02 Kinematics Lecture Notes

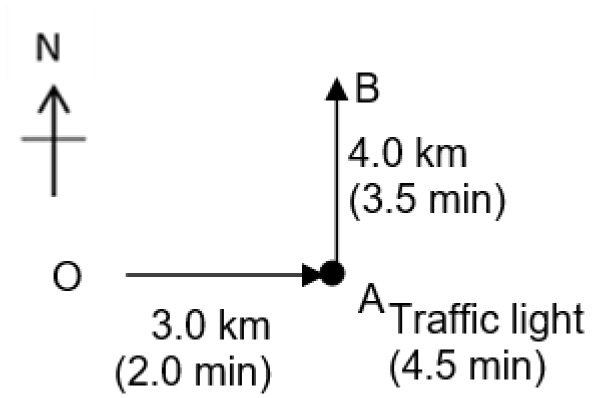
(Solutions for Worked Examples)

Example 1

A car travels 3.0 km due east from a point O to a point A for 2.0 minutes where it stops at a traffic light at A for 4.5 minutes. It then continues 4.0 km due north of A to point B for another 3.5 minutes. Find the

a) average speed

b) average velocity of the car



Average speed $= \dfrac{\text{total distance travelled}}{\text{time taken}}$

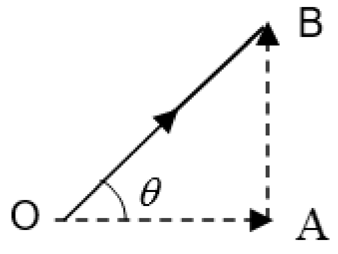
$= \dfrac{3.0+4.0}{2.0+4.5+3.5}$

$= 0.70 \text{ km min}^{-1}$

$= 12 \text{ m s}^{-1}$

Note that as velocity is a *vector* quantity, it is necessary to find the direction for average velocity

b) average velocity of the car



Average velocity $= \dfrac{\text{total change in displacement OB}}{\text{time taken}}$

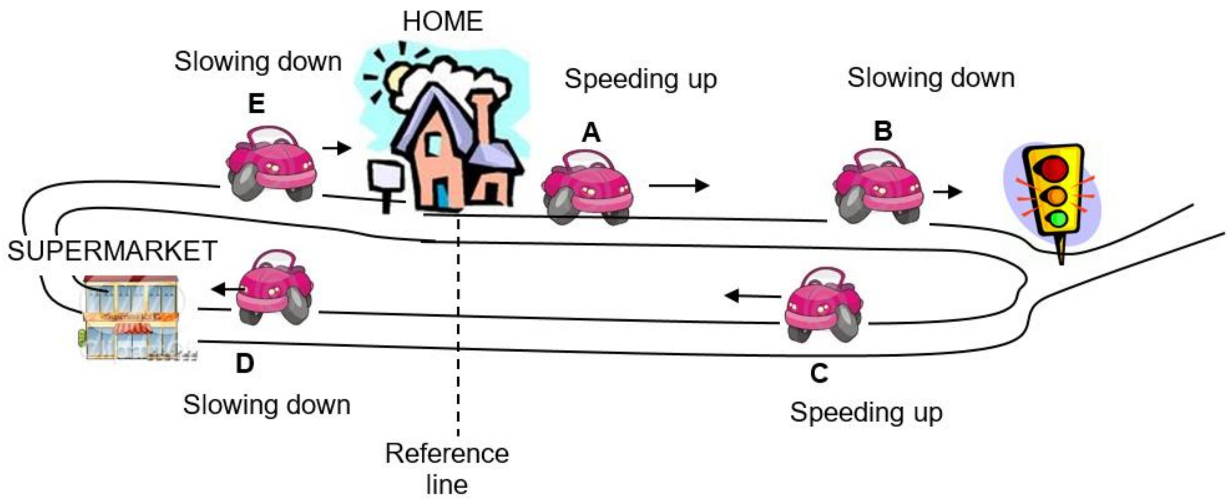
$= \dfrac{\sqrt{3.0^2+4.0^2}}{2.0+4.5+3.5}$

$= 0.50 \text{ km min}^{-1} \quad = 8.3 \text{ m s}^{-1}$

$\tan\theta = \dfrac{4}{3}$

$\theta = 53^{\circ}$

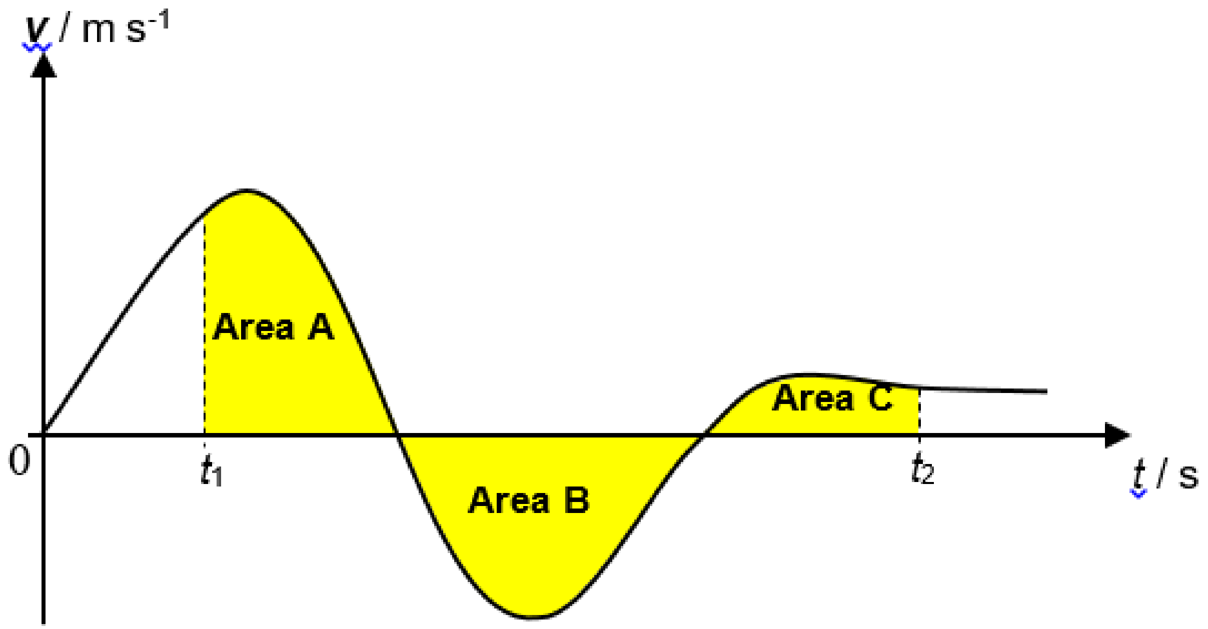
Example 2



	Displacement	Velocity	Acceleration
A (speeding up)	+	+	+
B (slowing down)	+	+	-
C (speeding up)	+	-	-
D (slowing down)	-	-	+
E (slowing down)	-	+	-

Example 3

What is the expression of the change in displacement, $\Delta \boldsymbol{s}$, of the object from t_1 to t_2?
If you need to find distance travelled by the object in the same time interval, what would be your answer?

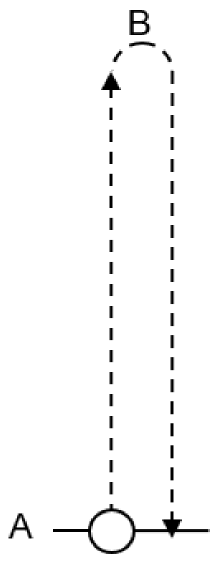


Displacement = A – B + C

Distance = A + B + C

Example 4

A student throws a ball vertically upwards at a speed of 20 m s^{-1} from level A. Neglect air resistance.



(a) Since *s, v* and *a* are vectors, we need to choose a positive sign convention before drawing the graphs. In this example, we will take upwards as positive.

All axes of the graph must be labelled as far as possible.
To determine maximum height (maximum displacement):

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - 20^2}{2(-9.81)} = 20.3 \text{ m}$$

Note: In the video, I solved this problem without the use of kinematics equations (instead using a more graphical approach). Both methods will give the same answer – as expected!

Example 4

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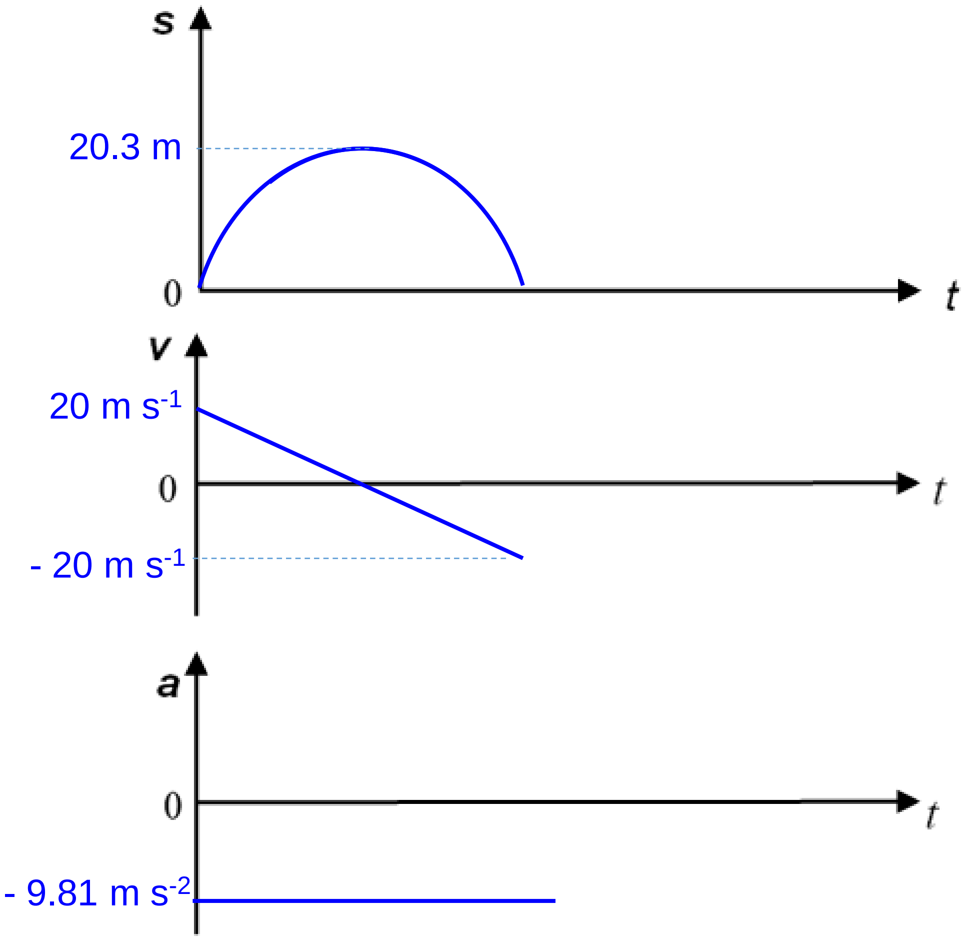
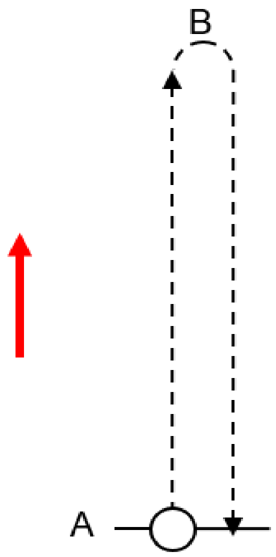




Example 4

(b) When the ball returns to the hand, its displacement is zero.

Taking upwards as positive:

$$s = ut + \frac{1}{2}at^2$$

$$0 = 20t + \frac{1}{2}(-9.81)t^2$$

$$\Rightarrow t = 0 \text{ s (NA) or } t = 4.1 \text{ s}$$

Example 5

A motorist traveling at 13 m s^{-1} approaches traffic lights, which turn red when he is 25 m away from the stop line. His reaction time is 0.70 s. If he brakes fully such that the car slows down at a rate of 4.5 m s^{-2},

On which side of the stop line will he stop, and how far from the stop line will he stop?

Distance travelled before reacting = 13 x 0.70 = 9.1 m

Distance travelled during deceleration:

Taking the direction of initial velocity to be positive

$$v^2 = u^2 + 2as$$

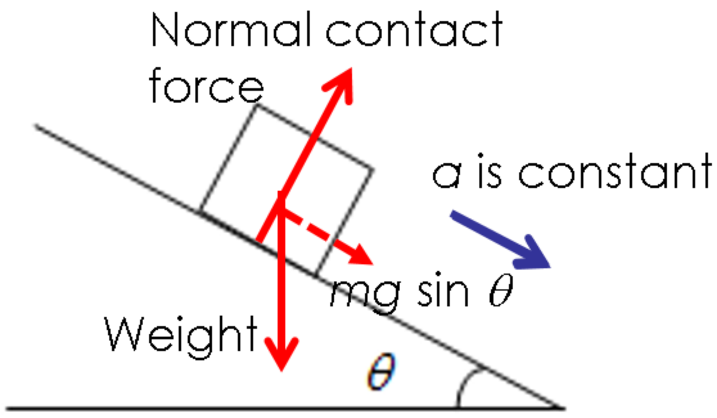
$$s = \frac{v^2 - u^2}{2a} = \frac{0 - 13^2}{2(-4.5)} = 18.8 \text{ m}$$

Total distance travelled = 9.1 + 18.8 = 27.9 m

Thus, he stops (27.9 – 25) = 2.9 m beyond the stop line.

Example 6

Considering the forces on the box,



a) **Net force**: the component of weight of box parallel to the slope

$$Mg\sin\theta = Ma$$

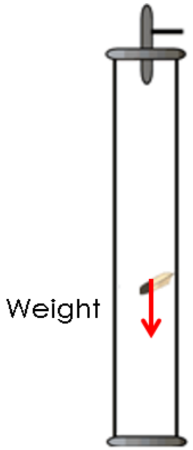
$$a = g\sin\theta$$

b) The acceleration is not *g*. It is **not** in free fall.

c) Since normal contact force and weight are constant, the resultant force acting on the box is constant. Hence, the acceleration is constant. Therefore, the equations of motion **can** be applied in case 1.

Example 6

Considering the force on the feather,



a) **Net force**: Weight of feather, *W*

$$mg = ma \rightarrow a = g$$

b) The acceleration is *g*. It is in free fall.

c) The acceleration is constant. Hence, the equations of motion can be applied in case 2.

Example 7

A steel ball bearing is released from a height above a deep pool of oil and it reaches a terminal velocity in the oil. Sketch the graph to show the variation of the velocity ***v*** of the ball bearing against time *t* when the ball enters the oil when the drag force acting on the ball is

a) smaller than the weight of ball (Case A)
b) larger than the weight of ball (Case B)

Indicate clearly on your diagram, t_1, the time when the ball bearing hits the oil. Assume that upthrust is negligible.

Solution

First and foremost, we should find out what makes the drag force to be smaller or larger than the weight.

We know that the *drag forces are proportional to the speed* of objects in a viscous medium. So, it must have to do with the **initial speed** of the ball before entering the oil. If the ball is released slightly above the surface of the oil then the ball will have a small speed just after entering the oil, hence the drag forces on it would be less than the weight. However, if the ball is released from a certain height before entering the oil then it will have a significantly large speed, which subsequently results in a larger drag force on the ball.

Example 7

Stage 1: Released from rest ($t = 0$ to t_1)

Before the ball bearing enters into the oil, only the weight acts on it and drag force (due to air) is very small compared to weight.

By Newton's 2nd Law,

$$(\downarrow)\Sigma F = ma$$
$$\Rightarrow mg = ma$$
$$\Rightarrow a = g$$

Stage 2 : Enters the oil after t_1

When the ball is released from a height *h* above the oil, its velocity before entering the oil would be

$$v^2 = u^2 + 2as \rightarrow v = \sqrt{2gh}$$

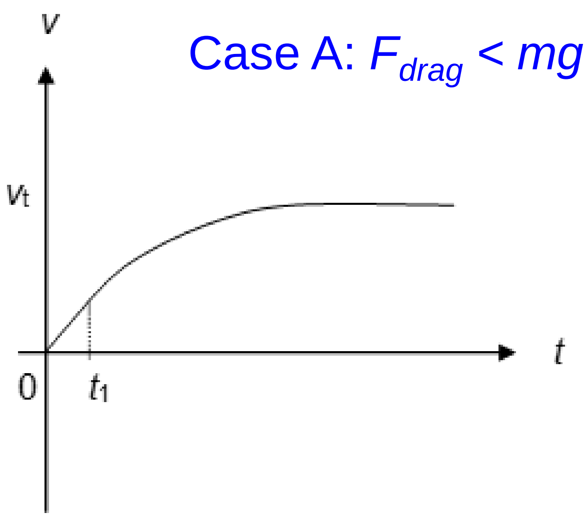
Enters with some initial velocity, drag force due to oil resistance is potentially large compared to the weight, leading to two possible cases:

Case A: Ball bearing is released just above the surface of the oil, entering the oil at low velocity. Its velocity increases at a decreasing rate until it reaches a terminal velocity.

Case B: Ball bearing is released high above the surface of the oil, entering the oil at high velocity. Its velocity subsequently decreases until it reaches a terminal velocity.

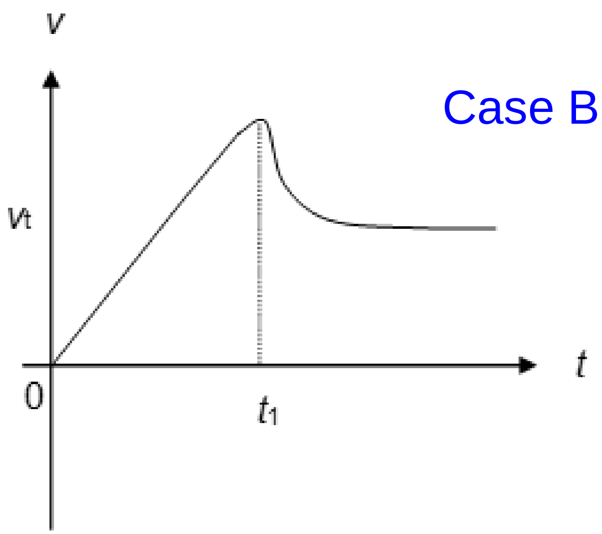
Example 7

Case A: $F_{drag} < mg$



- Resultant force is downwards
- The ball bearing accelerates (its velocity *v* increases) but at a value less than *g*.
- As *v* increases, F_{drag} increases until $F_{drag} = mg$
- Net force = 0
- Terminal velocity is reached

Case B: $F_{drag} > mg$



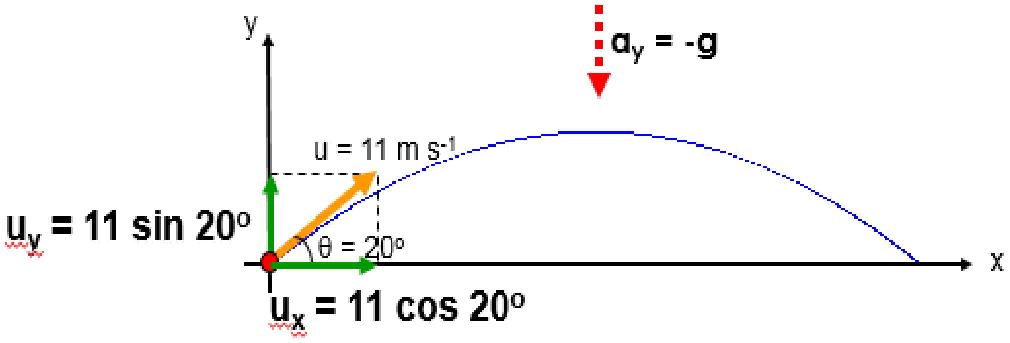
- Resultant force is upwards
- The ball bearing decelerates (slows down), its velocity *v* decreases
- As *v* decreases, F_{drag} decreases until $F_{drag} = mg$
- Net force = 0
- Terminal velocity is reached

Example 8

A long-jumper leaves the ground at an angle of 20° to the horizontal and at a speed of 11 m s^{-1}.

a) How far does he jump?

b) What is his maximum height reached?



(a) Taking upwards and rightwards as positive,

$$(\rightarrow)\ s_x = u_x t = \left(11\cos 20^o\right)t \ \ ....\ (1)$$

$$(\uparrow)\ s_y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow 0 = \left(11\sin 20^o\right)t + \frac{1}{2}(-9.81)t^2$$

$$\Rightarrow t = 0 \text{ s (NA) or } t = 0.767 \text{ s}$$

Substituting into equation (1):

$$s_x = \left(11\cos 20^o\right)(0.767) = 7.93 \text{ m}$$

(b)

$$v_y^{\ 2} = u_y^{\ 2} + 2a_y s_y$$

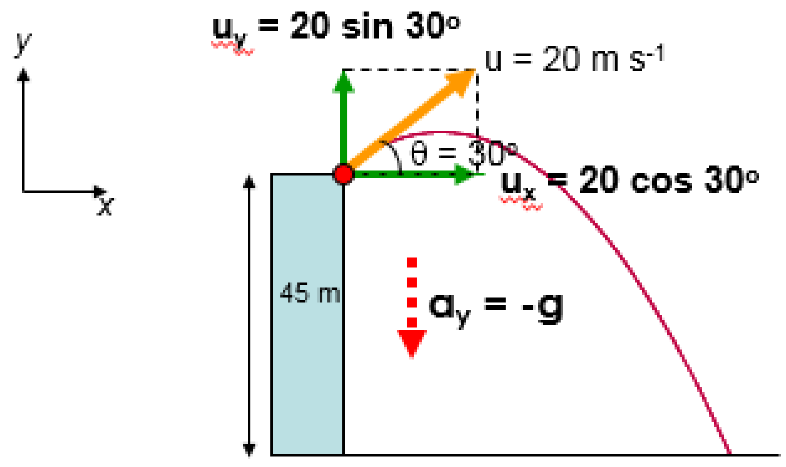
$$s_y = \frac{0-\left(11\sin 20^o\right)^2}{2(-9.81)} = 0.72 \text{ m}$$

Example 9

A stone is thrown from the top of a building upward at an angle of 30° to the horizontal with an initial speed of 20 m s^{-1}. If the height of the building is 45 m,

a) how long is the stone in flight?

b) what is the speed of the stone just before it strikes the ground?



(a) Taking upwards to be positive,

$$s_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow (-45) = \left(20\sin 30^o\right)t + \frac{1}{2}(-9.81)t^2$$

$$t = -2.81\text{ s} \quad \text{(NA) or } t = 4.22\text{ s}$$

(b) Taking to the right as positive,

$$v_x = u_x = 20\cos 30^o = 17.3\text{ m s}^{-1}$$

$$v_y^2 = u_y^2 + 2a_y s_y \Rightarrow v_y^2 = \left(20\sin 30^o\right)^2 + 2(-9.81)(-45)$$

$$v_y = -31.4\text{ m s}^{-1} \quad \text{or} \quad v_y = 31.4\text{ m s}^{-1}$$ (NA since object is moving downwards)

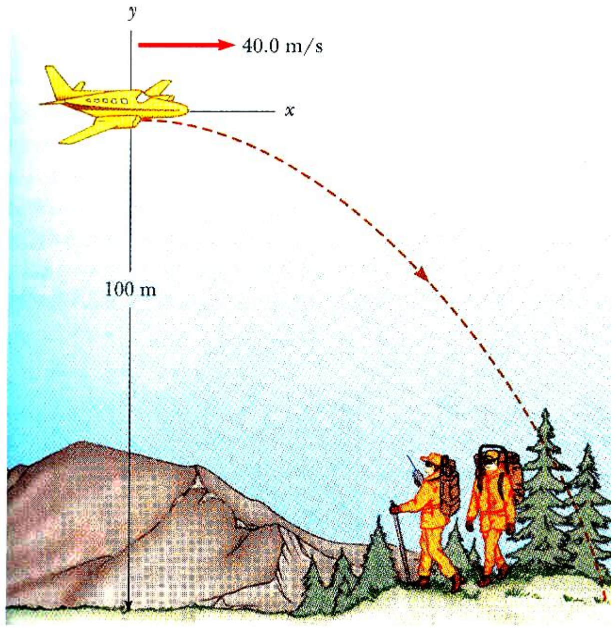
$$v = \sqrt{v_x^2 + v_y^2} = 35.9\text{ m s}^{-1}$$

Example 10

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in the figure below. The plane is travelling horizontally at 40.0 m s^{-1} and is 100 m above a point O on the ground.

a) Determine how far from the point O does the package strike the ground.

b) Determine the position of the package relative to the airplane when it strikes the ground.



a) Taking rightwards as positive,

$$s_x = u_x t = 40t \text{ ....} (1)$$

Taking downwards as positive

$$s_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 100 = 0 + \frac{1}{2}(9.81)t^2$$

$t = 4.52$ s (reject -ve time)

Substituting into equation (1):

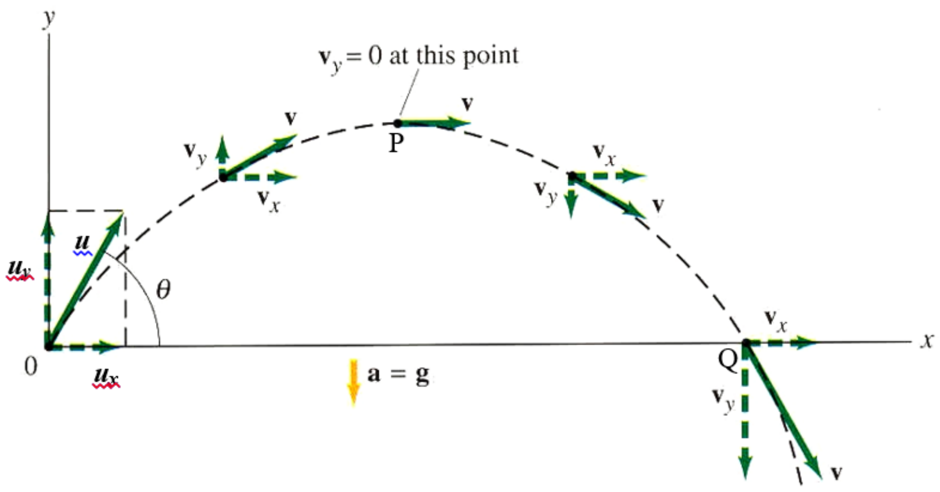
$$s_x = 40(4.52) = 181 \text{ m}$$

b) 100 m right below the airplane because both the airplane and the package have the same horizontal velocity

Example 11

Solution:

(a) Maximum Height, *H*



The maximum height reached is the maximum vertical displacement reached by the projectile (Point P on the diagram above). At the point of maximum height, the vertical component of the velocity is zero,

Taking upward direction as positive, consider vertical motion

$$s_y = H$$

$$a_y = -g$$

$$u_y = u\sin\theta$$

$$v_y = 0$$

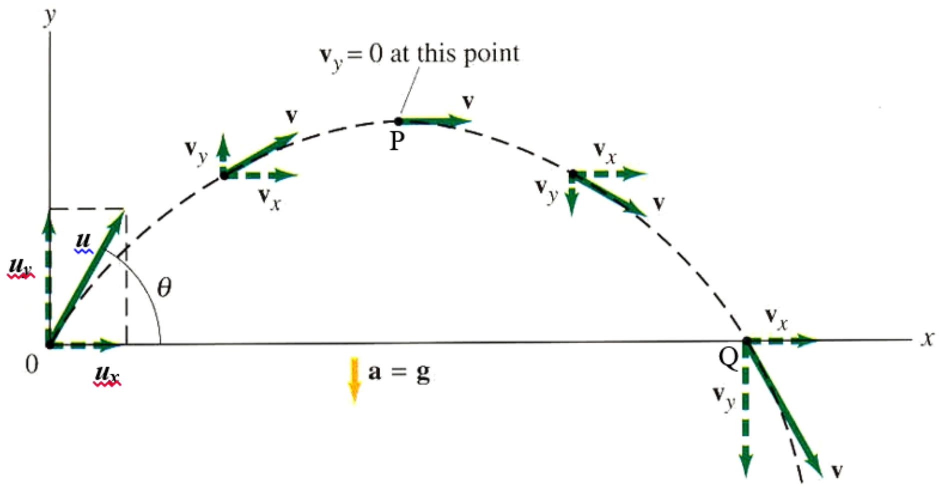
Using, $$v_y^2 = u_y^2 + 2a_y s_y$$

$$0 = (u\sin\theta)^2 + 2(-g)(H)$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

Example 11

(b) Duration of Flight, t_{flight} :



The time of flight of the projectile is the time the projectile spends in the air, i.e. the time interval between the point the projectile leaves the point of projection O to the point where it lands on the ground Q.

At point Q, the vertical displacement of the projectile from point O is zero, hence we have
Taking upward direction as positive, consider vertical motion,

$$s_y = 0$$

$$a_y = -g$$

$$u_y = u \sin\theta$$

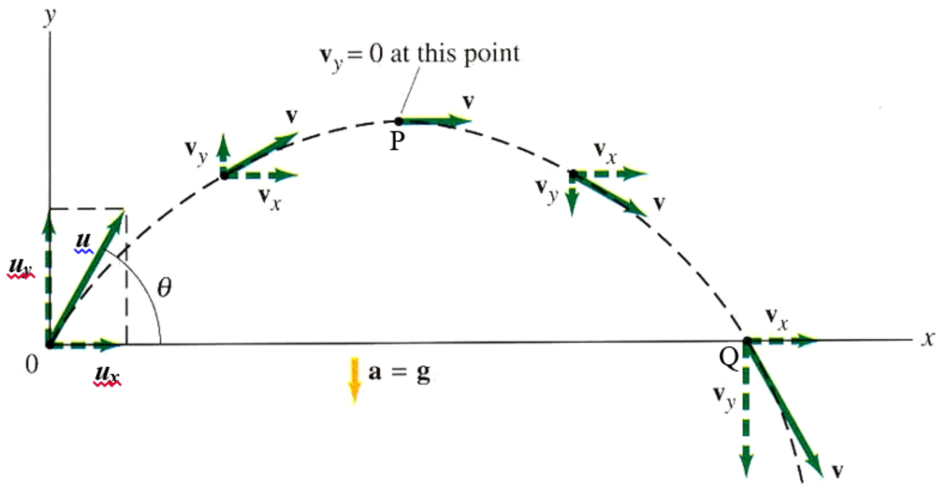
Using, $s_y = u_y\, t + \frac{1}{2} a_y\, t^2$

$$0 = (u \sin\theta)\, t_{flight} + \frac{1}{2}(-g)\, t_{flight}{}^2$$

$t_{flight} = \frac{2u\sin\theta}{g}$ or $t_{flight} = 0$ at the time of beginning of throw when $s_y = 0$

Example 11

(c) Horizontal Range, *R*:



Since there are no horizontal forces acting on the projectile, the horizontal velocity remains constant during the time of flight and the horizontal acceleration is zero. Using, the result for the time of flight that we derived in (b), hence, we have

Consider horizontal motion,

$$v_x = u_x = (u\cos\theta)$$

$$a_x = 0$$

$$t_{flight} = \frac{2u\sin\theta}{g}$$

$$s_x = R$$

Using, $s_x = u_x\, t + \frac{1}{2} a_x\, t^2$

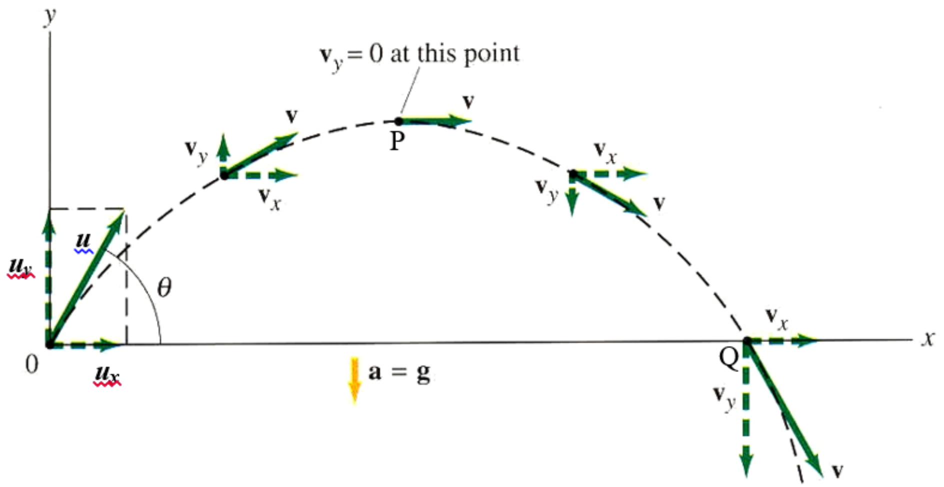
$$R = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right) + 0$$

Using the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$

$$R = \frac{u^2 \sin 2\theta}{g}$$

Example 11

(d) Optimal Angle of Projection θ_{max} (for maximum range):



Now $$R = \frac{u^2 \sin 2\theta}{g}$$

but $$-1 \leq \sin 2\theta \leq 1$$

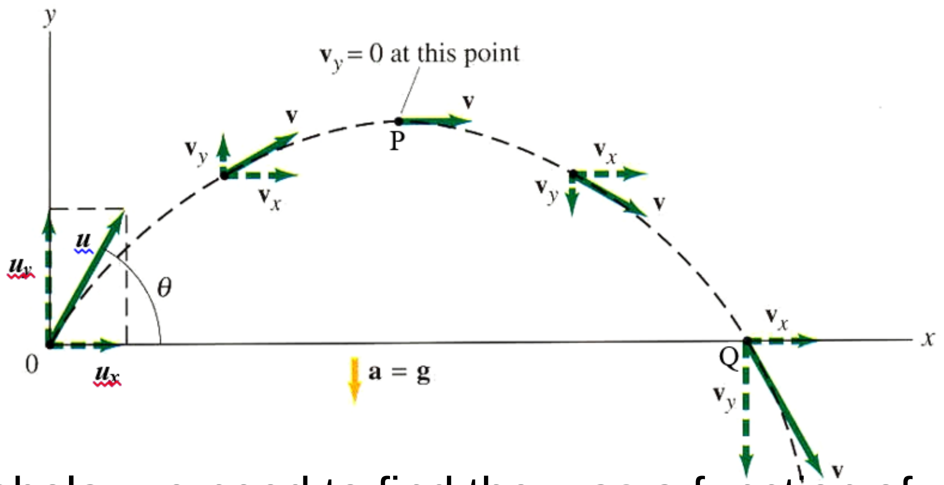
Maximum $$\sin 2\theta = 1$$

Therefore, the maximum possible range, $$R_{max} = \frac{u^2}{g}$$

This occurs when $\sin 2\theta = 1 \rightarrow 2\theta = 90^\circ \rightarrow \theta = 45^\circ$

Example 11

(e) The Trajectory Equation



To show that the path followed by any projectile is a parabola, we need to find the *y* as a function of *x* by eliminating *t* between the vertical motion and the horizontal motion.

We can write following equations for both the horizontal and vertical displacements:

$$x = u_x t \qquad (1)$$

$$y = u_y t - \frac{1}{2} g t^2 \qquad (2)$$

From (1), we have $t = x / u_x$ and we substitute it into (2) to obtain

$$y = \left(\frac{u_y}{u_x} \right) x - \left(\frac{g}{2 u_x^{\,2}} \right) x^2$$

If we write, $u_x = u \cos \theta$ and $u_x = u \sin \theta$, we can therefore express it as

$$y = (\tan \theta) x - \left(\frac{g}{2u^2 \cos^2 \theta} \right) x^2$$

We can see that *y* as a function of *x* has the form

$$y = ax - bx^2$$

This is an equation of a parabola.

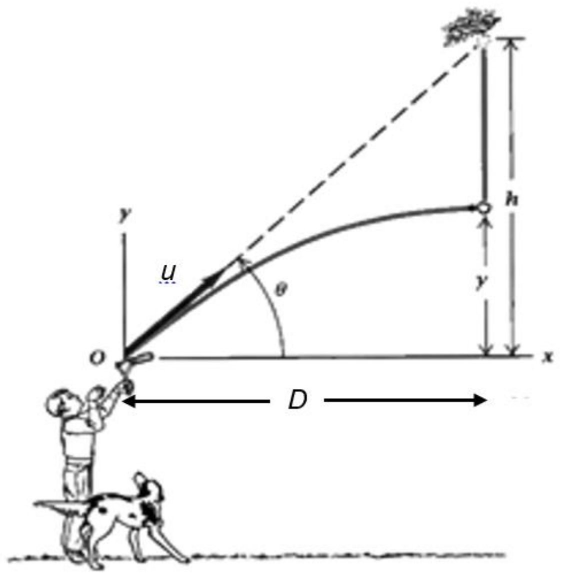
Example 12

A boy aims his slingshot directly at an apple hanging in a tree. At the moment he **shoots** a pebble, the apple **drops**.

The initial speed of the pebble is 25 m s^{-1}. The position of the apple is 5.0 m above the horizontal level, (h = 5.0 m). The pebble hits the apple 0.50 s later.

(a) Determine the angle at which the boy must aim his slingshot towards the apple.

(b) Find the horizontal distance D.



Solution

Let the initial position of slingshot be zero. Given that h = 5.0 m

Consider the vertical motion of the apple first. The vertical fall of the apple in 0.50 s is

$$s_{apple} = \frac{1}{2}gt^2 = \frac{1}{2}9.81(0.50)^2 = 1.23 \text{ m}$$

The pebble hits the apple at the height of $y = h - s_{\text{apple}}$ = 5.0 – 1.23 = 3.77 m

Now, let's look at the motion of the apple. The vertical position of the pebble **at the impact** could be written as

$$3.77 = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$3.77 = (25\sin\theta)0.50 - \frac{1}{2}9.81(0.50)^2$$

$$\rightarrow \sin\theta = \frac{0.20}{0.50} \rightarrow \theta = 24^\circ$$

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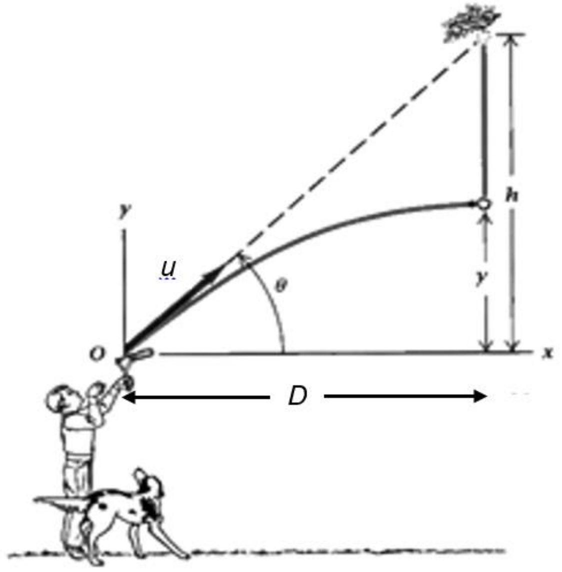

The horizontal distance is *D*

$$D = (u\cos\theta)t = (25\cos 24^\circ)0.50 = 11.4 \text{ m}$$

Think about the following scenario.

What would have happened if the boy aimed his slingshot higher than the pebble? Will the stone still hit the pebble?