$ Hence the chocolate is frozen. **<u>Alternatively</u>** At T = 16°C, $\lg T=1.2041$. x = 12.5 mins < 13mins Hence the chocolate is frozen.

3	**(a)**	$\frac{3x^2-1}{x^2(2x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x-1}$ By Comparison $3x^2-1=Ax(2x-1)+B(2x-1)+Cx^2$ Let x = 0, $-1=B(-1)$ B = 1 Let x = $\frac{1}{2}$, $-\frac{1}{4}=\frac{1}{4}C$ C = −1 Let x = 1, $2=A+1-1$ A = 2 $\frac{3x^2-1}{x^2(2x-1)}=\frac{2}{x}+\frac{1}{x^2}-\frac{1}{2x-1}$
	(c)	$2\ln x-\frac{1}{x}-\frac{1}{2}\ln(2x-1)+C$
4	**(a)**	$k-1<0$ $k<1$ $b^2-4ac<0$ $4^2-4(k-1)(k+2)<0$ $16-4(k^2+k-2)<0$ $4k^2+4k-24>0$ $4(k^2+k-6)>0$ $(k+3)(k-2)>0$ $k<-3$ or $k>2$ Ans $\therefore k<-3$
	(b)	$y=(k-1)x^2+4x+k+2---(1)$ $y=-2x+3-----------(2)$ Sub (1) into (2) $(k-1)x^2+4x+k+2=-2x+3$ $(k-1)x^2+6x+k-1=0$ $b^2-4ac=0$ $6^2-4(k-1)(k-1)=0$ $(k-1)^2=9$ $k-1=\pm3$ $k=4$ or 2

5	**(a)**	$T_{r+1}=\binom{8}{r}(3x)^{8-r}\left(-\frac{2}{x}\right)^{r}$ $=\binom{8}{r}3^{8-r}(-2)^{r}x^{8-2r}$ Since power of $x=8-2r=2(4-r)$. Power is a multiple of 2 for all integer values of *r*. Hence it is always even, no odd powers of *x*.
5	**(b)**	In $\left(3x-\frac{2}{x}\right)^{8}$, For x^4 power, $8-2r=4$ $r=2$ For x^2 power, $8-2r=2$ $r=3$ $(ax^2-1)(...+\binom{8}{2}3^6(-2)^2x^4+\binom{8}{3}3^5(-2)^3x^2+...)$ $-\binom{8}{2}(3)^6(-2)^2+a\binom{8}{3}(3)^5(-2)^3=0$ $-81648-108864a=0$ $a=-\frac{3}{4}$
6		$\frac{d^2y}{dx^2}=\sin x+1+\cos 2x$ $\frac{dy}{dx}=\int \sin x+1+\cos 2x\ dx$ $=-\cos x+x+\frac{1}{2}\sin 2x+C$ At $x=\frac{\pi}{2}$, $\frac{dy}{dx}=\frac{\pi}{2}$ $\frac{\pi}{2}=\frac{1}{2}\sin 2\left(\frac{\pi}{2}\right)-\cos\left(\frac{\pi}{2}\right)+\frac{\pi}{2}+C$ $\frac{\pi}{2}=\frac{\pi}{2}+C$ $C=0$ $y=\int \frac{1}{2}\sin 2x-\cos x+x\ dx$ $=-\frac{1}{4}\cos 2x-\sin x+\frac{1}{2}x^2+D$ At $x=\frac{\pi}{2}$, $y=-\frac{3}{4}$

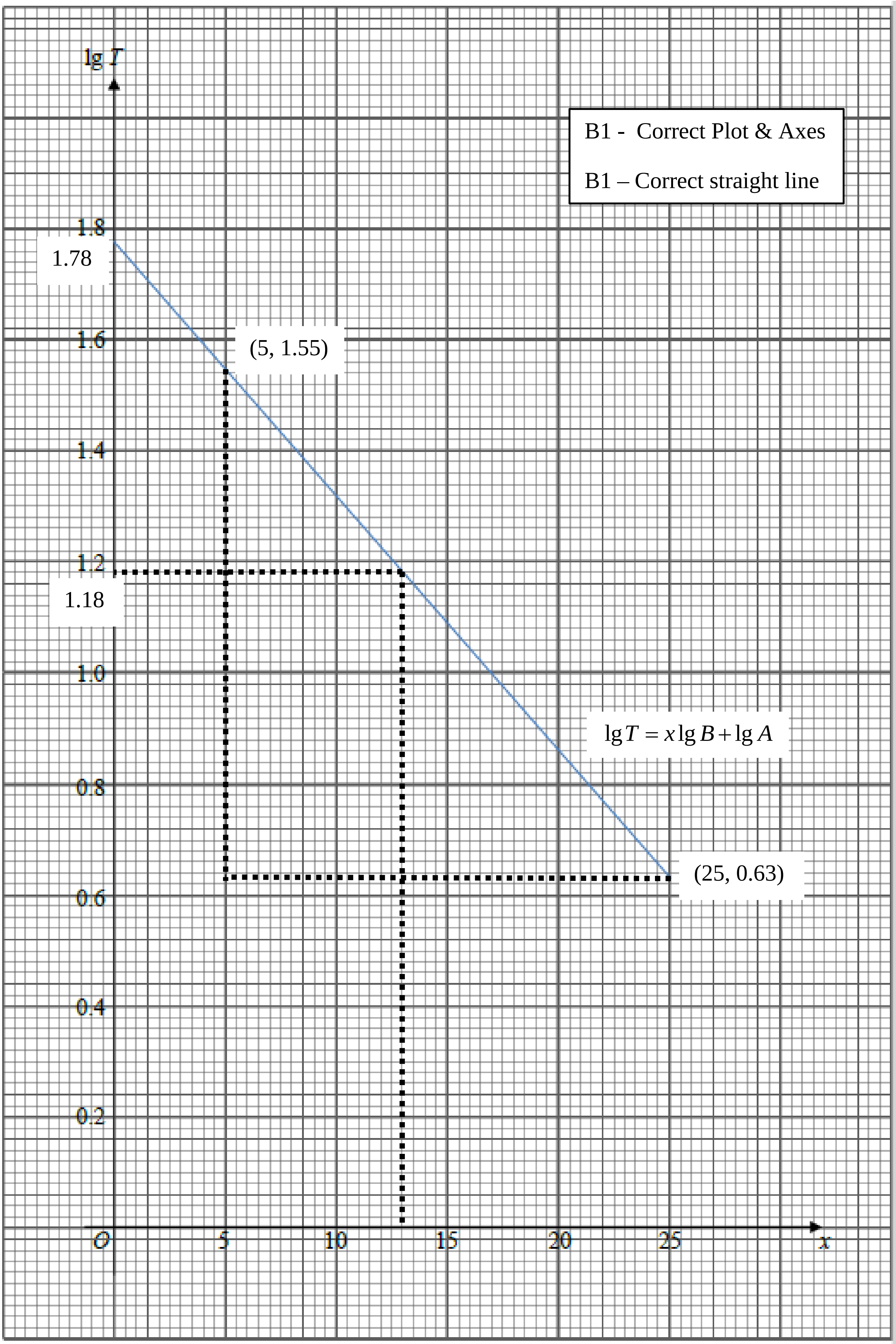
6		$-\frac{3}{4}=-\frac{1}{4}\cos 2\left(\frac{\pi}{2}\right)-\sin\left(\frac{\pi}{2}\right)+\frac{1}{2}\left(\frac{\pi}{2}\right)^2+D$ $D=-\frac{\pi^2}{8}$ $\therefore Eqn\ y=-\frac{1}{4}\cos 2x-\sin x+\frac{1}{2}x^2-\frac{\pi^2}{8}$
7	**(a)**	*R* is the midpoint of *AB*, *S* is the midpoint of *AC* By Midpoint Theorem, $RS//BC$ and $BC=2RS$ Hence, $\angle SRC=\angle RCB$ (Alternate angles, *RS//BC*)
	(b)	$\angle RTB=\angle CTR$ (Common angle) $\angle TRB=\angle TCR$ (Alternate Segment Theorem) $\therefore$ Triangle *TBR* and Triangle *TRC* are similar (AA Similarity)
	(c)	From (b) Triangle *TRB* and Triangle *TRC* are similar. $\frac{TR}{TC}=\frac{TB}{TR}$ (Corresponding sides of similar Δ s) $TR^2=TB\times TC$ $TR^2=TB\times(TB+BC)$ $TR^2=TB^2+TB\times BC$ From (a), $BC=2RS$ $TR^2=TB^2+TB(2RS)$ $TR^2-TB^2=2TB\times RS$
8	**(a)**	$\mathrm{f}(x)=\frac{x^2-x+1}{3x-3}$ $\mathrm{f}'(x)=\frac{(2x-1)(3x-3)-3(x^2-x+1)}{(3x-3)^2}$ $=\frac{6x^2-6x-3x+3-3x^2+3x-3}{(3x-3)^2}$ $=\frac{3x^2-6x}{(3x-3)^2}$ $=\frac{3(x^2-2x)}{9(x-1)^2}$ $=\frac{x^2-2x}{3(x-1)^2}$

8	**(b)**	$f'(x)=\dfrac{x^2-2x}{3(x-1)^2}$ $f''(x)=\dfrac{3(2x-2)(x-1)^2-6(x-1)(x^2-2x)}{9(x-1)^4}<0$ Since $9(x-1)^4>0$, $3(2x-2)(x-1)^2-6(x-1)(x^2-2x)<0$ $(x-1)[(6x-6)(x-1)-6x^2+12x]<0$ $(x-1)[(6x^2-12x+6-6x^2+12x]<0$ $6(x-1)<0$ $x<1$
9	**(a)(i)**	2
	(a)(ii)	π *or* $180°$
	(b)	
	(c)	4 solution
10	**(a)(i)**	$m_{AC}=-2$ *Eqn AC is* $y=-2x+6$
	(a)(ii)	$y=-2x+6--------(1)$ $x+5y=-6--------(2)$ Sub (1) into (2) $x+5(-2x+6)=-6$ $x=4$ $y=-2$ $C(4,-2)$

10	**(b)**	$D(-6,0)$ Area of $\Delta ACD = \frac{1}{2}\begin{vmatrix} 0 & -6 & 4 & 0 \\ 6 & 0 & -2 & 6 \end{vmatrix}$ $= \frac{1}{2}(12+24-(-36))$ = 36 units² Area of $\Delta ABC = \frac{36}{1.5}$ =24 units Let $B(k,6)$ $\frac{1}{2}\begin{vmatrix} 0 & 4 & k & 0 \\ 6 & -2 & 6 & 6 \end{vmatrix} = 24$ $\frac{1}{2}(24+6k-24+2k)=24$ $k=6$ $\therefore B(6,6)$ *Alternative Method* Area of $\Delta ACD = \frac{1}{2}\begin{vmatrix} 0 & -6 & 4 & 0 \\ 6 & 0 & -2 & 6 \end{vmatrix}$ $= \frac{1}{2}(12+24-(-36))$ = 36 units² Area of $\Delta ACD = \frac{36}{1.5}$ =24 units $\frac{1}{2}AB(8)=24$ $AB=6$ $\therefore B(6,6)$
11	**(a)**	$a=\frac{dv}{dt}=12t+k$ At t = 1, a = -6 $-6=12(1)+k$ $k=-18$ (Shown)
	(b)(i)	For min velocity, $a=0$ $12t-18=0$ $t=1.5$ At t = 1.5, $v=6(1.5)^2-18(1.5)+12$ $=-1.5$ m/s

	(b)(ii)	For instantaneous rest, v = 0 $6t^2-18t+12=0$ $6\left(t^2-3t+2\right)=0$ $\left(t-2\right)\left(t-1\right)=0$ $t=1\ or\ t=2$ $s=\int 6t^2-18t+12dt$ $\quad =2t^3-9t^2+12t+C$ At t = 0, s = 0 0 = 0 + C C = 0 $\therefore s=2t^3-9t^2+12t$ At t = 1, $s=2(1)^3-9(1)^2+12(1)=5$ At t = 2, $s=2(2)^3-9(2)^2+12(2)=4$ At t = 4, $s=2(4)^3-9(4)^2+12(4)=32$ Total distance = $5+1+(32-4)$ =34 m
12	**(a)**	$\frac{dy}{dx}=\frac{3\left(x+2\right)-(3x+1)}{\left(x+2\right)^2}$ $\quad =\frac{5}{\left(x+2\right)^2}$ For x > -2, $5>0$ $\left(x+2\right)^2>0$ $\frac{5}{\left(x+2\right)^2}>0$ $\frac{dy}{dx}>0\neq 0$ Hence, the curve does not have a stationary point.
12	**(b)**	Gradient of normal = -5 $\frac{dy}{dx}=\frac{1}{5}$ $\frac{5}{\left(x+2\right)^2}=\frac{1}{5}$ $25=\left(x+2\right)^2$ $x+2=\pm 5$ $x=3\ or\ -7(reject, x>0)$ $y=\frac{3\left(3\right)+1}{3+2}=2$ Coordinate of Q is (3, 2) Since $y=-5x+c$ passes through (3,2) $2=-5(3)+c$ $c=17$ Eqn AB is $y=-5x+17$

12	**(b)**	At $x = 0$, $y = 17$ At $y = 0$, $x = 3.4$ Area of triangle AOB = $\frac{1}{2}(17)(3.4)$ $= 28.9$ units^2
	(c)	$x+1\overline{\big)3x+1}$ with quotient 3 $\underline{3x+6}$ -5 $\frac{3x+1}{x+2} = 3 - \frac{5}{x+2}$ Given that $x > -2$ $\frac{5}{x+2} > 0$ $3 - \frac{5}{x+2} < 3$ $\therefore c \geq 3$ such that Line $y = c$ does not intersect the curve.


lg T
B1 - Correct Plot & Axes
B1 – Correct straight line
1.8
1.78
1.6
(5, 1.55)
1.4
1.2
1.18
1.0
lg T = x lg B + lg A
0.8
(25, 0.63)
0.6
0.4
0.2
O
5
10
15
20
25
x