2023 JC1 H1 Math MYA Solutions

Qn	Solutions	
1	$3\log_x 5-\dfrac{2}{\log_x 5}=5$ Let $u=\log_x 5$ $3u-\dfrac{2}{u}=5$ $3u^2-2=5u$ $3u^2-5u-2=0$ $(3u+1)(u-2)=0$ $u=-\dfrac{1}{3}$ or $u=2$ $\log_x 5=-\dfrac{1}{3}$ or $\log_x 5=2$ $x^{-\frac{1}{3}}=5$ or $x^2=5$ $x=5^{-3}$ or $x=\pm\sqrt{5}$ (reject $-\sqrt{5}$) $x=\dfrac{1}{125}$ or $x=\sqrt{5}$	
2	$2x+y=-1$ ----- (1) $y=x^2-mx+8$ ----- (2) From (1): $y=-1-2x$ ----- (3) Sub (3) into (2), $-1-2x=x^2-mx+8$ $x^2+2x-mx+9=0$ $x^2+(2-m)x+9=0$ Since the line and the curve intersect at two distinct points, Discriminant > 0 $(2-m)^2-4(1)(9)>0$ $(2-m)^2-36>0$ $(2-m-6)(2-m+6)>0$ $(-4-m)(8-m)>0$ $m<-4$ or $m>8$ Since the line is tangent to the curve, $m=-4$ or $m=8$	

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3(i)	Area of garden $=14\pi$ $\frac{1}{2}\pi\left(\frac{1}{2}y\right)^2-\frac{1}{2}\pi\left(\frac{1}{2}y-x\right)^2=14\pi$ $\left(\frac{1}{2}y\right)^2-\left(\frac{1}{2}y-x\right)^2=28$ $\frac{1}{4}y^2-\left(\frac{1}{4}y^2-xy+x^2\right)=28$ $xy-x^2=28$ ----- (1)	
3(ii)	$2x+\pi\left(\frac{1}{2}y\right)+\pi\left(\frac{1}{2}y-x\right)=10\pi$ $2x-\pi x+\pi y=10\pi$ $y=10+x-\frac{2}{\pi}x$ ----- (2)	
3(iii)	Sub (2) into (1): $x\left(10+x-\frac{2}{\pi}x\right)-x^2=28$ $10x-\frac{2}{\pi}x^2-28=0$ Using GC, $x=3.6465=3.65$ or $x=12.061=12.1$ $y=11.325=11.3$ or $y=14.383=14.4$ From the diagram, $\frac{1}{2}y-x>0$, $\therefore x=3.65, y=11.3$	

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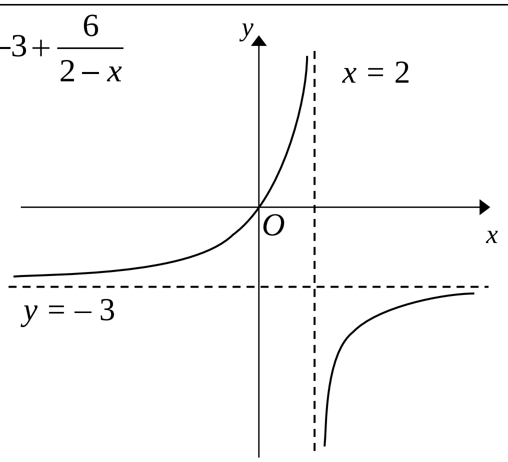
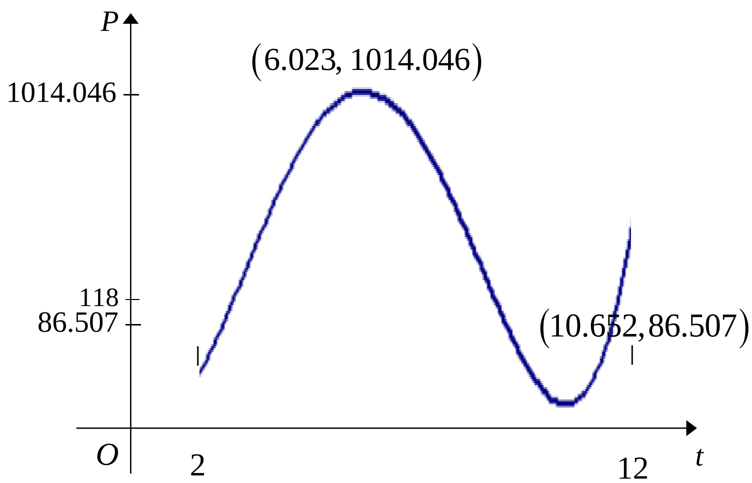
4(i)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\left(\dfrac{2x-\mathrm{e}}{\sqrt{x}}\right)^2\right]=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\dfrac{4x^2-4\mathrm{e}\,x+\mathrm{e}^2}{x}\right]$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[4x-4\mathrm{e}+\dfrac{\mathrm{e}^2}{x}\right]$ $=4-\dfrac{\mathrm{e}^2}{x^2}$	
4(ii)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{e}^{2-5x}+\sqrt{4-3x}\right]=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{e}^{2-5x}+(4-3x)^{\frac{1}{2}}\right]$ $=-5\mathrm{e}^{2-5x}+\dfrac{1}{2}(4-3x)^{-\frac{1}{2}}(-3)$ $=-5\mathrm{e}^{2-5x}-\dfrac{3}{2\sqrt{4-3x}}$	

Qn	Solutions	
5(i)	$y=-3+\dfrac{6}{2-x}$ 	
5(ii)	$y=-3+\dfrac{6}{2-x}\quad\Rightarrow\quad\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{6}{(2-x)^2}$ When $x=2+\sqrt{6}$, $y=-3-\sqrt{6}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$	

	Using $y - y_1 = m(x - x_1)$ or $y = mx + c$, Equation of tangent: $y - \left(-3 - \sqrt{6}\right) = 1\left(x - 2 - \sqrt{6}\right)$ $y + 3 + \sqrt{6} = x - 2 - \sqrt{6}$ $y = x - 5 - 2\sqrt{6}$ $\therefore m = 1$ and $c = -5 - 2\sqrt{6}$.	
5(iii)	Since curve C has a vertical asymptote at $x = 2$, and curve D has a vertical asymptote at $x = -\frac{b}{a}$, $\therefore -\frac{b}{a} = 2$ $b = -2a$ For curve D, when $y = 0$, $x = \frac{9}{4}$. $\therefore 0 = \ln\left(a\left(\frac{9}{4}\right) + b\right)$ $\left(a\left(\frac{9}{4}\right) + b\right) = 1$ $\left(a\left(\frac{9}{4}\right) - 2a\right) = 1$ $\Rightarrow a = 4,\ b = -8$	

Qn	Solutions	
6(i)	Using G.C, When $t = 2$, $P = 118$.	
6(ii)	$P = 2(t-5)^4 + (5-2t)^3 - 15t^2 + 470t - 925,$ for $2 \le t \le 12$ 	
6(iii)	Using G.C, when $P = 500$, $t = 3.378$ or $t = 8.689$. For $P \ge 500$, $3.38 < t < 8.69$	