RAFFLES INSTITUTION
Yr 5-6 PHYSICS DEPARTMENT

Tutorial 10 OSCILLATIONS

H2 Physics 9749

Self - Check Questions

S1 What do you understand by the terms displacement, amplitude, period, frequency and angular frequency of a simple harmonic motion?

S2 Express the period T in terms of frequency f and angular frequency ω.

S3 Define *simple harmonic motion*. State the defining equation of simple harmonic motion.

S4 Write down a solution for a simple harmonic oscillator which starts its motion from the equilibrium position. How do you express its velocity and acceleration in terms of time?

S5 Draw graphs to show the changes in displacement, velocity and acceleration with respect to time of the oscillator in **S4**.

S6 Draw graphs to show how the velocity and acceleration of the oscillator in **S4** vary with displacement.

S7 Describe the variation in between kinetic and potential energy with time during simple harmonic motion of a mass attached to a horizontal spring. Assume the mass is moving on a frictionless surface. What is the frequency of the energy variation as compared with that of the vibration itself?

S8 Give one practical example of a lightly damped oscillation. Why is critical damping in a car suspension system important?

S9 What do you understand by forced oscillations and resonance?

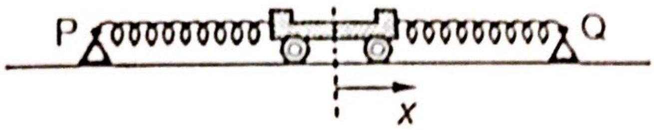
S10 Sketch a set of graphs, using the same axes, to show how the amplitude of forced oscillation varies with driving frequency for very light, moderate and heavy damping. Explain the features of your graphs.

S11 Describe two examples of resonance, one in which this phenomenon is useful and the other in which it is a nuisance.

Self - Practice Questions

SP1 A trolley of mass 2 kg with free-running wheels is attached to two fixed points P and Q by two springs under tension as shown in the figure below.

The trolley is displaced a small distance (0.05 m) towards Q by a force of 10 N and is then released. The equation of the subsequent motion is $\ddot{x} = -\omega^2 x$, where x is the displacement from the equilibrium position. What is the constant ω^2?



(Note: velocity = $\dot{x}$, acceleration = $\ddot{x}$)

A 0.25 rad^2 s^{-2}
B 1.0 rad^2 s^{-2}
C 4.0 rad^2 s^{-2}
D 100 rad^2 s^{-2}
E 400 rad^2 s^{-2}

J82/II/9

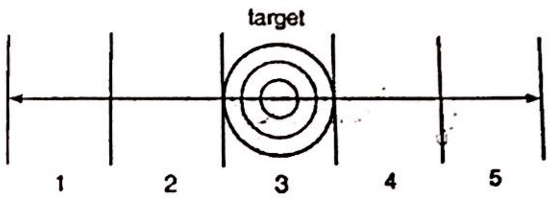
SP2 In which of the following lists are all three quantities constant when a particle moves in undamped simple harmonic motion?

A	acceleration	force	total energy
B	amplitude	angular frequency	acceleration
C	angular frequency	acceleration	force
D	force	total energy	amplitude
E	total energy	amplitude	angular frequency

J85/I/8 ; J92/I/9

SP3 In a fairground shooting game, a gun fires at a moving target. The gun fires by itself at random times. The player has to point the gun in a fixed direction, and the target moves from side to side with simple harmonic motion.

At which region should the player take a fixed aim in order to score the greatest number of hits?

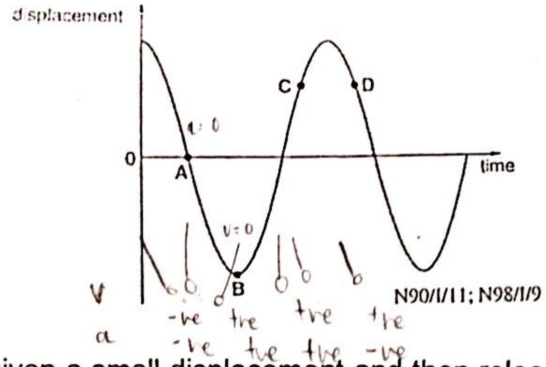


A 3
B either 1 or 5
C either 2 or 4
D any of 1, 2, 3, 4 and 5

J90/I/11 ; N95/I/9

SP4 The diagram shows the graph of displacement against time for a body performing simple harmonic motion.

At which point are the velocity and acceleration in opposite directions?

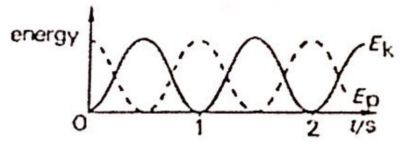


N90/I/11 ; N98/I/9

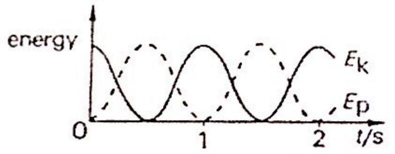
SP5 The bob of a simple pendulum of period 2 s is given a small displacement and then released at time $t = 0$.

Which diagram shows the variations with time of the bob's kinetic energy E_k and its potential energy E_p?

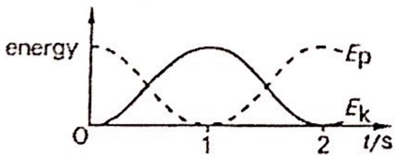
A



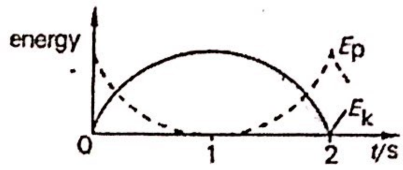
B



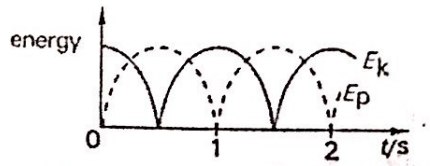
C



D



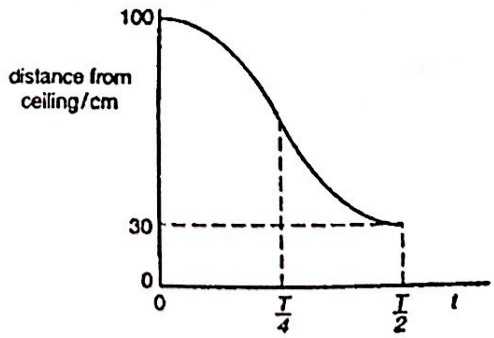
E



N92/I/9

SP6 A mass hanging from a spring suspended from a ceiling is pulled down and released. The mass then oscillates vertically with simple harmonic motion of period T. The graph shows how its distance from the ceiling varies with time t.

What can be deduced from this graph?



A The amplitude of the oscillation is 70 cm.

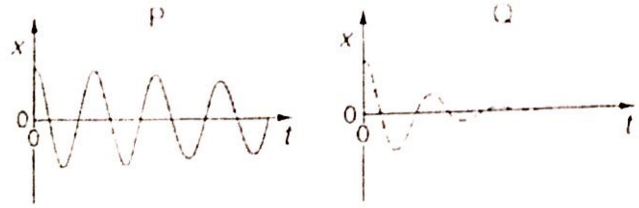
B The kinetic energy is a maximum at $t = \frac{T}{2}$.

C The restoring force on the mass increases between $t = 0$ and $t = \frac{T}{4}$.

D The speed is a maximum at $t = \frac{T}{4}$.

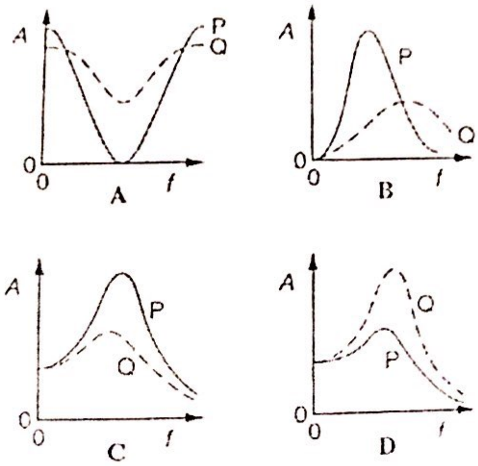
J2000/I/9

SP7 Two objects P and Q are given the same intial displacement and are then released. The graphs show the variation with time t of their displacements x.



P and Q are then subjected to driving forces of the same constant amplitude and of variable frequency f.

Which graph represents the variation with f of the amplitudes A of P and of Q?



J86/I/8 ; N93/I/7 ; J99/I/9

SP8 The displacement of a particle P which moves with simple harmonic motion can be described by the expression

$$x = (0.05) \sin 8\pi t$$

where x is in metres and t in seconds.

(a) What is the amplitude of the motion?

(b) What is the frequency of the motion?

(c) How long does it take for the particle to complete one oscillation?

(d) What is the velocity of the particle as it passes through its equilibrium position, and at the extreme end of the swing?

(e) What is the maximum acceleration of the particle during its motion?

Another particle Q also moves with simple harmonic motion of the same frequency. However, the motion of Q *lags* that of P by $\pi/2$ rad and the amplitude of Q is twice that of P.

Draw, using the same axes, the displacement-time graphs for motions of P and Q.

Write an equation to describe how the displacement of Q varies with time.

SP9 Discuss the energy changes which take place when a mass suspended from a spring is pulled downwards and released, such that it oscillates vertically.

Discussion Questions

D1 A light spring stretches 0.150 m when a 0.300 kg mass is hung from its lower end. The mass is pulled down 0.100 m below this equilibrium point and released. Determine

(a) the spring constant

(b) the amplitude of the oscillation

(c) the maximum velocity

(d) the magnitude of velocity when the mass is 0.050 m from equilibrium

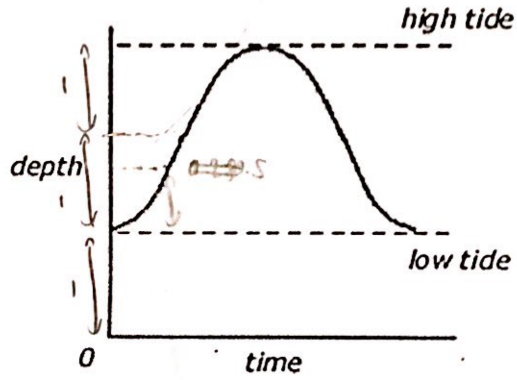
(e) the magnitude of the maximum acceleration of the mass

practical: don't extend more than e!

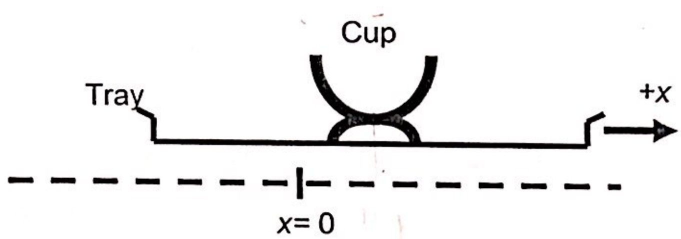
D2 **[J91/I/9]**

The rise and fall of water in a harbour is simple harmonic. The depth varies between 1.0 m at low tide and 3.0 m at high tide. The time between successive low tides is 12 hours.

A boat which requires a minimum depth of water of 1.5 m, approaches the harbour at low tide. How long will the boat have to wait before entering?



D3 A tray, holding an empty cup, is moved horizontally back and forth in simple harmonic motion. At one instant of time, the tray is displaced to the right of the equilibrium position ($x = 0$) as indicated by the arrow shown in the figure below.



(a) Draw the frictional force F acting on the cup for the instant of time shown.

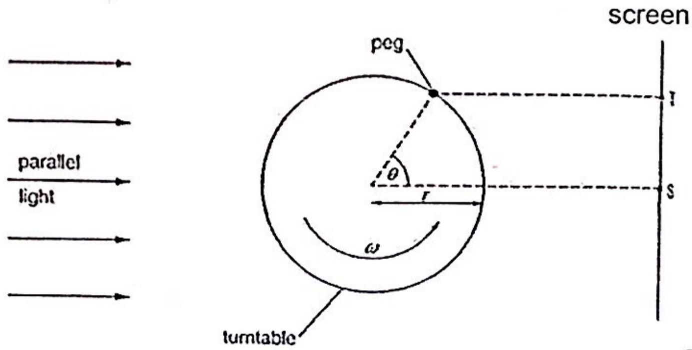
(b) Write an equation for F in terms of the mass m of the cup, the angular frequency ω of the motion, and the displacement x of the tray.

(c) Given that the maximum value of F is half the weight of the cup, explain why the cup will be observed to slip if the frequency of oscillation increases beyond a certain value.

(d) If the amplitude of the motion is 0.050 m, calculate the maximum possible frequency such that the cup does not slip.

D4 **[N96/II/2]**

A vertical peg is fixed to the rim of a horizontal turntable of radius r, rotating with a constant angular speed ω as shown in the figure.



Parallel light is incident on the turntable so that the shadow of the peg is observed on a screen which is normal to the incident light. At time $t = 0$, $\theta = 0$ and the shadow of the peg is seen at S. At some later time t, the shadow is seen at T.

(a) **(i)** Write down an expression for θ in terms of ω and t.

(ii) Derive an expression for the distance ST in terms of r, ω and t.

(b) By reference to your answer to **(a)(ii)**, describe the motion executed by the shadow on the screen.

explaining ✓
3m: (1) SHM
(2) double differentiation
(3) show ~~...~~ $a \propto x$

(c) The turntable has a radius r of 20 cm and an angular speed ω of 3.5 rad s^{-1}. Calculate, for the motion of the shadow on the screen,

(i) the amplitude,

(ii) the period,

(iii) the speed of the shadow as it passes through S,

(iv) the magnitude of the acceleration of the shadow when the shadow is instantaneously at rest.

D5 **[N03/II/4]**

(a) Define *simple harmonic motion*.

(b) A horizontal metre rule is clamped at one end. The free end oscillates vertically as shown in Fig. 1

Fig. 1

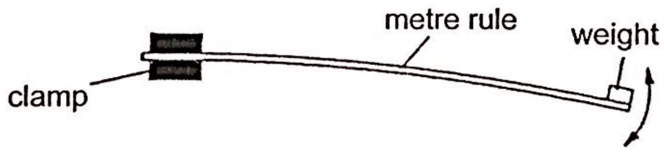


Fig. 2 shows the variation with time t of the velocity v of a point at the free end of the rule.

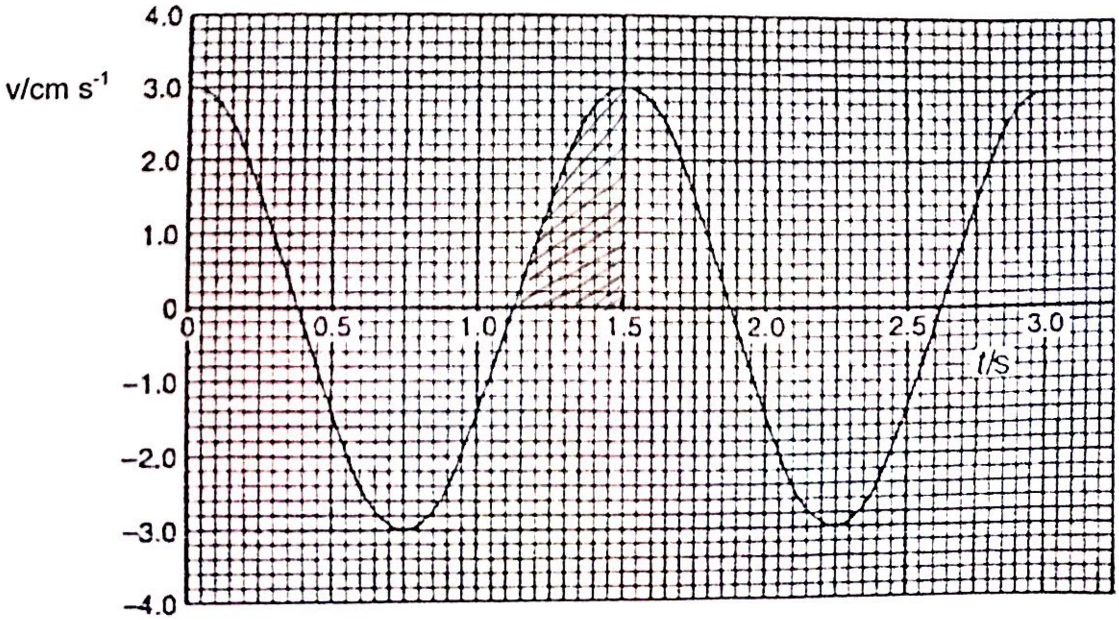


Fig. 2

(i) On Fig. 2, shade an area that represents the amplitude of the oscillations of the free end of the rule.

(ii) Determine, for these oscillations,

1. the frequency,
2. the amplitude.

(iii) On the axes of Fig. 3, sketch a graph to show the variation with displacement *d* of the velocity *v* of the end of the rule. Mark a scale on the d-axis.

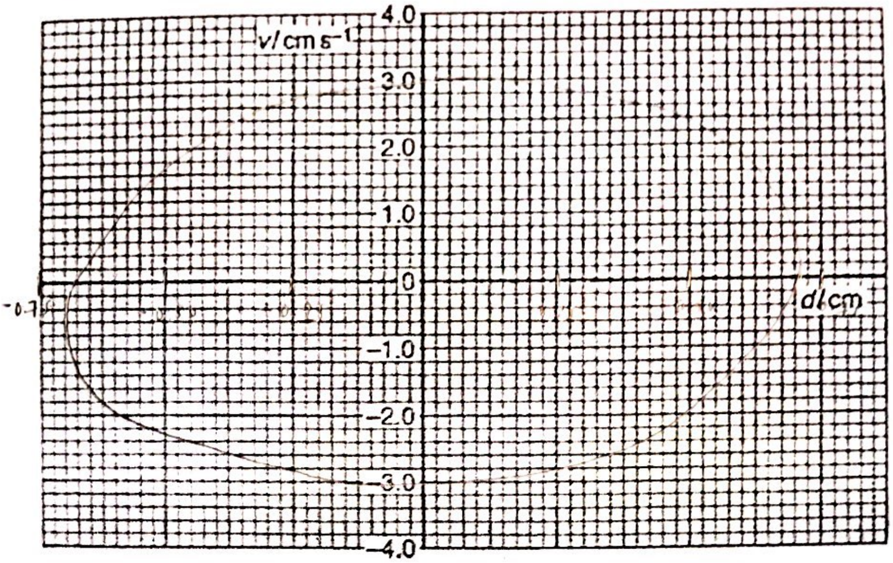
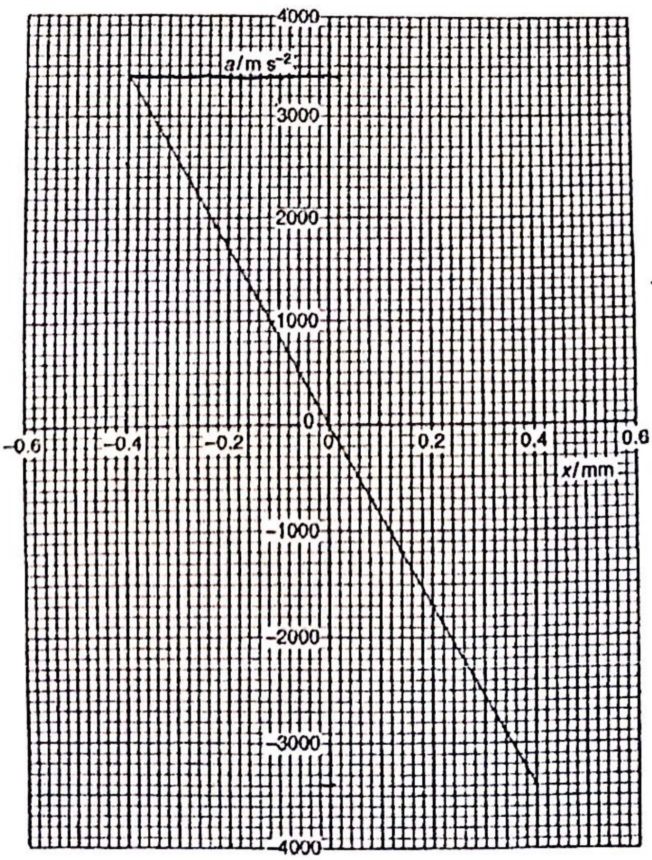


Fig. 3

D6 **[N06/II/3]**



The figure shows the variation with displacement x of the acceleration a of a particle P attached to the cone of a loudspeaker.

(a) Use the figure to

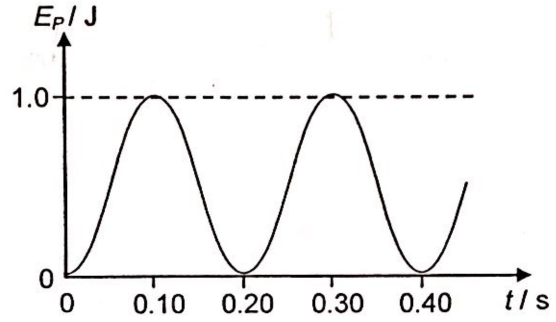
(i) explain why the motion of particle P is simple harmonic,

(ii) show that the frequency of oscillations of particle P is 460 Hz.

(b) (i) The magnitude of the gradient of the line in the figure is G. Show that, for a particle of mass m oscillating with amplitude A, its maximum kinetic energy E_{MAX} is given by $E_{MAX} = \frac{1}{2}mGA^2$.

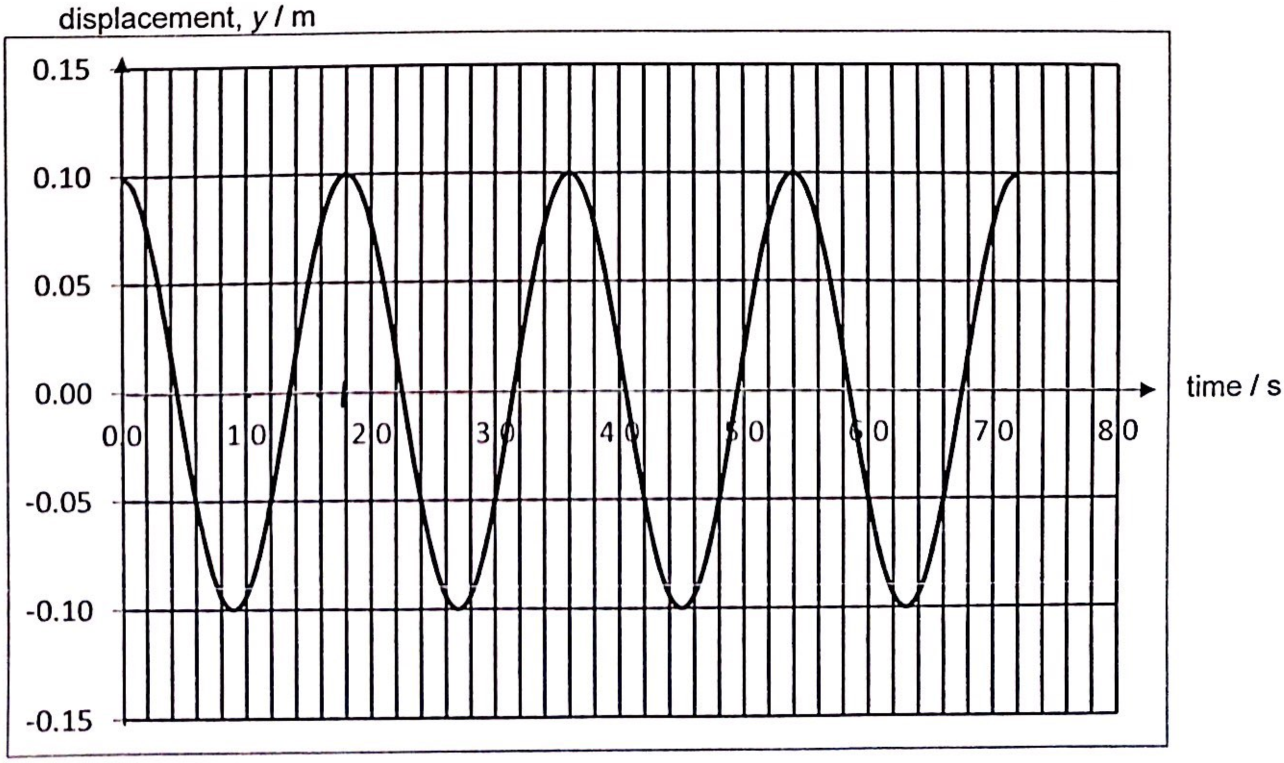
(ii) Determine E_{MAX} for particle P of mass 2.5×10^{-3} kg.

D7 An object undergoes simple harmonic motion with an amplitude of 0.30 cm. The graph shows the variation of its potential energy E_P with time t.



What is the maximum acceleration and mass of the object?

D8 An object undergoing a forced oscillation has displacement y, as shown.
Use the graph to determine the amplitude, period and angular frequency of this oscillation.



State, for each of the following, a time at which the oscillating object has

(a) maximum positive velocity,

(b) maximum positive acceleration,

(c) maximum negative acceleration,

(d) maximum kinetic energy,

(e) maximum potential energy.

D9 [J97/II/2]

Fig. 1 illustrates a mass which can be made to vibrate vertically between two springs.

The vibrator itself has constant amplitude. As the frequency is varied, the amplitude of vibration of the mass is seen to change as shown in Fig. 2.

Fig. 1

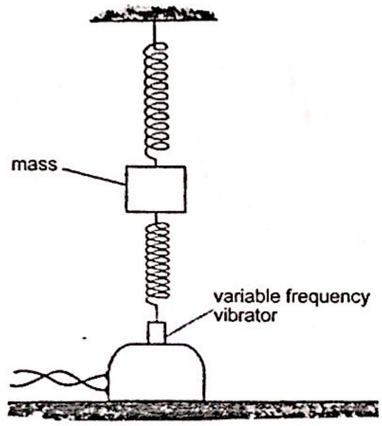
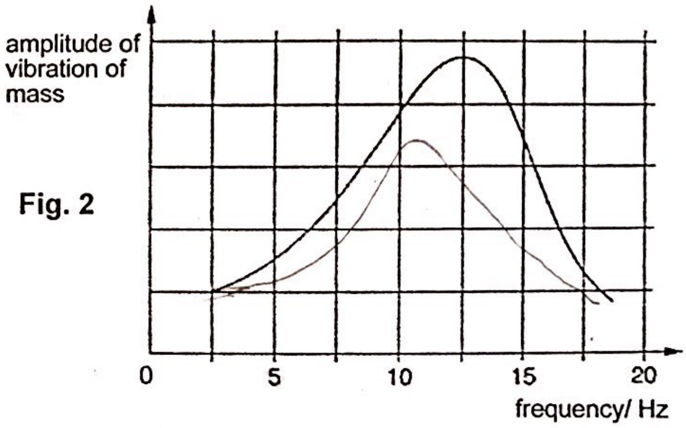


Fig. 2



(a) Name the phenomenon which is illustrated in Fig. 2.

(b) For the mass vibrating at maximum amplitude, calculate

 (i) the angular frequency,

 (ii) the period.

(c) A light piece of card is fixed to the mass with its plane horizontal. On Fig. 2, draw a line to show the variation with frequency of the amplitude of vibration of the mass.

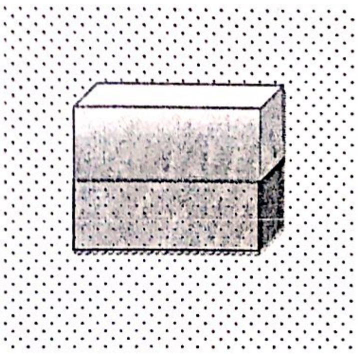
(d) State one situation in which the phenomenon illustrated in Fig. 2 is used to advantage.

D10 [N94/II/2]

A block of wood of mass m floats in still water as shown in the figure. When the block is pushed down into the water, without totally submerging it, and is then released, it bobs up and down in the water with a frequency f given by the expression:

$$f = \frac{1}{2\pi}\sqrt{\frac{28}{m}}$$

where f is measured in Hz and m in kg.



Surface water waves of speed 0.90 m s^{-1} and wavelength 0.30 m are then incident on the block. These cause resonance in the up-and-down motion of the block.

(a) Explain what is meant by the term *resonance*.

(b) Calculate

(i) the frequency of the water waves,

(ii) the mass of the block.

(c) Describe and explain what happens to the amplitude of the vertical oscillations of the block after the following changes are made independently:

(i) water waves of larger amplitude are incident on the block,

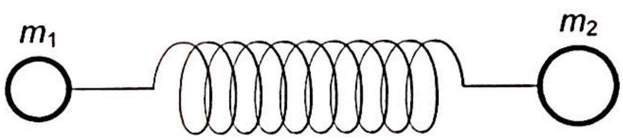
(ii) the distance between the wave crests increases,

(iii) the block has absorbed some water.

Note: for a wave, $v = f\lambda$ where v is the speed, f is the frequency and λ is the wavelength

Challenging Questions

C1 The figure below shows an isolated oscillatory system. Two bodies of mass m_1 and m_2 are joined by a light spiral spring. Each body oscillates along the axis of the spring, which obeys Hooke's law in both extension and compression.



(a) The bodies move in opposite directions and the centre of mass of the system is stationary. Explain why the periods of oscillations of both bodies are the same.

(b) Show that when the body on the left moves through a distance x, the change in length of the spring is:

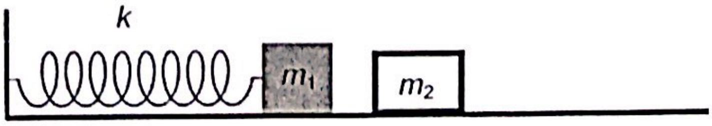
$$x\left(1+\frac{m_1}{m_2}\right)$$

(c) Hence, show that its period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m_1 m_2}{(m_1+m_2)k}}$$

where k is the spring constant of the spring.

C2 Two masses slide on a frictionless table. Mass m_1, but not m_2, is fastened to a spring. If now m_1 and m_2 are pushed to the left so that the spring is compressed a distance x, show that the amplitude of the oscillation of m_1 after the spring system is $x\sqrt{\frac{m_1}{m_1+m_2}}$.



Suggested Answers

SP1. D **SP2.** E **SP3.** B **SP4.** C **SP5.** A **SP6.** D **SP7.** C

D1.	19.6 N m^{-1}, 0.100 m, 0.808 m s^{-1}, 0.700 m s^{-1}, 6.54 m s^{-2}	**D6.**	1.7×10^{-3} J
D2.	2.0 h	**D7.**	0.74 m s^{-2}, 900 kg
D3.	1.58 Hz	**D8.**	1.35 s, 3.15 s, 4.95 s 0.90 s, 2.70 s, 4.50 s 0.00 s, 1.80 s, 3.60 s 1.35 s, 3.15 s, 4.95 s 0.00 s, 1.80 s, 3.60 s or 0.90 s, 2.70 s, 4.50 s
D4.	20 cm, 1.80 s, 0.700 m s^{-1}, 2.45 m s^{-2}	**D9.**	78.5 rad s^{-1}, 0.0800 s
D5.	0.6667 Hz, 0.72 cm	**D10.**	3.0 Hz, 0.079 kg

Solutions

Self - Check Questions

S1 displacement, x – distance in a specific direction from the equilibrium position

amplitude, x_o – the magnitude of maximum displacement from equilibrium position

period, T – time taken to complete one oscillation

frequency, f – number of oscillations per unit time

angular frequency, ω – rate of change of phase angle, it is equal to the product of 2π and frequency

S2 $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$

S3 Simple harmonic motion is defined as the motion of a particle about a fixed point such that its acceleration is proportional to its displacement from the fixed point and is always directed towards the point.

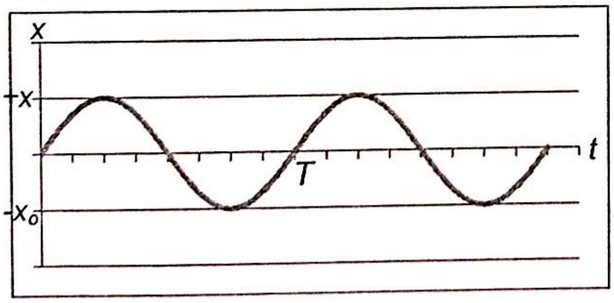
The defining equation is $a = -\omega^2 x$.

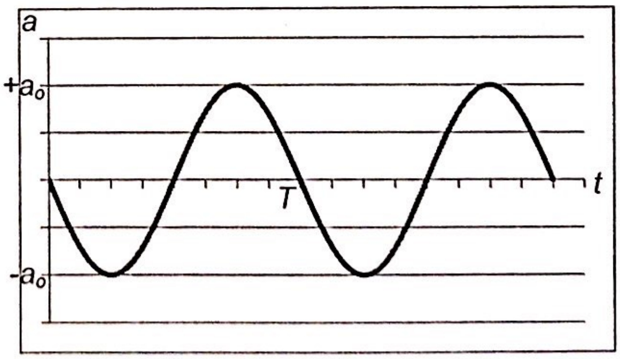
S4 $x = x_o \sin \omega t$

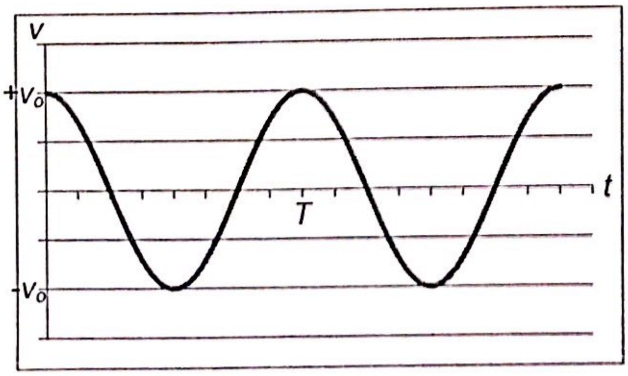
$v = x_o \omega \cos \omega t = v_o \cos \omega t$, where $v_o = x_o \omega$ is the maximum velocity

$a = -x_o \omega^2 \sin \omega t = -a_o \sin \omega t$, where $a_o = x_o \omega^2$ is the maximum acceleration

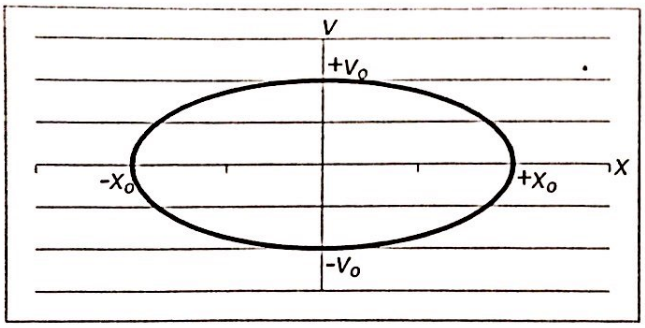
S5

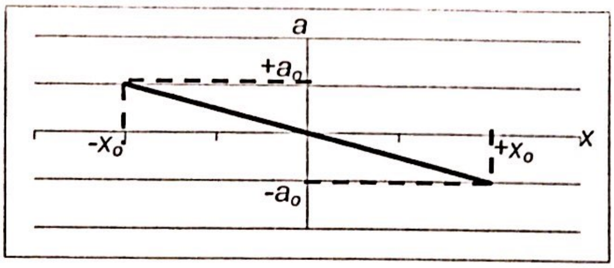




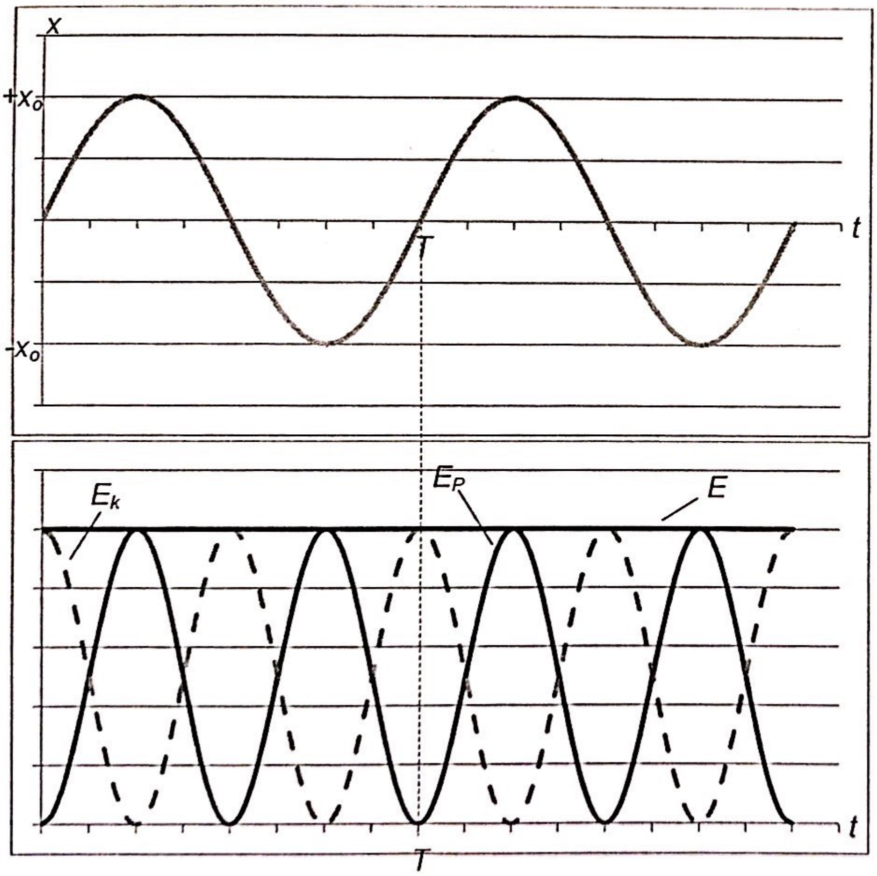


S6





S7



There is a continual change of energy from kinetic energy E_k to potential energy E_P and *vice-versa*. E_k is greatest and E_P is zero as the mass passes through the equilibrium position.

As the mass approaches the endpoints, its E_k decreases (E_P increases).

As the mass approaches the equilibrium point, its E_k increases (E_P decreases).

The frequency of energy variation is <u>twice</u> that of the oscillation itself.
(At any instant during the motion, the total energy E of the system is constant and equal to the sum of E_k and E_P. Note that E is proportional to x_o^2 and f^2.)

S8 A swinging pendulum or vibrating tuning fork are examples of lightly damped oscillations.

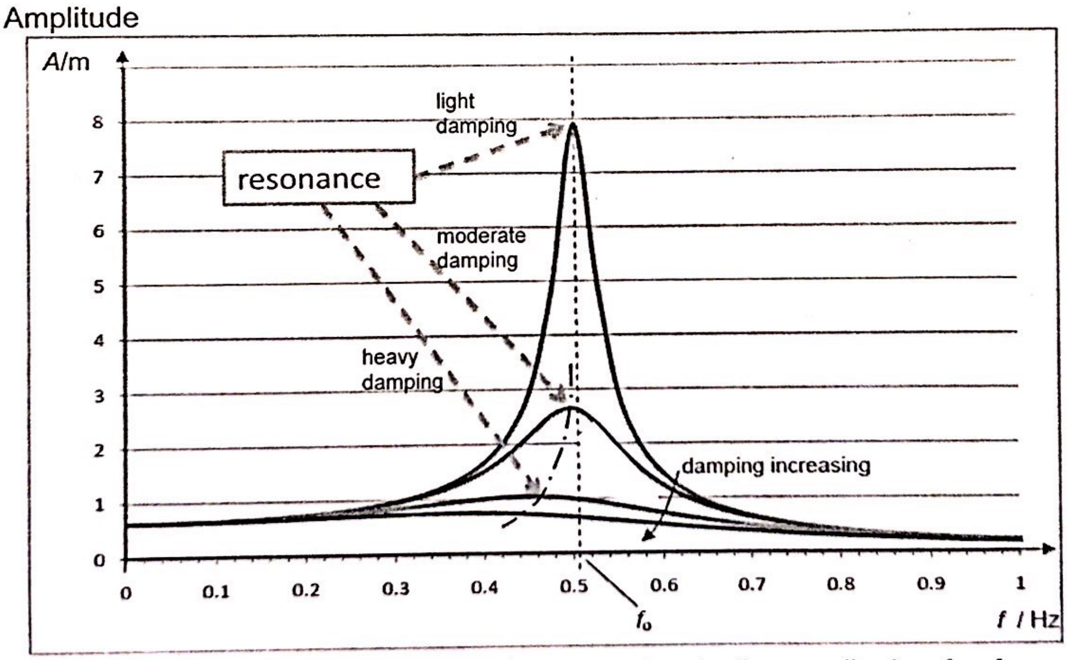
The suspension system of a car includes shock absorbers and springs. When a car goes over a bump, the springs will be displaced from their equilibrium lengths. The viscous oil in the shock absorbers provide damping to enable the springs to smoothly and quickly return to their equilibrium lengths without oscillating up and down. The springs are critically damped. This will reduce the discomfort of the passengers. Without critical damping, the body of the car will oscillate up and down after going over a bump, which is undesirable.

(*Refer to lecture notes on the car suspension system.*)

S9 Forced oscillations are produced when a body is subjected to an periodic external driving force.

Resonance occurs when a system responds at maximum amplitude to an external driving force. This occurs when the frequency of the driving force is equal to the natural frequency of the driven system.

S10



For a forced oscillation, when conditions are *steady*, the amplitude of a forced oscillation depends upon the damping of the system and the relative values of the driving frequency f to the natural frequency f_o of the system. Oscillations with the largest amplitude (i.e. resonance) occur when f is approximately equal to f_o.

The sharpness of resonance is determined by the degree of damping. When damping is light, the amplitude is large but falls off rapidly when the driving frequency of the body differs slightly from the natural frequency of the body. The resonance is sharp.

When damping is moderate, the amplitude at resonance decreases. The curve falls off gradually and maximum amplitude occurs at a frequency that is lower than the natural frequency of the body. When damping is heavy, the resonance is flat.

S11 *Refer to lecture notes on circumstances in which resonance is useful or should be avoided.*

Self - Practice Questions

1 Trolley is performing S.H.M. since it is given that its acceleration, $a = \ddot{x} = -\omega^2 x$.
At maximum displacement x_0,

$$F_{max} = ma_{max}$$

$$a_{max} = \frac{F_{max}}{m}$$

$$\omega^2 = \frac{F_{max}}{mx_0} = \frac{10}{2(0.05)} = 100 \text{ rad}^2 \text{ s}^{-2}$$

Ans: D

2 In an undamped S.H.M., total energy of the oscillating system is a constant though kinetic and potential energies are constantly being transformed from one form to the other.
Since total energy and maximum energy is constant, amplitude is also constant.
Period of the oscillation is also constant, hence angular frequency is also constant.
Ans: E

3 Player should aim at the regions where the moving target is moving the slowest i.e. at the extreme ends of the S.H.M. (Target is momentarily stationary at the amplitude of the oscillation.)
Ans: B

4 At point C, the body's velocity is positive (gradient positive) and since its displacement positive, its acceleration is negative ($a = -\omega^2 x$).
Ans: C

5 At t = 0, kinetic energy of the bob is zero and potential energy is maximum. This will happen again at t = 1 s and t = 2 s when the bob is at the points of maximum displacement.
Hence the frequency of the energy variations is twice that of the oscillation itself (i.e. the period of the energy variations is half that of the oscillation itself).
Ans: A

6 From t = 0 to t = $T/4$ (a quarter through its oscillation), the speed of the mass increases from zero to a maximum at the equilibrium position and acceleration/restoring force decreases from a maximum to zero. The amplitude is thus (100 –30) / 2 = 35 cm.
At t = T/2, the mass is at its maximum displacement from the equilibrium position. It is momentarily at rest. So speed and kinetic energy is zero.
Ans: D

7 P experiences a smaller damping force than Q because Q comes to rest in a shorter time. Hence under forced oscillations, P should exhibit larger amplitudes and a larger resonance frequency.
Ans: C

8 Compare $x = (0.05) \sin 8\pi t$ with $x = x_o \sin \omega t$.

(a) amplitude, $x_o = 0.05$ m

(b) frequency, $f = \omega/2\pi = 8\pi/2\pi = 4$ Hz

(c) period, $T = 1/f = ¼$ s $= 0.25$ s

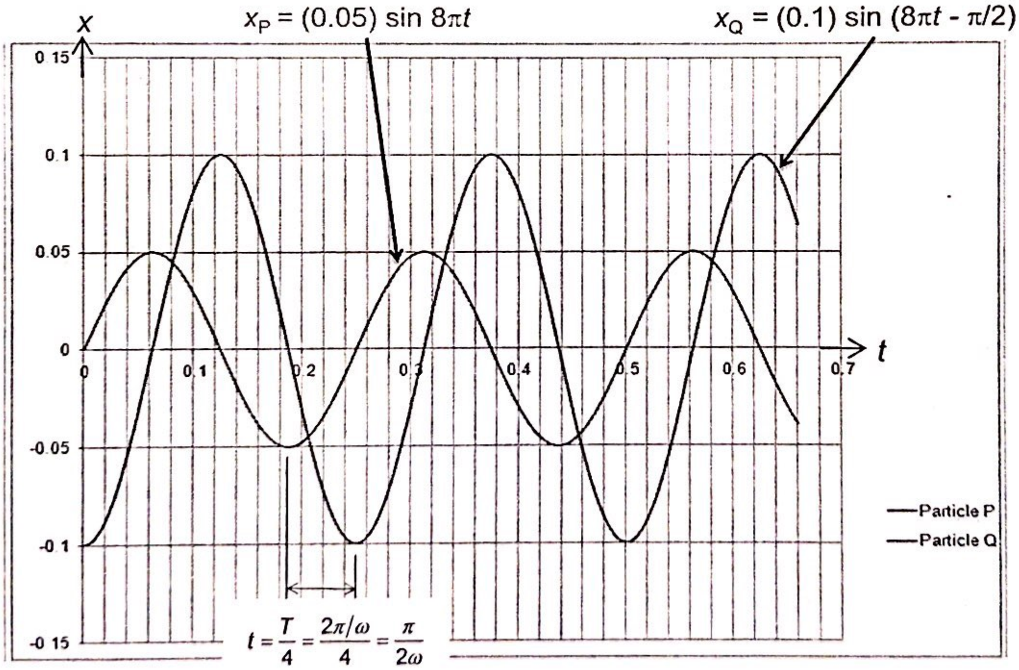
(d) velocity of the particle as it passes through its equilibrium position,

$|v_o| = \omega x_o$
$= (8\pi)(0.05 \text{ m})$
$= 1.26 \text{ m s}^{-1}$;

velocity of the particle at the extreme end of the swing = 0 m s^{-1}

(e) maximum acceleration of the particle during its motion,

$|a_o| = \omega v_o$
$= (8\pi)(8\pi)(0.05 \text{ m})$
$= 31.6 \text{ m s}^{-2}$



9 Assume that the spring is not compressed during the oscillation of the mass, ie the mass is not pulled down a distance which is more than the equilibrium extension.

Take the lowest position of the oscillating mass to be the zero gravitational potential energy level.

At the lowest position of the oscillation, the gravitational potential energy and kinetic energy of the system is at a minimum while its elastic potential energy is at a maximum.

As the mass moves upwards, elastic potential energy is converted into gravitational potential energy and kinetic energy.

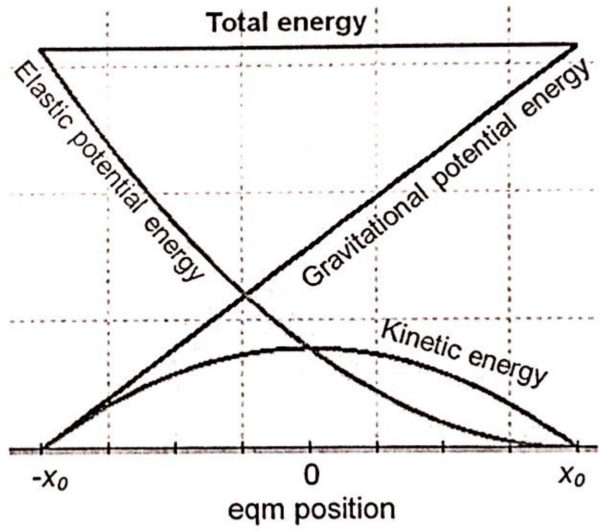
When the mass is at the centre of its oscillation, the system has maximum kinetic energy.

As it continues to move upwards, kinetic and elastic potential energy of the system is converted into gravitational potential energy.

At the highest position of the oscillation, the system has the highest gravitational potential energy, lowest elastic potential energy and zero kinetic energy.

The interchange between these three forms of energy continues as the mass oscillates but there will be energy lost in the form of internal energy due to air resistance.

Assuming the spring is displaced elastically by a distance equal to the equilibrium extension, the following graph showing the variation of energies with displacement will be obtained.



(Refer to notes E.g. 5)

If the spring is displaced elastically by a distance equal to the equilibrium extension and the zero level for gravitational potential energy is taken to be midway between the un-stretched and equilibrium positions instead, the following graph will be obtained.