Statistics Tutorial 1: Permutations and Combinations

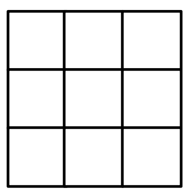
Additional Practice Questions

1. (a) In how many ways can five copies of a book be distributed among ten people if no one gets more than one copy?

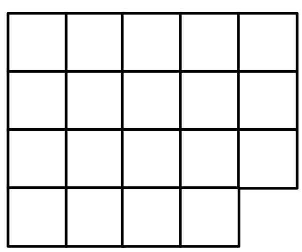
   (b) In how many ways can five different books be distributed among ten people if each person can get any number of books?

> (a) Assuming the books are identical, this is equivalent of choosing 5 people from 10 who get a book.
>
> $$\binom{10}{5} = 252$$
>
> (b) Each book has 10 choices of person it could go to.
>
> $$10^5 = 100\ 000$$

2. (a) How many squares are there in the grid below:



   (b) How many squares are there in the grid below:



> (a) Split into cases.
> 1x1 squares: 3x3
> 2x2 squares: 2x2
> 3x3 squares: 1x1
> Total number of squares = 14
>
> (b) Consider:
> Number of squares in 4x5 block – number of squares involving bottom right tile.
> Number of squares in a 4x5 block:
> 1x1 squares: 4x5

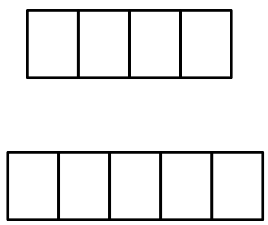
2x2 squares: 3x4
3x3 squares: 2x3
4x4 squares: 1x2
Squares using bottom right tile = 1 (1x1), 1 (2x2), 1 (3x3), 1 (4x4)
Number of squares = 40 – 4 = 36

3. **SRJC/2012/H2 Prelim Paper II/Q6**

(a) Find the number of ways in which 6 different pens can be distributed among 5 students such that each student can receive any number of pens. [1]

(a) Number of ways
$= 5^6$
$= 15625$

(b) A fleet of 9 different cars are to be parked at a reserved bay of 9 identical lots of which 4 lots are on one side and 5 lots are on the other.



Find the number of possible parking arrangements of these 9 cars

(i) if two particular cars cannot be parked side by side. [3]

(b)(i) Number of parking arrangements

$= 9! - (\text{cases where the two cars are taking the side with 4 lots})$
$- (\text{cases where the two cars are taking the side with 5 lots})$

$= 9! - {}^7C_2 3!2!5! - {}^7C_3 4!2!4! = 292320$

(ii) if two particular cars can only be parked at corner lots. [2]

(ii) Number of parking arrangements

$= {}^4C_2(2!)(7!) = 60480$

4. **ACJC/2012/Prelim Paper II/Q6**

Four boys and three girls are at a playground.

(a) In one of their games, all seven of them have to stand in a straight line.

(i) If all the girls are separated, find the number of ways in which this can be done. [2]

$4! \times {}^5P_3 = 24 \times 60 = 1440$

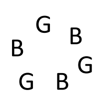
Or $4! \times ({}^5C_3 \times 3!) = 24 \times 60 = 1440$

(ii) If all the boys are to be together, find the number of ways in which this can be done. [2]

$4! \times 4! = 576$

(b) In another game, only six of them can play at a time and the six players have to stand in a circle with all the girls separated. Find the number of ways in which this can be done. [3]

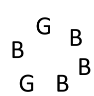
Case (i): 3 boys and 3 girls



${}^4C_3 \times (3-1)! \times 3!$

$= 4 \times 2 \times 6 = 48$

Case (ii): 4 boys and 2 girls



$(4-1)! \times {}^3C_2 \times {}^4P_2$

$= 6 \times 3 \times 12$

$= 216$

Or $(4-1)! \times {}^3C_2 \times {}^4C_2 \times 2! = 216$

Total number of arrangements = 48 + 216 = 264

5. **MI/2012/Prelim Paper II/Q5**

(a) An interior designer is designing a tile pattern of 3 blue, 3 red, 3 green, 1 yellow and 1 purple tiles in a line. All the tiles are identical except for the colour. Find the number of different possible ways to arrange the tiles if

(i) the green, yellow and purple tiles must be placed next to each other, [3]

Consider the green, yellow and purple tiles as 1 unit. Number of ways to arrange the green, yellow and purple tiles within the unit

$= \frac{5!}{3!}$

Arranging the unit with the rest of the tiles

$= \frac{7!}{3!3!}$

Thus number of possible arrangements for the tiles

$= \frac{5!}{3!} \times \frac{7!}{3!3!}$

$=2800$

(ii) no blue tiles are placed next to each other, [3]

Number of ways to arrange the 3 red, 3 green, 1 yellow and 1 purple tile = $\frac{8!}{3!3!}$

Number of ways to slot in the blue tiles = $\binom{9}{3}$

Number of possible arrangements such that no blue tiles are placed next to another

$= \frac{8!}{3!3!} \times \binom{9}{3}$

$=94080$

(iii) a red tile to be placed at the beginning and at the end of the line. [3]

Number of possible arrangements such that a red tile at the beginning and another red tile at the end of the line

$= \frac{(3+3+3)!}{3!3!} = 10080$

(b) A group of 10 people consists of 9 men and 1 woman. Find the number of ways which the group can be seated at a round table with identical chairs if

(i) there is no restriction, [1]

Number of ways = $(10-1)! = 362880$

(ii) 2 particular men, Caleb and James, do not want to sit beside the woman, but will like to sit together. [3]

Number of ways $= (7-1)! \times {}^7C_2 \times 2! \times 2! = 60480$

The chairs at the table are replaced with 10 chairs of different colours.

(iii) Find the number of ways which the group can be seated at a round table if Caleb and James must still sit together, but they need not be separated from the woman. [3]

Number of ways $= (9-1)! \times 10 \times 2! = 806400$

6. **JJC/2012/Prelim Paper II/Q6**

Find the number of ways in which 4-letter code-words can be obtained from the word ENDANGERED if

(i) there are no repeated letters, [1]

ENDANGERED: 3E, 2N, 2D, 1A, 1G, 1R

No of 4-letter code-words from E, N, D, A, G, R

$= {}^6C_4 \times 4!$
$= 360$

(ii) there are three "E's" [2]

Select 1 letter from N, D, A, G, R.

No of 4-letter code-words with the chosen letter and 3 "E"s
$= {}^5C_1 \times \frac{4!}{3!}$ = 20

(iii) there is at least one repeated letter. [3]

Case I: 3 same letters

No. of such code-words = 20

Case II: 1 pair of same letters

No. of such code-words $= {}^{3}C_1 \times {}^{5}C_2 \times \dfrac{4!}{2!} = 360$

Case III: 2 pairs of same letters

No. of such code-words $= {}^{3}C_2 \times \dfrac{4!}{2!2!} = 18$

No. of 4-letter code-words that contain at least 1 repeated letter
$= 20 + 360 + 18 = 398$

7. **NYJC/2013/H2 Prelim Paper II/Q9**

Find the number of ways in which the letters of the word **SYSTEMATIC** can be arranged if

(i) there are no restrictions, [1]

(ii) the two 'T's must not be next to each other, [2]

(iii) there must be exactly 3 letters between the two 'T's, [2]

(iv) the first letter is 'Y' and the last letter is a consonant. [3]

Solution:

<table>
<tr><td>(i)</td><td>$\dfrac{10!}{2!2!} = 907200$</td></tr>
<tr><td>(ii)</td><td><u>Method 1</u> (Complementation)
$\begin{pmatrix}\text{No of ways}\\ \text{without restriction}\end{pmatrix} - \begin{pmatrix}\text{No of ways the two Ts}\\ \text{are next to each other}\end{pmatrix}$
$= 907200 - \dfrac{9!}{2!}$
$= 725760$

<u>Method 2</u> (Insertion method)
$\dfrac{8!}{2!} \cdot \dbinom{9}{2} = 725760$</td></tr>
<tr><td>(iii)</td><td>X X X X X X X X
T T
T T
T T
T T
T T
T T
(Insertion method)
$\dfrac{8!}{2!} \cdot 6 = 120960$</td></tr>
</table>

(iv)	Case 1: Last letter is S or T No of ways $=2\cdot\frac{8!}{2!}=40320$ Case 2: Last letter is M or C No of ways $=2\cdot\frac{8!}{2!2!}=20160$ Total number of ways $=60480$ **<u>Alternative method</u>** **Y _ _ _ _ _ _ _ _ _** No of ways $=\binom{6}{1}\cdot\frac{8!}{2!2!}=60480$

8. **TJC/2013/H2 Prelim Paper II/Q7**

(a) (i) At a wedding dinner, 6 men and 4 women are to be seated at a round table with 10 identical seats. Find the number of different arrangements if there are no restriction. [1]

(ii) At another round table, 2 of the 10 identical seats are each tied with a red ribbon and are adjacent to each other. How many ways can 7 people be seated at the table? [2]

(b) Eight students participate in the semi-final round of a Mathematics Quiz. They are randomly paired up to compete with each other and the winner of each pair will advance to the final round.

(i) Find the number of possible sets of results from the semi-final round. For example, one possible set of results is *A* beats *C*, *B* beats *H*, *D* beats *F*, and *E* beats *G*. [2]

(ii) How many ways can the prizes be awarded to the four finalists if there are one \$500, one \$200 and two \$100 book prizes? [1]

Solution:

(a) (i) No of ways = (10 – 1)! = 9! =362880

(ii) No of ways = ${}^{10}P_7$ = 604800

(b) **(i)** Number of possible sets of results = $\frac{{}^{8}C_2\times{}^{6}C_2\times{}^{4}C_2\times{}^{2}C_2}{4!}\times 2^4=1680$

(Split the students into pair groupings first, then decide on the results)

OR

Number of possible sets of results = $4!\binom{8}{4}=1680$

(There are $\binom{8}{4}$ ways of choosing 4 winners. And there are 4! ways of matching these winners to losers.

(ii) Number of ways to award the four finalists = $\frac{4!}{2!} = 12$

9. **RI/2018/MYE/Q9**

The eleven letters in the word INSTITUTION are individually printed on eleven identical cards.

(i) The eleven cards are arranged in a line.

(a) Find the number of different arrangement of the eleven cards that can be made. [1]

(b) Find the number of different arrangements that can be made if S, U and O are separated from one another. [2]

(ii) Three cards are to be chosen from the eleven cards; the order in which they are chosen does not matter. Find the number of different possible selections of three cards. [3]

(i)(a)	11 letters with 3T, 3I, 2N and 3 other distinct letters. Number of ways = $\frac{11!}{3!3!2!} = 554\ 400$
(i)(b)	Arrange 8 letters I,N,T,I,T,T,I,N then slot S, U, O in 9 spaces. Number of ways = $\frac{8!}{3!3!2!} \times \binom{9}{3} \times 3! = 282\,240$ <u>ALTERNATIVELY</u> (Complement method – more tedious!) Total no. of ways without restrictions – No. of ways with S, U and O all together – No. of ways 2 of S, U, O together and the remaining separated $= 554\ 400 - \frac{9!}{3!3!2!} \times 3! - \binom{3}{2} \times \frac{8!}{3!3!2!} \times \binom{9}{2} \times 2! \times 2!$ $= 282\,240$
(ii)	Number of ways to choose three cards = Number of ways to choose three cards with either all three cards identical (T or I), 2 cards identical (from T, I or N) or all three cards are different (from I,N,S,T,U,O) $= \binom{2}{1} + \binom{3}{1}\binom{5}{1} + \binom{6}{3}$ $= 2 + 15 + 20$ $= 37$

10. **ACJC/2021/PRELIM/Q7**

A group of 12 students consists of 3 students from class *A*, 4 students from class *B* and 5 students from class *C*.

(i) Find the number of ways in which a committee of 8 students can be chosen from the 12 students if it includes at least 1 student from each class. [2]

(ii) The 12 students from the 3 classes sit at random at a round table. Albert is a student from class *A* and Bob is a student from class *B*. Find the the number of ways that Albert and Bob are seated together and no two students from class *A* are next to each other. [2]

(iii) Each of the 12 students attends one of 3 leadership programmes *X*, *Y* and *Z*. The table below shows the number of students from each class attending the various leadership programmes.

	Leadership Programmes		
	X	*Y*	*Z*
Class *A*	3	0	0
Class *B*	0	3	1
Class *C*	0	0	5

4 students are selected from the 12 students to participate in a group interview about the leadership programmes. They are arranged to sit in a row of 8 labelled seats such that there is exactly one empty seat between every 2 students as part of safe management measures.

Find the number of possible arrangements if each arrangement must include students from all 3 classes with representation from all 3 leadership programmes. [4]

(i)	No. of ways $= {}^{12}C_8 - {}^{9}C_8 - {}^{8}C_8$ $= 495 - 9 - 1$ $= 485$
(ii)	$(9-1)! \times 2 \times {}^{8}C_2 \times 2! = 4515840$
(iii)	Case 1: 2A, 1B, 1C No of choices of 4 students = ${}^{3}C_2 \times {}^{3}C_1 \times {}^{5}C_1$ (AX, AX, BY, CZ) $= 45$ Case 2: 1A, 2B, 1C No of choices of 4 students = ${}^{3}C_1 \times {}^{3}C_2 \times {}^{5}C_1$ (AX, BY, BY, CZ) $+ {}^{3}C_1 \times {}^{3}C_1 \times {}^{1}C_1 \times {}^{5}C_1$ (AX, BY, BZ, CZ) $= 45 + 45 = 90$ Case 3: 1A 1B 2C No of choices of 4 students = ${}^{3}C_1 \times {}^{3}C_1 \times {}^{5}C_2$ (AX, BY, CZ, CZ) $= 90$ Total no of arrangements = $(45 + 90 + 90) \times 4! \times 2 = 10800$