AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **7 August 2023**

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

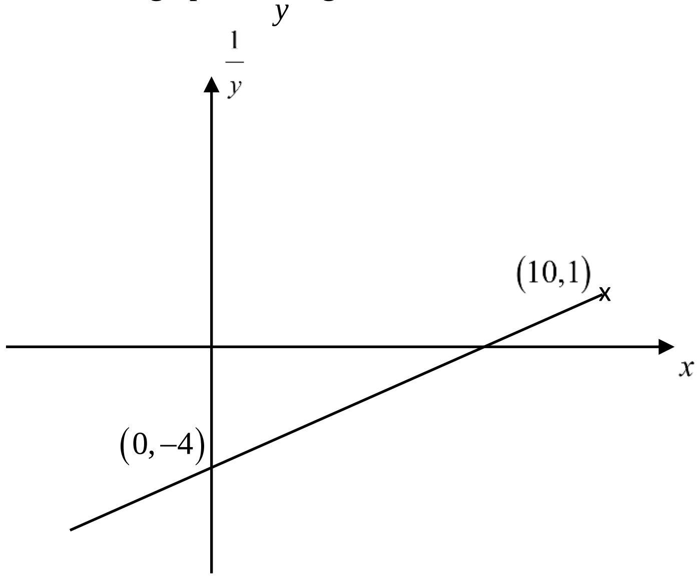
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The variables x and y are related by the equation $y = \dfrac{h}{2x-k}$. The diagram below shows the graph of $\dfrac{1}{y}$ against x.



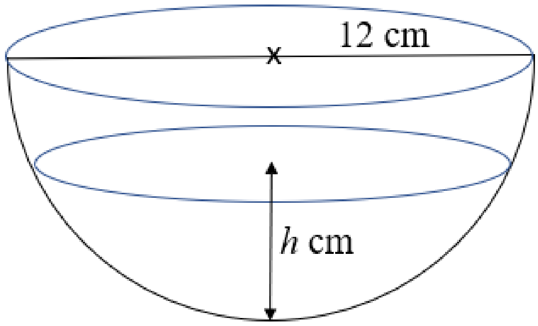
Calculate the value of h and of k. [4]

2 The Richter scale measures the intensity of an earthquake using the formula $M = \lg\left(\frac{I}{I_0}\right)$, where M is the magnitude of the earthquake, I is the intensity of the earthquake, and I_0 is the intensity of the smallest earthquake that can be measured.

(a) Calculate the magnitude of an earthquake if its intensity is 1000 times the intensity of the smallest earthquake that can be measured. [1]

(b) In February 2011, an earthquake with magnitude 6.2 was recorded in Christchurch, New Zealand. Few weeks later, an earthquake with magnitude 9.0 was detected in Fukushima, Japan. How many times stronger in intensity was the Japan's earthquake as compared to the New Zealand's earthquake? Give your answer to 2 decimal places. [3]

3 The diagram shows a hemispherical bowl of radius 12 cm. Water is poured into the bowl and at any time t seconds, the height of the water level from the lowest point of the hemisphere is h cm. The rate of change of the height of the water level is 0.4 cm/s.



(a) Show that the area of the water surface, A, is given by $A = \pi h(24 - h)$. [2]

(b) Find the rate of change of A when $h = 5$ cm.
Leave your answer in terms of π. [3]

4 **(a)** Explain why there is only one solution to the equation $\log_5\left(13-4x\right)=\log_{\sqrt{5}}\left(2-x\right)$. [5]

(b) Solve the simultaneous equations

$$4^{x+3} = 32\left(2^{x+y}\right) ,$$

$$9^{x} + 3^{y} = 10 .$$

[7]

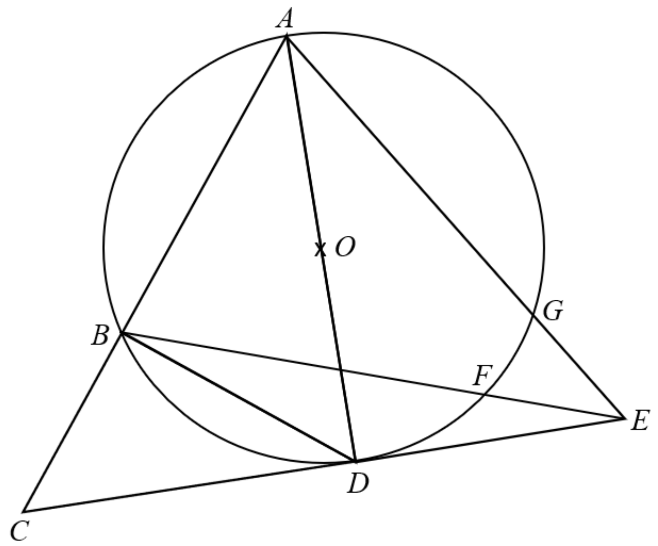
5 **(a)** Prove the identity $\cot 2x = \dfrac{1}{2\tan x} - \dfrac{1}{2}\tan x$. [2]

(b) Hence solve the equation $\tan x\left(3 - 4\cot 2x\right) = 3$ for $0° \le x \le 360°$. [5]

(c) Without further solving, explain why there are 6 roots to the equation $\tan\dfrac{x}{2}\left(3 - 4\cot x\right) = 3$ for $-360° \le x \le 720°$. [2]

6 The curve $y = e^{2x}\sqrt{1-3x}$ intersects the y-axis at the point P. The tangent and the normal to the curve at P meet the x-axis at A and B respectively. Find the exact area of triangle PAB. [7]

7 In the diagram, *CE* is a tangent that touches the circle of centre *O* at *D*.
AD is the diameter of the circle, *EA* cuts the circle at points *G* and *A*, and *EB* cuts the circle at points *F* and *B*.



(a) Given that *ABC* is a straight line, show that triangle *ABD* and triangle *DBC* are similar. [3]

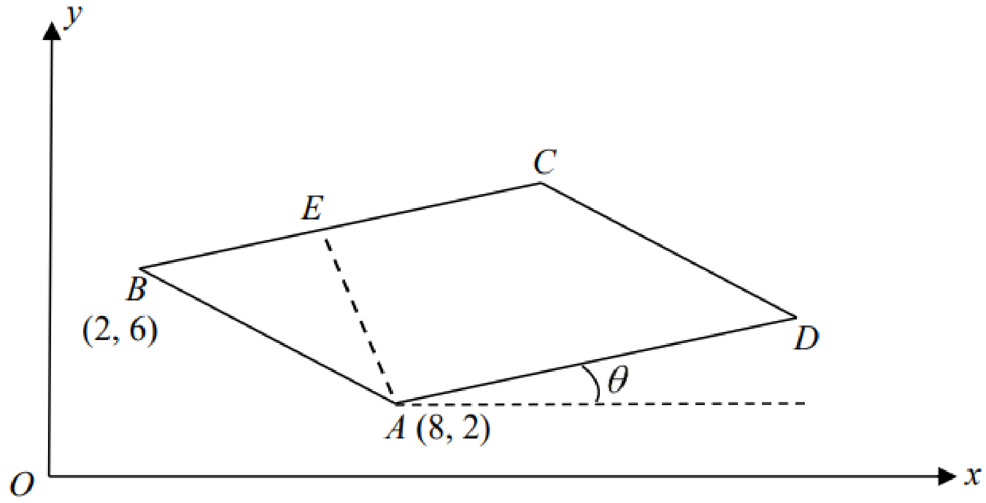
(b) If $BE = AE$, show that $EF = EG$. [4]

8 **(a)** Write down the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{4}\right)^n$, where n is a positive integer greater than 2. [3]

(b) The first two terms in the expansion, in ascending powers of x, of $(1+x)^2\left(2-\frac{x}{4}\right)^n$ are $a+bx^2$, where a and b are constants.
Find the value of n. [3]

(c) Hence find the value of a and of b. [3]

9 The diagram shows a parallelogram $ABCD$ in which the coordinates of the points A and B are (8, 2) and (2, 6) respectively. The line AD makes an angle θ with the horizontal and $\tan\theta = 0.5$. The point E lies on BC such that AE is the shortest distance from A to BC.



(a) Show that the equation of line BC is $2y = x + 10$. [2]

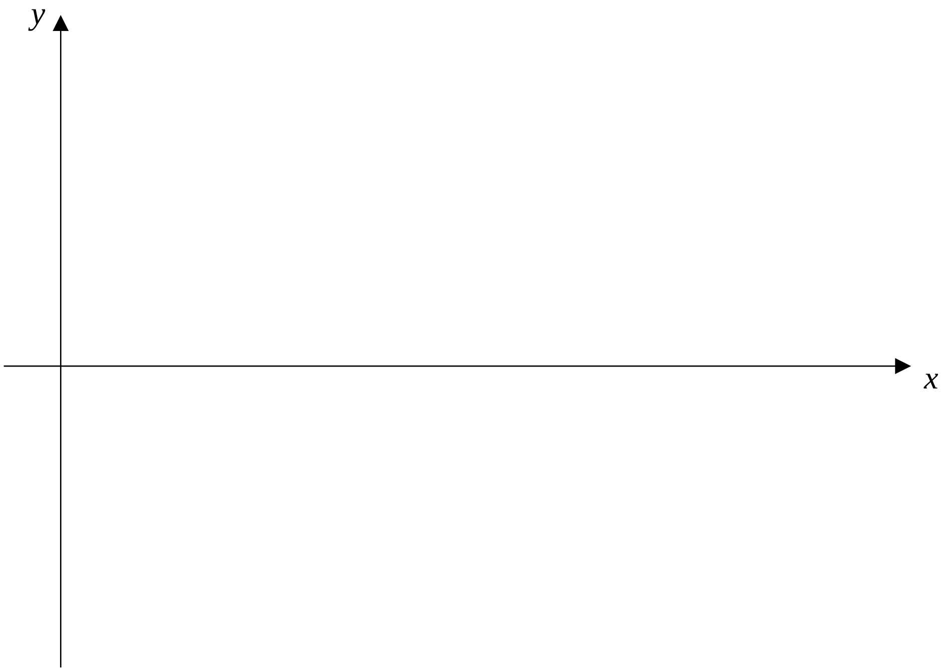
(b) Find the equation of line AE and the coordinates of E. [3]

(c) Given that $\frac{BE}{BC} = \frac{1}{5}$, find the coordinates of C and D. [4]

(d) Find the area of the figure $OBEA$, where O is the origin. [2]

10 **(a)** Solve the equation $2\cos 3x + 1 = 0$ for $0 \le x \le \pi$. [3]

(b) Sketch the graph of $y = 2\cos 3x + 1$ for $0 \le x \le \pi$. [3]



(c) The equation of a curve is $y = \dfrac{\sin 3x}{2 + \cos 3x}$, where $0 \le x \le \pi$.

Using **(a)** and **(b)**, find the range of values of x for which y is a decreasing function. [5]

11 **(a)** Express $\dfrac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}$ in partial fractions. [5]

(b) Differentiate $\ln\left(x^2+4\right)$ with respect to x. [2]

(c) The gradient function of a curve is $\dfrac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}$.

Given that the y-intercept of the curve is $\left(0, \ln 4\right)$, using part **(a)** and **(b)**, find the equation of the curve. [4]

END OF PAPER