Class	Index Number	Name
20S		

ST. ANDREW'S JUNIOR COLLEGE
JC 2 2021
Preliminary Examination

PHYSICS 9814/01

Paper 1 **17th September 2021**

3 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and Civics Group on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

The use of an approved scientific calculator is expected, where appropriate.

Section A

Answer **all** questions.

You are advised to spend about 1 hour and 50 minutes on Section A

Section B

Answer **two** questions only.

You are advised to spend about 35 minutes on each question in Section B

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **30** printed pages including this page.

Data

speed of light in free space	c	=	3.00×10^{8} m s^{-1}
permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	ε_0	=	8.85×10^{-12} F m^{-1}
			$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	e	=	1.60×10^{-19} C
the Planck constant	h	=	6.63×10^{-34} J s
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron	m_e	=	9.11×10^{-31} kg
rest mass of proton	m_p	=	1.67×10^{-27} kg
molar gas constant	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant	k	=	1.38×10^{-23} J K^{-1}
gravitational constant	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$
moment of inertia of rod through one end	$I = \frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	$I = \frac{1}{2}M\left(r_1^2 + r_2^2\right)$
moment of inertia of solid sphere through centre	$I = \frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	$I = \frac{2}{3}MR^2$
work done on/by a gas	$W = p\Delta V$
hydrostatic pressure	$p = \rho gh$
gravitational potential	$\phi = -\frac{Gm}{r}$
Kepler's third law of planetary motion	$T^2 = \frac{4\pi^2 a^3}{GM}$
temperature	T/K = T/°C + 273.15
pressure of an ideal gas	$p = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$
mean translational kinetic energy of an ideal gas molecule	$E = \frac{3}{2}kT$

displacement of particle in s.h.m.	$x = x_0 \sin \omega t$
velocity of particle in s.h.m.	$v = v_0 \cos \omega t$
	$= \pm \omega\sqrt{x_o^2 - x^2}$
electric current	$I = Anvq$
resistors in series	$R = R_1 + R_2 + \ldots$
resistors in parallel	$1/R = 1/R_1 + 1/R_2 + \ldots$
capacitors in series	$1/C = 1/C_1 + 1/C_2 + \ldots$
capacitors in parallel	$C = C_1 + C_2 + \ldots$
energy in a capacitor	$U = \frac{1}{2}CV^2$
electric potential	$V = \frac{Q}{4\pi\varepsilon_o r}$
electric field strength due to a long straight wire	$E = \frac{\lambda}{2\pi\varepsilon_o r}$
electric field strength due to a large sheet	$E = \frac{\sigma}{2\varepsilon_o}$
alternating current/voltage	$x = x_0 \sin \omega t$
magnetic flux density due to a long straight wire	$B = \frac{\mu_o I}{2\pi d}$
magnetic flux density due to a flat circular coil	$B = \frac{\mu_o NI}{2r}$
magnetic flux density due to a long solenoid	$B = \mu_o nI$
energy in an inductor	$U = \frac{1}{2}LI^2$
RL series circuits	$\tau = \frac{L}{R}$
RLC series circuits (underdamped)	$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
radioactive decay	$x = x_0 \exp(-\lambda t)$
decay constant	$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$

Section A

Answer **all** questions in this section

You are advised to spend about 1 hour 50 minutes on this section.

1 An alpha particle of mass 4 *u* moving with a velocity 2000 ms^{-1} collides elastically with a carbon atom of mass 12 *u* at rest. The alpha particle is scattered through an angle of 30° from the original direction.

(a) Calculate the magnitude of the velocity of the centre of mass of the 2 particles before the collision.

velocity = _______________ $m\ s^{-1}$ [1]

(b) Construct a vector diagram relating the velocity of the alpha particle in the centre of mass frame and the laboratory frame after the collision.

Calculate the magnitude of the velocities of the alpha particle after the collision in the centre of mass reference frame, and the lab frame.

velocity in lab frame = ____________ $m\ s^{-1}$

velocity in centre of mass frame = ____________ $m\ s^{-1}$

[4]

(c) Hence determine the velocities of the carbon atom after the collision in the centre of mass reference frame and the laboratory frame, and the velocity direction in the laboratory frame.

direction in the lab frame = ________________ ^{o}

velocity in lab frame = ____________ m s^{-1}

velocity in centre of mass frame = ____________ m s^{-1}

[3]

[Total: 8]

2 Consider a circular sheet of radius 2*R* having a uniform density. A circular hole of radius *R* is made at a distance *R* from the center of the first circle, as shown in Fig. 2.1.

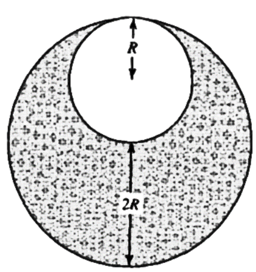

Fig. 2.1

(a) Determine the position of the centre of mass of the circular sheet.

[2]

(b) The centre of mass of the circular sheet is projected with a speed of 22.6 ms^{-1} at an angle of 45° from the horizontal while rotating forward with an angular velocity of 5π rad s^{-1} about a horizontal axis passing through the centre of mass. The value of R is 0.600 m.

(i) Sketch a path showing how the outer most edge of the hole moves until it hits the ground.

[3]

(ii) Calculate the number of times the outer most edge of the hole moves backwards horizontally before the disc reaches the highest point of its path.

number of times = ______________ [4]

(iii) Calculate the vertical velocity of the outermost edge of the hole when its horizontal velocity is zero for the first time.

vertical velocity = ______________ m s^{-1} [1]

[Total: 10]

3 A spacecraft in low Earth orbit has to be put into an orbit around Venus using Hofmann transfer orbit. The Earth and Venus are in coplanar circular orbit with the Sun as the centre of the circle. Use the data provided below to answer parts **(a)** and **(b)**.

1 AU = 1.496×10^{11} m

Planets	Distance from the Sun / AU	Period / s
Earth	1.000	3.156×10^{7}
Venus	0.7233	1.941×10^{7}

(a) Calculate the change in magnitude of velocity required at the start of the Hofmann transfer and at the destination orbit.

change in speed at the start = ____________ m s^{-1}

change in speed at the destination orbit = ____________ m s^{-1}

[4]

(b) Calculate the time taken to complete the transfer.

time = ____________ days [1]

(c) Calculate the angle between Venus and Earth at the start of the transfer.

angle between Earth and Venus = ___________ ^{o} [2]

[Total: 7]

4 The circuit in Fig. 4.1 shows a capacitor of capacitance C connected to a switch **S** that can swing to the left or to the right to make a contact. Switch is initially open. The emf of the source is E and the resistance connected to it is R. The variable resistor R_2 is initially set to zero. Each inductor has an inductance of L and they are connected in such a way that they are cumulatively coupled with a mutual inductance of M.

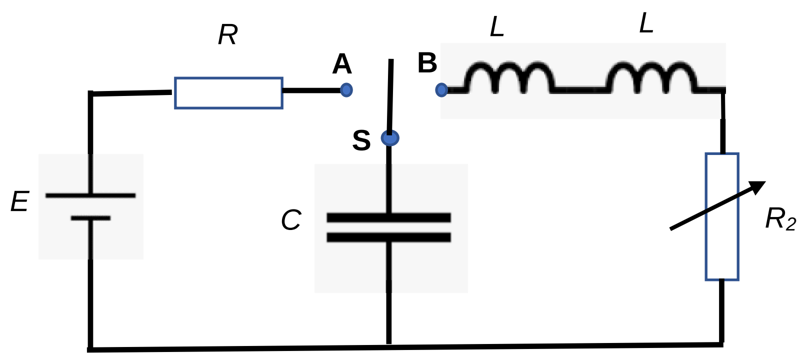

Fig. 4.1

(a) At t = 0, the switch **S** makes contact to **A**.
Derive and expression for the time taken for the voltage across the capacitor to reach $E/2$, in terms of RC.

[2]

(b) After a sufficiently long time, switch **S** is disconnected from **A** and makes contact with **B**.

(i) State the natural frequency of the signal of the current in the circuit in terms of *L*, *C* and *M*.

[2]

(ii) The resistance of R_2 is now increased from zero to *R* so that the charge on the capacitor has the solution of the form $q = Q_o \exp(-\gamma t) \sin(\omega t + \phi)$.
Determine expressions for the constants ω and γ in terms of *C*, *R*, *L* and *M* by considering the change in voltage in the circuit.

[4]

[Total: 8]

5 A parallel-plate capacitor is made of two square plates 25.0 cm on a side and 1.00 mm apart. The capacitor is connected to a 50.0 V battery.

(a) Calculate the energy stored in the capacitor.

energy = ______________ J [2]

(b) With the battery still connected, the plates are pulled apart to a separation of 2.00 mm. Calculate the energy stored in the capacitor and account the change in energy compared to part **(a)**.

energy = ________________ J

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………………………………………………………………………………………………………... [2]

(c) The plates are placed back 1.00 mm apart. The battery is disconnected after charging the capacitor. The plates are now pulled apart to a separation of 2.00 mm.
Calculate the energy stored in the capacitor and account for the change in energy compared to part **(a)**.

energy = ________________ J

……

……………………………………………………………………………………………………….. [2]

(d) A piece of dielectric is inserted into the capacitor plates in part **(c)**.
Explain how this affects the energy stored in the capacitor.

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....

.... [2]

[Total: 8]

6 Sound waves of frequency *f* are emitted from the siren of an ambulance with velocity *c*.

(a) **(i)** A motorcyclist moves towards the stationary ambulance with a velocity *u*. Both *u* and *c* are being measured with respect to the land.
By considering the speed of the sound relative to the observer or otherwise, show that the motorcyclist hears a frequency f_O given by

$$f_O = \frac{u + c}{c} f$$

[2]

(ii) The motorcyclist travels on a road that runs parallel to the road that the ambulance is situated. A graph of the frequency measured by the motorcyclist with time is show in Fig. 6.1

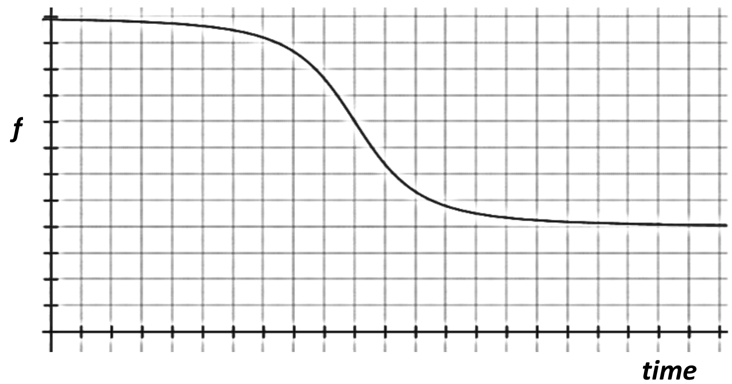

Fig. 6.1

Explain how the graph is consistent with the equation in part **(i)**.

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…………………………………………………………………………………………… [4]

(b) The ambulance now moves with a speed of v_s towards a stationary observer. The wavelength perceived by an observer will change, as illustrated by Fig. 6.2.

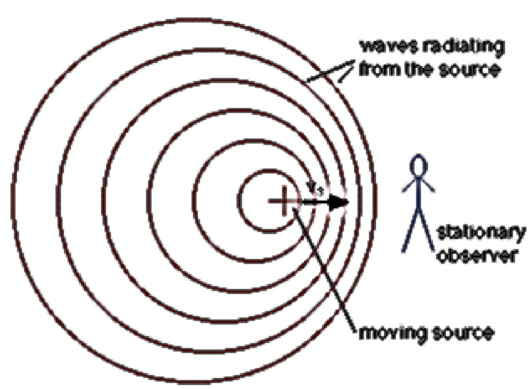

Fig. 6.2

By consider the wavelength change due to the moving siren or otherwise, show that if the air is stationary, the frequency f_O detected by the observer is given by

$$f_O = \frac{c}{c - v_s}\, f$$

[2]

(c) If the motorcyclist in part **(a)** moves at a speed of u towards the moving ambulance that travels in opposite direction at a speed of v_s, state the frequency detected by the motorcyclist in terms of c, f, u and v_s.

[1]

(d) A wind now blows against the direction of motion of the motorcyclist with a speed of v_w, suggest how your expression in part **(c)** is affected.

[2]

(e) (i) Show that for part **(b)**, if $c >> v_s$ in a stationary air, the value of $\Delta f = f_O - f$ is given by

$$\frac{\Delta f}{f} \approx \frac{\boldsymbol{v_s}}{\boldsymbol{c}}$$

[1]

(ii) Show that the observed frequency of an absorption line of emitted frequency *f* in the spectrum of the Sun should vary between $f + \delta f$ and $f - \delta f$ where

$$\delta f = \frac{2\pi R f}{Tc}$$

in which *R* is the radius of the Sun, *T* is the period of rotation and *c* is the speed of light.

[2]

(iii) Given that the density of the Sun is 1.4×10^3 kg m^{-3}, the mass is 2.0×10^{30} kg and the equatorial rotation period is 24.5 days. Calculate δf / f for the peak in the visible spectrum at 550 nm.

fractional frequency shift = ____________ [2]

(iv) The gas molecules producing solar spectrum are in random thermal motion. As a result of the random motion, the width of the lines can also become broader. If the ratio (width of line)/wavelength of a hydrogen line spectrum observed is 8 x 10^{-5}, estimate the temperature of the Sun, assuming that the gas follows the kinetic theory of gases.

Molar mass of hydrogen is 1.00 x 10^{-3} kg mol^{-1}.

temperature = _______________ K [2]

[Total: 18]

Section B

Answer **two** questions from this section

You are advised to spend about 35 minutes on each question.

7 A yoyo is made from two circular discs, each of radius 2r, attached to the ends of an axle r so that the axis of the axle is perpendicular to the discs and passes through their centres. The yoyo lies on an inclined plane which makes an angle of 30^o with the horizontal. A light inextensible string has one end attached to the curved surface of the axle of the yoyo and is wound several rounds around the axle. The string leaves the axle in a direction perpendicular to the axle and parallel to the inclined plane, passes over a smooth peg and carries a mass 2M at its other end, as shown in Fig. 7.1.

The mass of the yoyo M and their moment of inertia is Mr^2, where r = 0.025 m.

On being released the disc roll without slipping on the inclined plane. The acceleration of the 2M mass is a, the tension in the string is T, and the frictional force between the yoyo and the inclined plane is f.

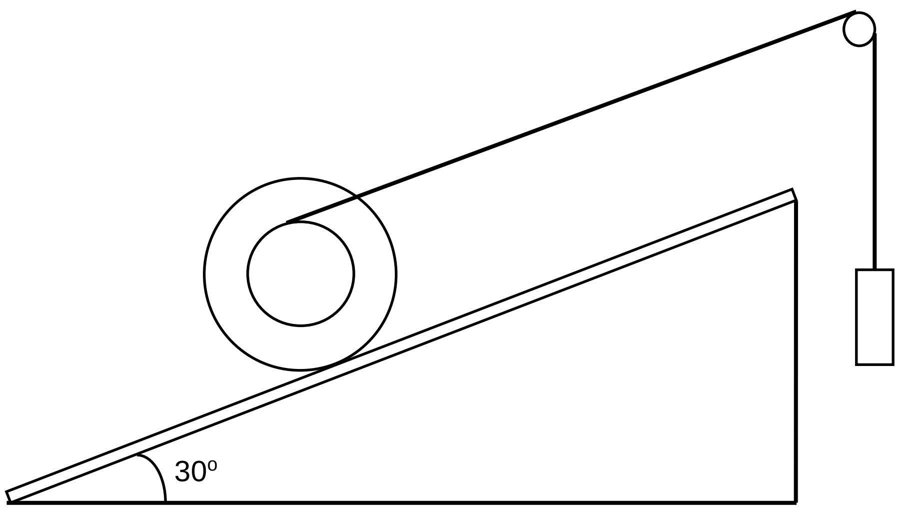

Fig. 7.1

(a) Express the acceleration of the centre of mass of the yoyo in terms of a.

[1]

(b) Derive the angular acceleration of the yoyo from part **(a)**

[2]

(c) State the linear equations of motion for 2*M* and the yoyo in terms of *T*, *f* and *a*.

[2]

(d) State the angular equation of motion of the yoyo in terms of *T*, *f* and *a*.

[2]

(e) Hence determine the values of *T*, *f* and *a* in terms of *M* and *g*.

[3]

(f) Determine the least possible value of the coefficient of static friction between the yoyo and the inclined plane.

coefficient of static friction = ____________ [2]

(g) Determine the angular displacement of the yoyo from rest in 0.500 s.

angular displacement = ____________ rad [2]

(h) The string breaks when the 2*M* mass is travelling at a speed of 2.50 ms^{-1}. Determine

(i) the gain in vertical height of the yoyo to the highest point,

vertical height = ____________ m [2]

(ii) the time taken by the yoyo to reach its highest point and

time = _____________ s [2]

(iii) the angular retardation of the yoyo.

angular retardation = ____________ rad s^{-2} [2]

[Total: 20]

8 (a) An infinite cylindrical wire with radius 2*R* carries a uniform current per unit area *J* flowing out of the page, except inside an infinite cylindrical hole parallel to the wire's axis, as shown in Fig. 8.1a and Fig. 8.1b. The hole has radius *R* and is tangent to the exterior of the wire.

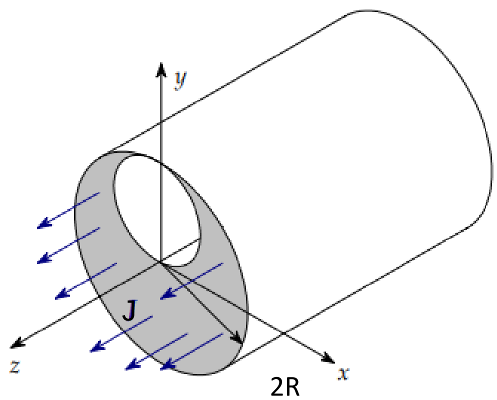

Fig. 8.1a

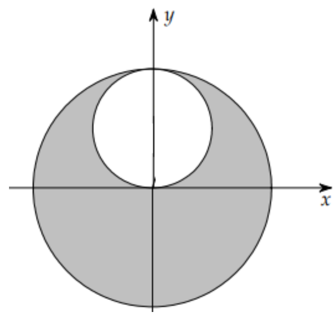

Fig. 8.1b

(i) State the direction and derive an expression for the magnitude of the magnetic flux density inside the hole along the Y-axis, in terms of *J* and *R*.

[3]

(ii) Sketch a graph showing how the magnitude of the flux density varies with distance along the Y-axis from -2*R* to 0.

[2]

(b) A current I flows through a toroidal coil wrapped on a hollow rectangular cross-sectional circular tube as a holder. The inner and outer radius of the coil is *a* and *b* respectively. The height of the coil is *H*, and it has *N* uniform turns, as shown in Fig. 8.2.

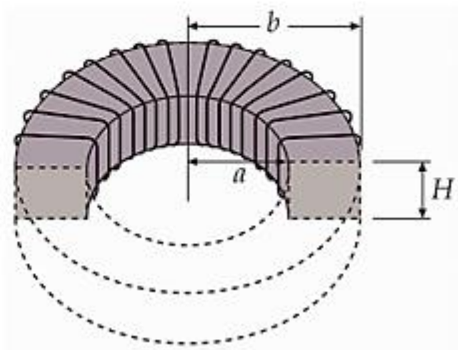

Fig. 8.2

(i) Show that the magnetic flux density inside the toroidal coil at a radial distance of *r* from the centre of the coil is given by

$$B = \frac{\mu_0 N I}{2\pi r}$$

[2]

(ii) Show that the magnetic flux through all the coils is given by

$$\frac{\mu_0 NIH}{2\pi}\ln\frac{b}{a}$$

[2]

(iii) Hence derive an expression for the self-inductance of the toroidal coil.

[2]

(c) A current-carrying coil with magnetic dipole moment 5.50 N m T^{-1} makes an angle of 110° with a 2.50 T magnetic field.

(i) Calculate the torque acting on the coil.

torque = ______________ N m [2]

(ii) Calculate the maximum energy that can be extracted from the interaction of the magnetic moment with the magnetic field.

energy = ______________ J [2]

(iii) The coil can perform simple harmonic motion about its equilibrium position. Given that the moment of inertia of the coil about the axis is 0.150 kgm^2, calculate the frequency of oscillation.

frequency = _______________ Hz [5]

[Total: 20]

9 A non-conducting sphere of radius a is placed at the centre of a spherical conducting sphere of inner radius b and outer radius c, as shown in Fig. 9.1

A charge +Q is distributed uniformly throughout the inner sphere. The outer shell is charged in such a way that the outer surface of the outer shell carries a charge of –Q.

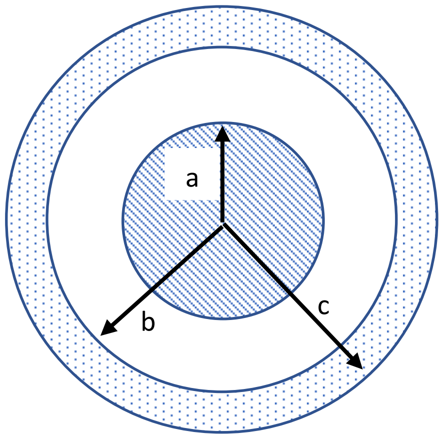

Fig. 9.1

(a) State the quantity of charge on the inner surface of the spherical shell.

charge = _______________ [1]

(b) State the expression of the electric field in terms of Q, the radial distance r from the centre of the sphere at a point for each of the cases below.

(i) Within the non-conducting sphere.

[3]

(ii) Between the non-conducting sphere and the conducting shell.

[1]

(iii) Inside the conducting shell.

[1]

(iv) Outside the conducting shell.

[1]

(c) (i) Sketch a graph showing the variation of the electric field with radial distance *r* from the centre of the shell.

[3]

(ii) Sketch a graph showing the variation of electric potential with radial distance *r* from the centre of the shell.

[3]

(d) A metal plate with perpendicular edge is brought near to the shell, as shown in Fig 9.2. The potential of the plate is opposite in sign but of the same magnitude as the potential of the metal shell.

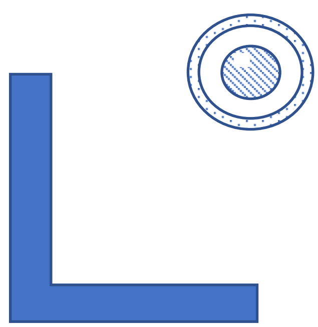

Fig. 9.2

Sketch on Fig 9.2 the electric field lines and equipotential lines in the space between the plate and the shell. [3]

(e) The metal plate and outer spherical shell are removed. The non-conducting sphere still carries a charge of Q. A small cylindrical hole is drilled through a diameter length through the centre of the sphere. The total charge on the non-conducting sphere is unaffected.

Explain how a negatively charged particle with a charge of q and mass m, released from rest at the opening of the cylindrical hole will behave. Derive any quantity that are relevant to the motion.

[4]

[Total: 20]

<End of Paper>