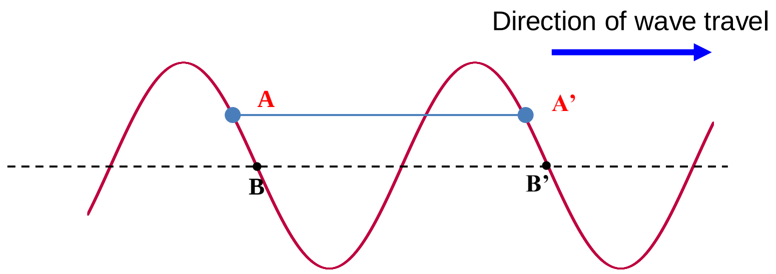
Wave Lecture Examples Suggested Solution

Example 9.3.1:

Consider the snapshot of the wave shown below. Other than the distance between the 2 successive crests or troughs,

(i) identify another point (B') that pairs with B that will correspond to a single wavelength.

(ii) Choose another pair of points on the wave that correspond to one wavelength.



(iii) What do you notice about the pairs of points that represent one wavelength?

Each pair of points not only have the same distance but also the same displacement from the equilibrium position. They are said to be in the same state of oscillation or in phase.

<u>Example 9.3.2 : Transverse wave</u>

The diagram below shows an instantaneous position of a string as a transverse progressive wave travels along it from left to right.

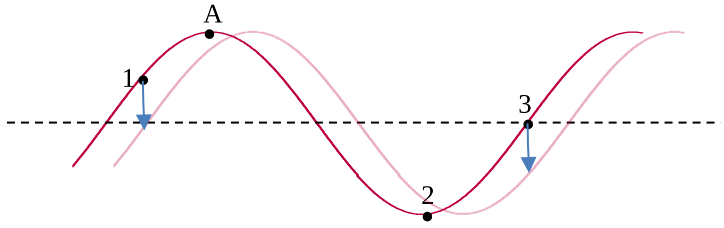


Fig. 9.3.8.

(a) Taking upwards to be positive, for each of the points 1, 2, and 3 on the string,

(i) state whether the displacement of each point is positive or negative.

1: positive
2: negative
3: neither (zero displacement)

(ii) What are the directions of the velocities of the points?

By drawing the wave when it is at the next instance (refer to diagram above), you can deduce that:

1: downwards
3: downwards
For 2, its velocity is zero since it is at maximum displacement.

(iii) Of the three points, which one has the largest speed?

3 has the highest speed since it is at the equilibrium position.

(b) Mark on the string a point A with zero velocity.

Refer to diagram above.

(c) What is the direction of acceleration of point A?

The direction of the acceleration is downwards, towards the equilibrium position.

(Recall SHM equation $a = -\omega^2 x$)

Example 9.3.3: Longitudinal Wave

Fig. 9.3.11 shows a 'snapshot' of a horizontal slinky spring when it is (a) at rest and (b) carrying a longitudinal wave moving from left to right. There is no energy dissipation along the slinky spring.

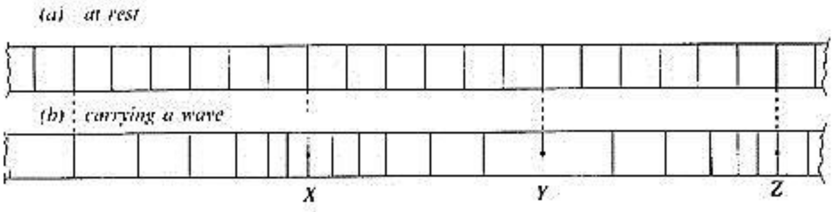


Fig. 9.3.11

State and explain whether each of the statement is true or false.

Statement A: The distance between X and Y is one wavelength.

False. XY is distance between centre of compression C and rarefaction R.
Hence XY corresponds to 1/2 λ. (XZ corresponds to 1λ)

Statement B: The amplitude of oscillation of Y is greater than the amplitude of oscillation of X.

False. Since no energy is dissipated, amplitude of oscillation for all points is the same for a 1-D progressive wave. Hence amplitude of X = amplitude of Y.

Statement C: The displacement of X at this instant is zero.

True. X is at its equilibrium position. Hence its displacement at this instant is zero. (same at Y, Z)

Example 9.3.4: Speed of a Wave

A fisherman notices that wave crests passes the bow of his anchored boat every 6.0 s. He measures the distance between the two crests to be 15.0 m. How fast are the waves travelling?

Time taken for a crest to travel by 1 wavelength, T = 6.0 s
Distance between 2 crests, λ = 15.0 m

$$v = f\lambda = \frac{\lambda}{T} = \frac{15.0\text{ m}}{6.0\text{ s}} = 2.5\text{ m s}^{-1}$$

Example 9.3.5

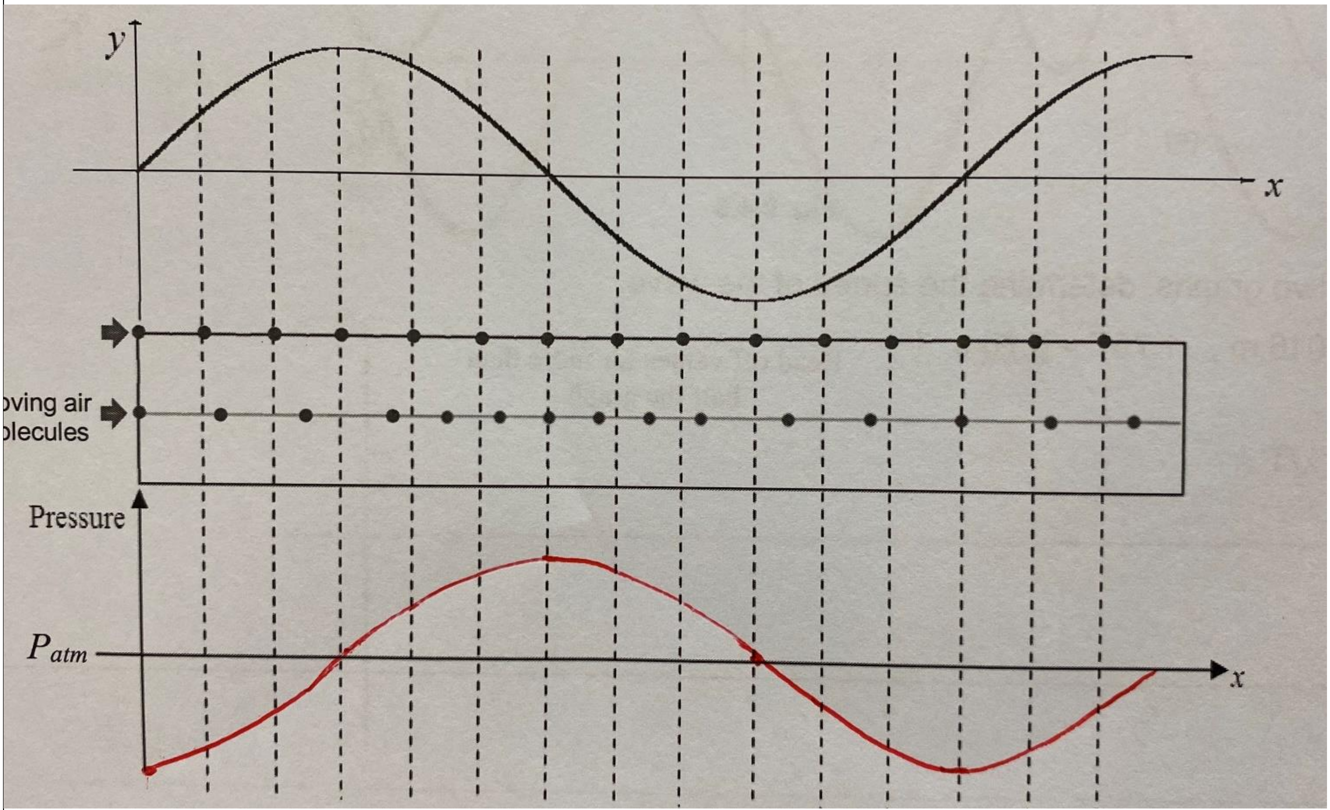
Calculate the frequency of a green light of wavelength 0.60 μm.

Speed of light, c = 3.0 x10^8 m s^{-1}

Using $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{3.00\times10^{8}\text{ m s}^{-1}}{0.60\times10^{-6}\text{ m}} = 5.0\times10^{14}\text{ Hz}$

Example 9.4.1: Displacement-Position Graph for a Longitudinal Wave

A displacement–position graph can also be drawn for a longitudinal wave. Consider the following displacement-position graph for the air molecules along a line when a sound wave passes through a region of space. Sketch the corresponding pressure variation of this region.



Note: Points of **compression and rarefactions** corresponds to points where **displacements of air molecules are zero.**

Compressions are regions of **high air pressure**, while **rarefactions** are regions of **low air pressure**.

Example 9.4.2: Determining speed of wave

A vibrating bar makes waves in a ripple tank. Fig. 9.4.5(a) shows the displacement of the wave as it travels out from the bar. The position of the floating cork in the tank varies with time as shown in Fig. 9.4.5(b).

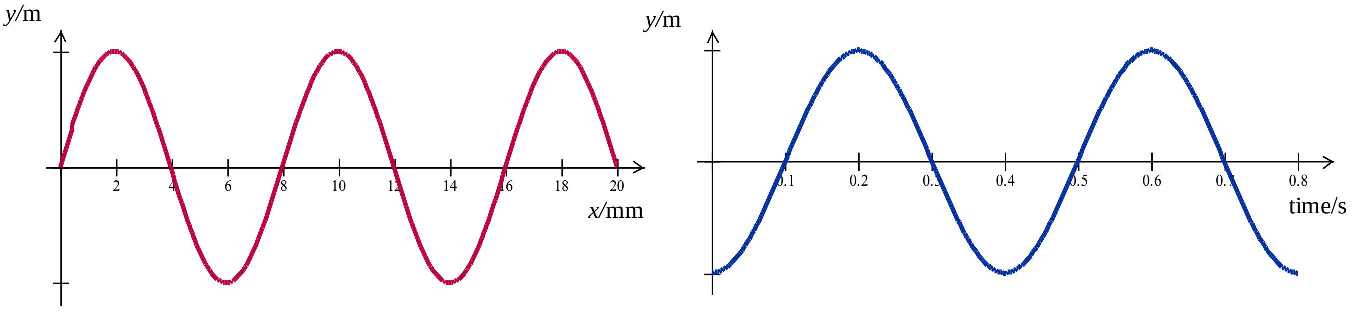


Fig. 9.4.5

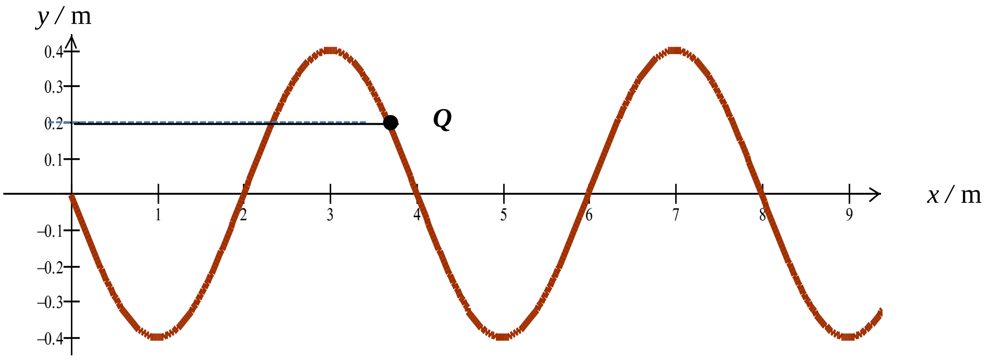
From the above two graphs, determine the speed of the wave.

λ = 0.0080 m , T = 0.40 s

speed $v = f\lambda = \lambda/T = \frac{0.0080}{0.40} = 0.020$ m s^{-1}

Example 9.4.3

The following graph shows a displacement vs. position graph of a progressive transverse wave at t = 0. For the point **Q** on the wave, sketch the corresponding displacement vs. time graph, given that the wave is moving in the positive x direction at a speed of 1.0 m s^{-1}.

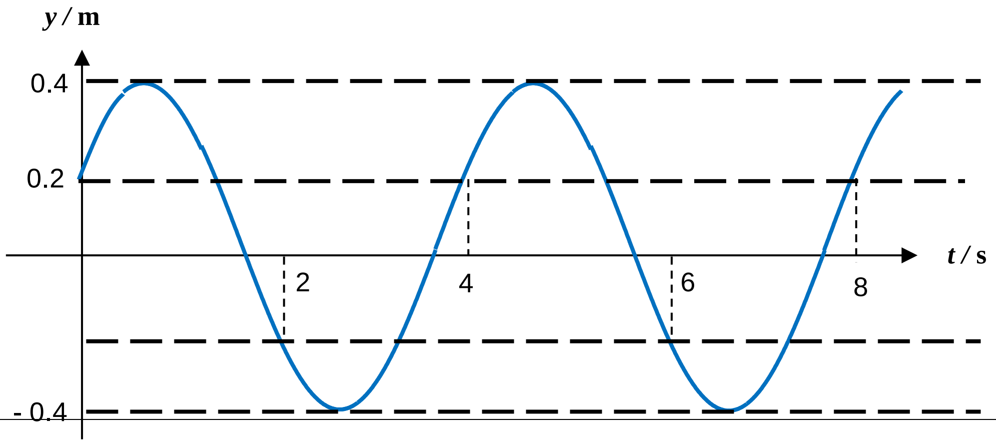


λ = 4.0 m

2λ = 8.0 m

$v = f\lambda = \lambda/T$

$T = \lambda/v$ = 4.0/1.0
= 4.0 s



Example 9.5.1:

(a) The following graph shows the displacement vs. position of two waves **P** and **Q** at a particular instant of time, what is the phase difference between the two waves **P** and **Q**?

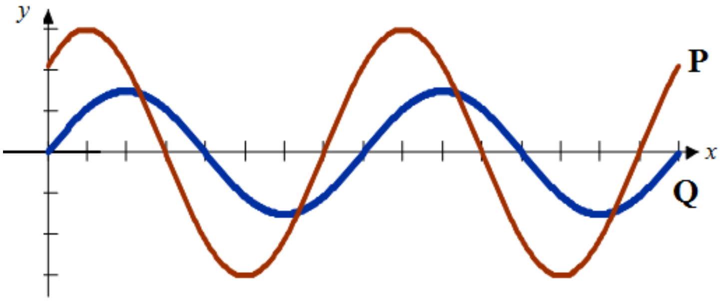


Fig. 9.5.9

Answer:

$\Delta\phi = (\Delta x/\lambda)(2\pi) = (1/8)(2\pi) = \pi/4$ rad

Wave P lags Wave Q by a phase difference of $\pi/4$ rad.

OR

Wave Q leads Wave P by a phase difference of $\pi/4$ rad.

(b) Is it possible to compare the phase difference between two waves of different wavelengths? What is the phase difference between the two waves shown below?

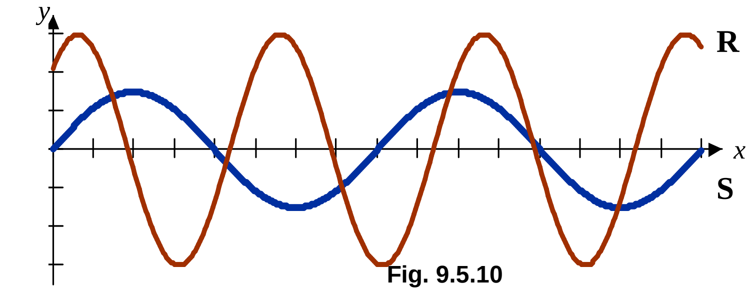


Fig. 9.5.10

Answer:

No. As Δx is not constant for the 2 waves, the phase difference between the two waves keeps changing value.

It is only meaningful to talk about phase difference between 2 waves only if they have the same wavelength or same frequency.

Example 9.6.1: (N94/I/10 – modified)

A sound wave of amplitude 0.20 mm has an intensity of 3.0 W m^{-2}.

(a) What will be the intensity of a sound wave of the same frequency which has an amplitude of 0.40 mm?

(b) How will the intensity change if the frequency of the source of wave is doubled but the amplitude remains unchanged? Treat the source as a simple harmonic oscillator.

(a) Assume spherical source

$$I \propto A^2$$

$$\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2$$

$$\frac{I_2}{3.0} = \left(\frac{0.40}{0.20}\right)^2$$

(b) Recall from Oscillations chapter,

$$E = \frac{1}{2}m\omega^2 A^2$$

$$E \propto \omega^2 \propto f^2$$

Since $I = \frac{Power}{area} = \frac{energy}{time(area)}$

$$I \propto f^2$$

Since the frequency f is doubled, the intensity is four times of the initial intensity, i.e. 12 W m^{-2}.

Example 9.6.2:

If the intensity of an earthquake *P*-wave is 1.0 x 10^6 W m^{-2} at 100 km from the source, what is the intensity of the wave 400 km from the source?

Answer:
Assume spherical source

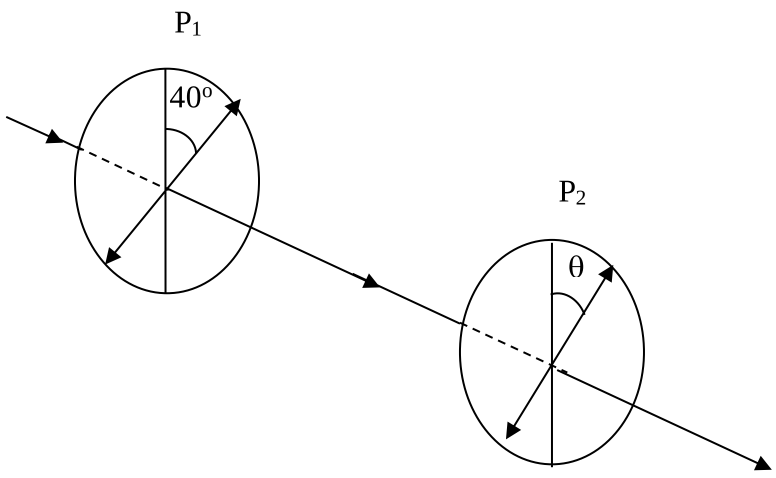
$$I \propto \frac{1}{r^2}$$

$$\frac{I}{1.0\times10^6} = \left(\frac{100}{400}\right)^2$$

$$I = \left(\frac{100}{400}\right)^2 \times (1.0\times10^6) = 6.3\times10^4 \text{ Wm}^{-2}$$

Example 9.7
The figure below shows a beam of initially unpolarised light passing through two polarizing filters P_1 and P_2. The polarising axis of P_1 is fixed at 40° with respect to the vertical axis.

The polarizing axis of P_2 is then rotated clockwise from its vertical axis. At what values of θ will intensity minima of the emergent light occur?



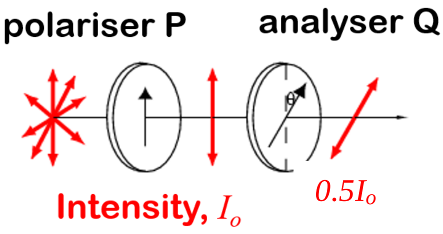
For intensity minima to occur (no emergent light), the two polarizing axes must be **mutually perpendicular** to each other.

Rotate P_2 through **90°** with respect to P_1 , therefore θ = 40°+90° =130°

Rotate P_2 through **270°** with respect to P_1, therefore θ = 40° +270° =310°

Example 9.8

Two polarising sheets have their polarizing directions parallel so that the intensity I_o of the transmitted light is a maximum. Through what angle must either sheet be turned if the intensity is to drop by half?



From Malus' Law,

$$I = I_o \cos^2 \theta$$

$$\tfrac{1}{2} I_o = I_o \cos^2 \theta$$

$$\theta = 45^o$$