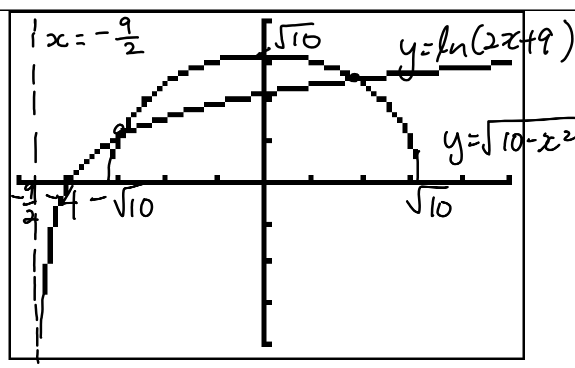
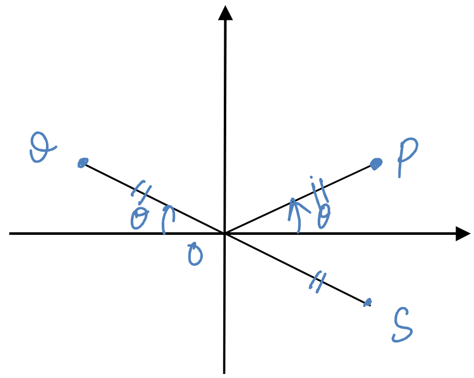
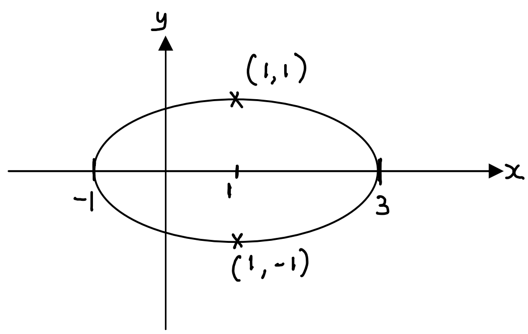
Qn	
1(i)	Let the number of units of sand, stone and brick required by the company be x, y and z respectively. $15x+10.5y+8.1z=205.2$ $11x+17.3y+7z=229.4$ $12x+13y+10z=208$ From GC, $x=7,\quad y=8,\quad z=2$. The number of units of sand, stone and brick required is 7, 8 and 2 units respectively.
1(ii)	Total amount that the company must pay $=\$0.9\left[11(7)+10.5(8)+7(2)\right]$ $=\$157.50$
2(ii)	$\frac{d^2y}{dx^2}=x\sin x$ $\frac{dy}{dx}=x(-\cos x)-\int -\cos x\,dx$ $=-x\cos x+\sin x+C$ $=-x\cos x+\sin x+C$ $y=-x\sin x-\int(-1)(\sin x)\,dx-\cos x+Cx+D$ $y=-x\sin x-2\cos x+Cx+D$ Given $f(0)=0,\ f'(0)=3$ $y=\mathrm{f}(x)$ passes through the origin $\Rightarrow f(0)=0\ \Rightarrow D=2\qquad f'(0)=3;\ C=3$ $y=-x\sin x-2\cos x+3x+2$
3	Let P_n be the statement $\sum_{r=1}^{n} r^3=\frac{1}{4}n^2(n+1)^2$ for $n\in\mathbb{Z}^+$. Prove that P_1 is true, i.e. LHS: $\sum_{r=1}^{1} r^3=(1)^3=1$ RHS: $\frac{1}{4}(1)^2(1+1)^2=1$ P_1 is true. Assume P_k is true, i.e. $\sum_{r=1}^{k} r^3=\frac{1}{4}k^2(k+1)^2$ for $k\in\mathbb{Z}^+$. Prove that P_{k+1} is true, i.e. $\sum_{r=1}^{k+1} r^3=\frac{1}{4}(k+1)^2(k+2)^2$ for $k\in\mathbb{Z}^+$. LHS: $\sum_{r=1}^{k+1} r^3=\sum_{r=1}^{k} r^3+(k+1)^3$

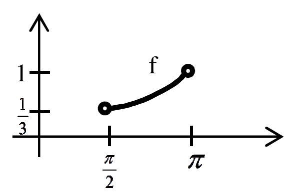
	$=\frac{1}{4}k^2(k+1)^2+(k+1)^3$ $=\frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right]$ $=\frac{1}{4}(k+1)^2\left[k^2+4k+4)\right]$ $=\frac{1}{4}(k+1)^2(k+2)^2$ $=RHS$ Thus, P_k is true $\Rightarrow$ P_{k+1} is true Since P_1 is true and P_k is true $\Rightarrow$ P_{k+1} is true, by mathematical induction, P_n is true for all $n\in\mathbb{Z}^+$. (Shown) $v_r=\ln(2a^{r^3})=\ln 2+r^3\ln a$ $S_n=\sum_{r=1}^{n}v_r$ $=\sum_{r=1}^{n}\left(\ln 2+r^3\ln a\right)$ $=n\ln 2+(\ln a)(1^3+2^3+3^3+...+4^3)$ $=\frac{n}{4}\ln 2^4+(\ln a)\left(\frac{1}{4}n^2(n+1)^2\right)$ $=\frac{n}{4}\left(\ln 16+\ln(a^{n(n+1)^2})\right)$ $=\frac{n}{4}\left(\ln 16a^{n(n+1)^2}\right)(proven)$
4(i)	$\frac{d}{dx}\left(xy-2y^2+4x^2\right)=\frac{d}{dx}66$ $x\frac{dy}{dx}+y-4y\frac{dy}{dx}+8x=0$ $\frac{dy}{dx}\left(x-4y\right)=-8x-y$ $\frac{dy}{dx}=\frac{8x+y}{4y-x}$ For tangent parallel to y-axis, $4y-x=0$ $x=4y$

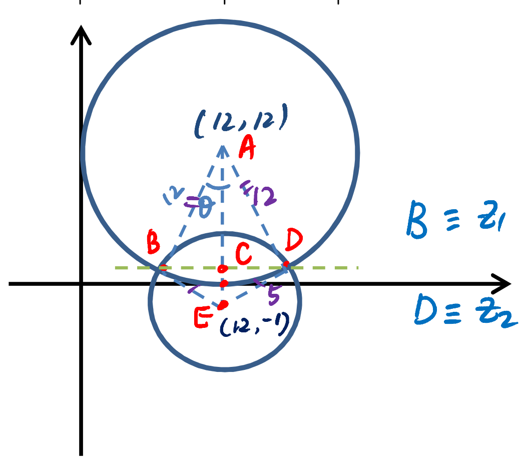
	Substitute $x=4y$ into equation of curve, $(4y)y-2y^2+4(4y)^2=66$ $66y^2=66$ $y^2=1$ $y=\pm1$. When $y=1$, $x=4$ When $y=-1$, $x=-4$ Coordinates are $(4,1)$, $(-4,1)$
4(ii)	Substitute $y=k$ into equation of the curve, $kx-2k^2+4x^2=66$ $4x^2+kx+(-2k^2-66)=0$ Considering the discriminant, $k^2-4(4)(-2k^2-66)$ $=33k^2+1056$ >0 for all real values of k The line $y=k$ cuts the curve for all real values of k.
5	 From the GC, $y=\ln(2x+9)$ intersects $y=\sqrt{10-x^2}$ at $x=-2.9539$, 1.8760 Hence for $\ln(2x+9)\ge\sqrt{10-x^2}$, $-\sqrt{10}\le x\le-2.9539$ *or* $1.8760\le x\le\sqrt{10}$ $-3.16\le x\le-2.95$ *or* $1.88\le x\le3.16$ Using previous result to solve for $\ln(2\lvert x\rvert+9)=\sqrt{10-x^2}$, $-\sqrt{10}\le\lvert x\rvert\le-2.9539(rej)$ *or* $1.8760\le\lvert x\rvert\le\sqrt{10}$ $-3.16\le x\le-1.88$ *or* $1.88\le x\le3.16$

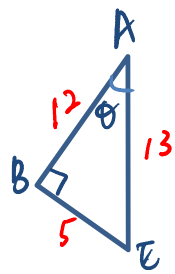
6(i)	Since a and b are the radiuses of a circle, $\|\boldsymbol{a}\| = \|\boldsymbol{b}\|=2$ $\boldsymbol{a}\cdot\boldsymbol{b} = \|a\|\|b\|\boldsymbol{cos 12}0^o = r^2\left(-\frac{1}{2}\right)$ Let $AN{:}NB = k{:}(1-k)$, hence $\overrightarrow{ON} = k\boldsymbol{a} + (1-k)\boldsymbol{b}$ Since ON is perpendicular to OB, $\overrightarrow{ON}.\overrightarrow{OB} = 0$ $\boldsymbol{b}\cdot\left(k\boldsymbol{a}+(1-k)\boldsymbol{b}\right)=0$ $k\boldsymbol{a}.\boldsymbol{b}+(1-k)\|\boldsymbol{b}\|^2=0$ $kr^2\left(-\frac{1}{2}\right)+(1-k)r^2=0$ $-\frac{1}{2}k+(1-k)=0$ $k=\frac{2}{3}$ Hence, $\overrightarrow{ON}=\frac{1}{3}(2\boldsymbol{a}+\boldsymbol{b})$ <u>Alternatively,</u> Use of geometry (various method) Area of $\triangle OAN$ $=\frac{1}{3}\left\|\boldsymbol{a}\times\frac{1}{3}(2\boldsymbol{a}+\boldsymbol{b})\right\|$ $=\frac{1}{6}\|2\boldsymbol{a}\times\boldsymbol{a}+\boldsymbol{a}\times\boldsymbol{b}\|$ $=\frac{1}{6}\|\boldsymbol{a}\times\boldsymbol{b}\|$
6(ii)	$\tan 30^\circ = \frac{ON}{r}$ $ON=\frac{r}{\sqrt{3}}$ $\overrightarrow{OC}=-\frac{\overrightarrow{ON}}{\left(r/\sqrt{3}\right)}r$ $=-\frac{\sqrt{3}}{3}(2\boldsymbol{a}+\boldsymbol{b})$

7(a)	
7(i)	$R \equiv -z$
7(ii)	$Area = (2\times 2\cos\theta)(2\times 2\sin\theta)$ $= 16\sin\theta\cos\theta$ $= 8\sin 2\theta$
7(b)	$3+3e^{i\theta} = 3(1+e^{i\theta})$ $= 3e^{i\frac{\theta}{2}}(e^{-i\frac{\theta}{2}} + e^{i\frac{\theta}{2}})$ $= 3e^{i\frac{\theta}{2}}(2\cos\frac{\theta}{2})$ $= 6e^{i\frac{\theta}{2}}\cos\frac{\theta}{2}$ $6e^{i\frac{\theta}{2}}\cos\frac{\theta}{2} = \frac{a}{\sqrt{2}}(1+i)$ $6e^{i\frac{\theta}{2}}\cos\frac{\theta}{2} = \frac{a}{\sqrt{2}}\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)$ $6e^{i\frac{\theta}{2}}\cos\frac{\theta}{2} = ae^{i\frac{\pi}{4}}$ $6\cos\frac{\theta}{2} = a, \qquad e^{i\frac{\theta}{2}} = e^{i\frac{\pi}{4}}$ $\theta = \frac{\pi}{2},\ a = 3\sqrt{2}$
8(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-x}\left(\frac{-2}{1-2x}\right) - \mathrm{e}^{-x}\ln(1-2x)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{e}^{-x}(1-2x)^{-1} - y$ $(1-2x)\frac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{e}^{-x} - y(1-2x)$ (shown) Differentiating with respect to x, $(1-2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{-x} + 2y - (1-2x)\frac{\mathrm{d}y}{\mathrm{d}x}$ $(1-2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2\mathrm{e}^{-x} - \left(-2\mathrm{e}^{-x} - (1-2x)y\right) + 2y$

	$\left(1-2x\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\dfrac{\mathrm{d}y}{\mathrm{d}x}+4\mathrm{e}^{-x}+\left(3-2x\right)y$ (shown)
8(ii)	Differentiating with respect to x, $\left(1-2x\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}-2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-4\mathrm{e}^{-x}+\left(3-2x\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y$ When $x=0,\ \ y=0,\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=-2,\ \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0,\ \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=-10$. $\therefore\ y=-2x-\dfrac{5}{3}x^3+\ldots$
8(iii)	$y=\left(1-x+\dfrac{x^2}{2}-\dfrac{x^3}{6}+\ldots\right)\left(-2x-2x^2-\dfrac{8x^3}{3}+\ldots\right)$ $y=-2x-2x^2-\dfrac{8}{3}x^3+2x^2+2x^3-x^3+\ldots$ $y=-2x-\dfrac{5}{3}x^3+\ldots$ (verified).
9 (i)	
(ii)	$\pi\int_{-1}^{3}y^2\ \mathrm{d}x=-2\pi\int_{\pi}^{0}\sin^3 t\ \mathrm{d}t$ $=8.38$ (3 s.f.)
(iii)	$\dfrac{x-1}{2}=\cos t$ $\cos^2 t=\dfrac{\left(x-1\right)^2}{2^2}$, $\sin^2 t=y^2$ Since $\sin^2 t+\cos^2 t=1$ $\dfrac{\left(x-1\right)^2}{2^2}+y^2=1$
(iv)	Stretch parallel to the y axis with stretch factor 2. Translate -1 unit in the direction of the x axis.
10 (a)(i)	$R_h=(a,\infty)$ $D_g=[a,\infty)$ Since $R_h\subseteq D_g$, gh exists.

(ii)	$\text{gh}(x)=\sqrt{x^2+a-a}$ $\text{gh}(x)=\sqrt{x^2}$ $\text{gh}(x)=\lvert x\rvert$ $\text{gh}(x)=-x$ since $x<0$ $\text{gh}: x\mapsto -x \quad x<0$
10(b) (i)	$\text{f}(x)=\dfrac{1}{1+2\sin x}, \quad \dfrac{\pi}{2}<x<\pi$ $\text{f}'(x)=\dfrac{-2\cos x}{(1+2\sin x)^2}$ Since $(1+2\sin x)^2>0$ and $0<-2\cos x<2 \quad \text{for } \dfrac{\pi}{2}<x<\pi$, $\text{f}'(x)=\dfrac{-2\cos x}{(1+2\sin x)^2}>0 \quad \text{for } \dfrac{\pi}{2}<x<\pi$ Thus $\text{f}(x)$ is an increasing function. **(Shown)**
(ii)	$y=\dfrac{1}{1+2\sin x}$ $1+2\sin x=\dfrac{1}{y}$ $x=\sin^{-1}\left(\dfrac{1}{2}\left(\dfrac{1}{y}-1\right)\right)$ $\text{f}^{-1}(x)=\sin^{-1}\left(\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)\right)$ $\text{D}_{\text{f}^{-1}}=\text{R}_\text{f}=\left(\dfrac{1}{3},1\right)$ 
11(a) (i)	$M_1=\{5\}$, $M_2=\{10, 15\}$ and $M_3=\{20, 25, 30\}$… M_n has n elements Hence, the number of elements in each set up to and including M_n is an AP with $a=1$, $d=1$ and n number of terms $S_n=\dfrac{n}{2}[2(1)+(n-1)(1)]=\dfrac{n}{2}(n+1)$ As the elements of the sets are multiples of 5, Last element of M_n = $5\times\dfrac{n}{2}(n+1)=\dfrac{5}{2}n(n+1)$ **(Proven)**

(a)(ii)	The elements in M_{n+1} follow an AP with first term $= \frac{5}{2}n(n+1)+5$, common difference = 5, and n+1 number of terms. Hence, sum of all the elements in M_{n+1} $=\frac{n+1}{2}\left(\frac{5n(n+1)}{2}+5+\frac{5(n+1)(n+2)}{2}\right)$ $=\frac{n+1}{2}\left(\frac{5(n+1)}{2}(2n+2)+5\right)$ $=\frac{5}{2}(n+1)(n^2+2n+2)$ or $\frac{5}{2}(n+1)\left[(n+1)^2+1\right]$
(b)(i)	It is a GP with r = 1.15 $a(1.15)^3=150$ $a=\frac{150}{1.15^3}=98.627$ $S_6=\frac{98.627(1.15^6-1)}{1.15-1}=863.35$ Hence, David will take 863s (or14 mins 23s) to run 2.4km.
(b)(ii)	The time taken for Tommy to run each round is equivalent to a GP with a = 100, r = 1.1 and n = 6. Time taken for Tommy to run 2.4km $=\frac{110(1.1^6-1)}{1.1-1}=848.72$ Since 848.72 + 60 > 863.35 , Tommy will not complete the 2.4km run before David.
12(i) &(ii)	$\lvert iz-1-12i\rvert=5$ $\lvert i\rvert\lvert z-12+i\rvert=5 \quad \Rightarrow \lvert z-(12-i)\rvert=5$ 

	 $\tan\theta = \frac{5}{12}, \ \Rightarrow \theta = 0.39491$ $BC = 12\sin(0.39491) \qquad \Rightarrow BC = 4.6167$ $AC = 12\cos(0.39491) \qquad \Rightarrow AC = 11.0764$ $z_1 = (12 - BC) + i(12 - AC)$ $z_1 = 7.38 + 0.924i$ $z_2 = (12 + BC) + i(12 - AC)$ $z_2 = 16.6 + 0.924i$ Alternatively, $\left(x-12\right)^2 + \left(y-12\right)^2 = 12^2 \quad --(1)$ $\left(x-12\right)^2 + \left(y+1\right)^2 = 5^2 \qquad --(2)$ $(1)-(2):$ $\left(y-12\right)^2 - \left(y+1\right)^2 = 144 - 25$ $y = \frac{12}{13}$ subst $y = \frac{12}{13}$ into (1): $x = \frac{96}{13} \ \text{ or } \ \frac{216}{13}$ $z_1 = \frac{96}{13} + \frac{12}{13}\mathrm{i} \quad \text{and} \quad z_2 = \frac{216}{13} + \frac{12}{13}\mathrm{i}$
(iii)	$a = 12, \ \ b = -10.2$
(iv)	$\min \arg\left(w - 30 - 12\mathrm{i}\right) = -\left[\pi - \tan^{-1}(12/18)\right]$ $= -2.55 rad$