TEMASEK JUNIOR COLLEGE
2022 JC2 Preliminary Examination
Higher 1

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

MATHEMATICS **8865/01**

Paper 1 **26 August 2022**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

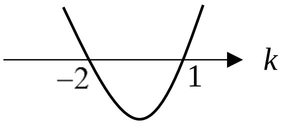
Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

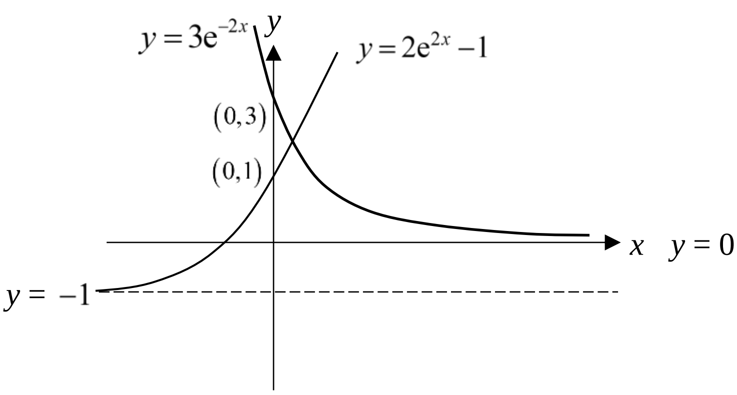
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

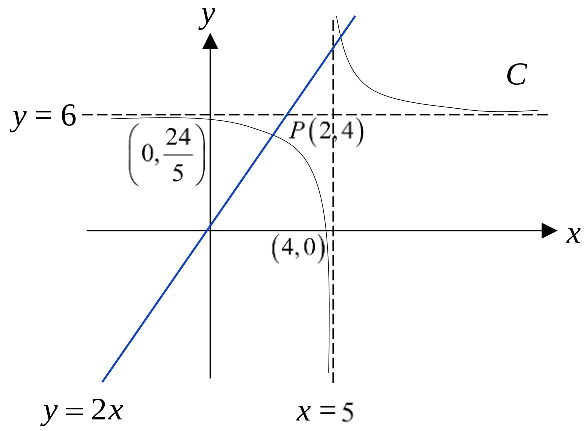
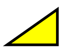
For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Marks	

Section A: Pure Mathematics [40 marks]

Qn 1	Solution
	For $2kx^2-4x+(k+1)>0$ for all real values of *x*, $2k>0$ and Discriminant < 0 $k>0$ and $16-4(2k)(k+1)<0$ $16-8k(k+1)<0$ $16-8k^2-8k<0$ $k^2+k-2>0$ $(k-1)(k+2)>0$  $k>0$ and $k>1$ or $k<-2$ Combining, $k>1$

Qn 2	Solution
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(9x^{\frac{1}{3}}-2x^2\right)^3=3\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(9x^{\frac{1}{3}}-2x^2\right)$ $=3\left(\frac{1}{9x^{\frac{1}{3}}-2x^2}\right)\frac{\mathrm{d}}{\mathrm{d}x}\left(9x^{\frac{1}{3}}-2x^2\right)$ $=\frac{3\left(3x^{-\frac{2}{3}}-4x\right)}{9x^{\frac{1}{3}}-2x^2}$ or $\frac{9x^{-\frac{2}{3}}-12x}{9x^{\frac{1}{3}}-2x^2}$
(b)	$\int\frac{(2x-3)^2}{\sqrt{x}}\,\mathrm{d}x=\int\frac{4x^2-12x+9}{\sqrt{x}}\,\mathrm{d}x$ $=\int 4x^{\frac{3}{2}}-12x^{\frac{1}{2}}+9x^{-\frac{1}{2}}\,\mathrm{d}x$ $=\frac{8}{5}x^{\frac{5}{2}}-8x^{\frac{3}{2}}+18x^{\frac{1}{2}}+C$

Qn 3	Solution
(i)	$2e^{2x}-1=3e^{-2x}$ $2e^{2x}-1-\frac{3}{e^{2x}}=0$ Let $u=e^{2x} \Rightarrow 2u-1-\frac{3}{u}=0$ $2u^2-u-3=0$ $(2u-3)(u+1)=0$ $e^{2x}=\frac{3}{2}$ or $e^{2x}=-1$ (reject $\because e^{2x}>0$) $x=\frac{1}{2}\ln\left(\frac{3}{2}\right)$
(ii)	
(iii)	$2e^{2x}-3e^{-2x}-1>0$ $2e^{2x}-1>3e^{-2x}$ From (i) and (ii), $x>\frac{1}{2}\ln\left(\frac{3}{2}\right)$
(iv)	Replace x by $-\frac{p}{2}$ $-\frac{p}{2}>\frac{1}{2}\ln\left(\frac{3}{2}\right)$ $p<-\ln\left(\frac{3}{2}\right)=\ln\left(\frac{2}{3}\right)$

Qn 4	Solution
(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{a}{(x-5)^2}$ At $x=2$, equation of tangent is $y=-\dfrac{2}{3}x+\dfrac{16}{3}$. (gradient is $-\dfrac{2}{3}$) $\dfrac{a}{(2-5)^2}=-\dfrac{2}{3}\Rightarrow a=-6$ $y=-6\int\left(x-5\right)^{-2}\mathrm{d}x$ $=-6\left[\dfrac{1}{-1}\left(x-5\right)^{-1}\right]+C$ $=\dfrac{6}{x-5}+C$ At $x=2,\ y=4$ $\therefore 4=\dfrac{6}{-3}+C\Rightarrow\ C=6$ Equation of curve *C*: $y=\dfrac{6}{x-5}+6=6\left(\dfrac{1}{x-5}+1\right)$. (Shown)
(ii)	
(iii)	Equation of line which passes through origin and *P*: $y=2x$ Sketch line (see **ii**)
(iv)	Exact area of the curve = Area of triangle  + Area under *C*  $=\dfrac{1}{2}\times 2\times 4+\int_2^4\left(\dfrac{6}{x-5}+6\right)\mathrm{d}x$ $=4+\left[6\ln\left\|x-5\right\|+6x\right]_2^4$ $=4+\left[6\ln 1+24-6\ln 3-12\right]$ $=16-6\ln 3$

Qn 5	Solution

(i)

Let n represent cost price of a nineth scale War-Robot
Let s represent cost price of a sixth scale War-Robot
Let q represent cost price of a quarter scale War-Robot

$$20n-4s=30 \quad \text{..............(1)}$$
$$30n+16s+6q=8517 \quad \text{..............(2)}$$
$$1.2n+s+0.8q=661.5 \quad \text{..........(3)}$$

By GC,
Cost price for a nineth scale War-Robot is \$57.00
Cost price for a sixth scale War-Robot is \$277.50
Cost price for a quarter scale War-Robot is \$394.50

(ii)

$$C=\frac{w}{20}-\ln(3w-60)+11$$
$$\frac{\mathrm{d}C}{\mathrm{d}w}=\frac{1}{20}-\frac{3}{3w-60}=\frac{1}{20}-\frac{1}{w-20}$$
$$\text{For } \frac{\mathrm{d}C}{\mathrm{d}w}=\frac{1}{20}-\frac{1}{W-20}=0$$
$$\frac{1}{20}=\frac{1}{w-20}$$
$$w-20=20$$
$$w=40$$

Method 1

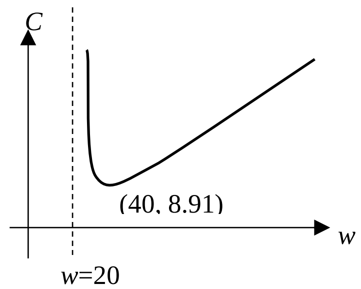
$$\frac{\mathrm{d}^2C}{\mathrm{d}w^2}=\frac{1}{\left(w-20\right)^2}>0 \text{ for } w>20$$

Method 2

w	39	40	41
$\frac{\mathrm{d}C}{\mathrm{d}w}$	–0.00263<0	0	0.00238>0
Shape of graph	\	—	/

C is minimum when w = 40

(iii)



Estimated minimum cost is \$8905.66

(iv)

$$P=0.42-\frac{C}{w}=0.42-\frac{1}{w}\left(\frac{w}{20}-\ln(3w-60)+11\right)$$

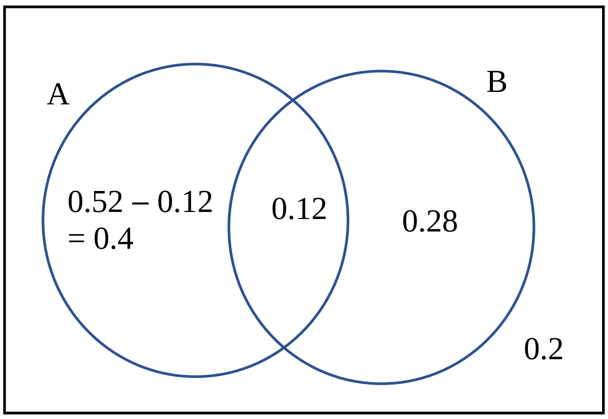
From GC, the manufacturer should only accept order with at least 24 sixth scale War-Robots.

Section B: Probability and Statistics [60 marks]

Qn 6	**Solution**
(i)	Number of different committees can be formed $=\binom{14}{5}-\binom{9}{5}-\binom{5}{5}$ $=1875$
(ii)	Number of ways $=3!\times\binom{3}{2}\times 2!$ $=36$

<table>
<tr><th>Qn 7</th><th>Solution</th></tr>
<tr><td>(i)</td><td>$\mathrm{P}\left(A'\cap B\right)$ represents the probability that B occurs and A does not occur.

$\mathrm{P}\left(A|B\right)=\dfrac{\mathrm{P}\left(A\cap B\right)}{\mathrm{P}(B)}=0.3$
$\mathrm{P}(A\cap B)=0.3\times\mathrm{P}(B)=0.3\times0.4=0.12$

$\mathrm{P}(A'\cap B)=\mathrm{P}(B)-\mathrm{P}(A\cap B)=0.4-0.12=0.28$</td></tr>
<tr><td>(ii)</td><td>$\mathrm{P}(A)=\mathrm{P}(A\cup B)-\mathrm{P}(A'\cap B)$
$=0.8-0.28$
$=0.52$
Or
$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$
$\Rightarrow 0.8=\mathrm{P}(A)+0.4-0.12$
$\Rightarrow \mathrm{P}(A)=0.52$</td></tr>
<tr><td>(iii)</td><td>
</td></tr>
<tr><td>(iv)</td><td>Since $\mathrm{P}(A)\cdot\mathrm{P}(B)=0.52\times0.4=0.208\neq0.12=\mathrm{P}(A\cap B)$,
A and B are not independent.</td></tr>
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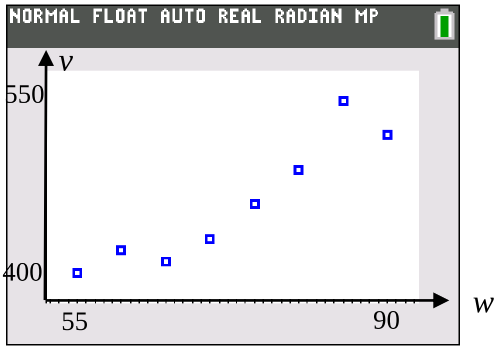
Qn 8	Solution
(i)	$X \sim \mathrm{B}(60, p)$ $\mathrm{P}(X \le 1) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.61$ $\binom{60}{0} p^0 (1-p)^{60} + \binom{60}{1} p^1 (1-p)^{59} = 0.61$ $(1-p)^{60} + 60p(1-p)^{59} = 0.61$ Using GC, $p = 0.0224$ (3 s.f.)
(ii)	$X \sim \mathrm{B}(60, 0.02)$ $\mathrm{P}(X \ge 2) = 1 - \mathrm{P}(X \le 1) = 0.33810 \approx 0.338$ (3sf)
(iii)	Let Y be the number of days with day's production accepted as satisfactory out of a month of 30 days. $Y \sim \mathrm{B}(30, 0.6619)$ $\mathrm{E}(Y) = 30(0.6619) = 19.857$ $\mathrm{Var}(Y) = 30(0.6619)(1-0.6619) = 6.7137$ Required probability $= \mathrm{P}(19.857 - \sqrt{6.7137} < Y < 19.857 + \sqrt{6.7137})$ $\approx \mathrm{P}(17.27 < Y < 22.45)$ $= \mathrm{P}(18 \le Y \le 22)$ $= \mathrm{P}(Y \le 22) - \mathrm{P}(Y \le 17)$ ≈ 0.66593 $= 0.666$ (3 s.f.)

Qn 9	Solution
(i)	
(ii)	From GC, $r = 0.93259 \approx 0.933$ (3 s.f.) Since $r = 0.933$ is close to 1, there is a strong positive linear correlation between the weekly advertising costs (in dollars) and the number of items sold weekly.
(iii)	$v = 156.19 + 4.1904w$ i.e, $v = 156 + 4.19w$ $w = -22.972 + 0.20755v$ i.e, $w = -23.0 + 0.208v$
(iv)	Use $v = 156.19 + 4.1904w$ $500 = 156.19 + 4.1904w$ $w \approx 82$ An estimate of the advertising cost is $82 when the number of items sold in the week was 500. Use the line v on w as v is the dependent variable and w is the independent/controlled variable or v depends on w. The estimate is reliable as 1. $v = 500$ is within the data range of 400 to 550. 2. $r \approx 0.933$ is close to 1, so the points will lie close to a straight line, thus estimate is reliable.
(v)	The value of r found in **(ii)** will remain unchanged.

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Qn 10	Solution
(i)	$\text{P}\left(\text{1st spin not star} \mid \text{spins 2 stars}\right)=\dfrac{\text{P}\left(\text{1st spin not star and 2nd,3rd spin stars}\right)}{\text{P}\left(\text{spins 2 stars}\right)}$ $=\dfrac{\frac{5}{8}\times\frac{3}{8}\times\frac{3}{8}}{\frac{3}{8}\times\frac{3}{8}\times\frac{5}{8}+\frac{3}{8}\times\frac{5}{8}\times\frac{3}{8}+\frac{5}{8}\times\frac{3}{8}\times\frac{3}{8}}$ $=\dfrac{1}{3}$
(ii)	P(wins a \$6 voucher) $=\text{P}\left(\text{3 spins same shape}\right)$ $=\left(\dfrac{3}{8}\right)^3+\left(\dfrac{2}{8}\right)^3+\left(\dfrac{2}{8}\right)^3+\left(\dfrac{1}{8}\right)^3$ $=\dfrac{11}{128}$
(iii)	P(wins a \$2 voucher) $=\dfrac{3}{8}\times\dfrac{3}{8}\times\dfrac{5}{8}\times 3+\dfrac{3}{8}\times\dfrac{5}{8}\times\dfrac{5}{8}\times 3$ $=\dfrac{45}{64}$
	$\text{P}\left(\text{Anand wins \$10 in total}\right)=\dfrac{11}{128}\times\dfrac{45}{64}\times\dfrac{45}{64}\times 3$ $=0.12746$ $\text{P}\left(\text{Charlie wins nothing}\right)=\left(1-\dfrac{11}{128}-\dfrac{45}{64}\right)^3$ $=0.0093856$ $\text{P}\left(\text{Anand wins \$10 in total and Charlie wins nothing}\right)=0.12746\times 0.0093856$ $=0.0011963$ Required probability $=0.12746+0.0093856-2\left(0.0011963\right)$ $=0.134$

Qn 11	Solution
(i)	Unbiased estimate of μ, $\bar{x}=\dfrac{260}{100}+30=32.6$ Unbiased estimate of $\sigma^2, s^2=\dfrac{1}{99}\left(7844-\dfrac{260^2}{100}\right)$ $=72.40404\approx 72.4\ (3\text{sf})$
(ii)	$\text{H}_0: \mu=31.4$ $\text{H}_1: \mu>31.4$ Level of Significance: 5% Under H_0, since n = 100 is large, by Central Limit Theorem, $\bar{X}\sim \text{N}\left(31.4, \dfrac{72.40404}{100}\right)$ approximately and $Z=\dfrac{\bar{X}-31.4}{\sqrt{\dfrac{72.40404}{100}}}\sim \text{N}(0,1)$ approximately. Using a one-tailed test, $\bar{x}$ = 32.60 gives $p\text{-value}=0.079225>0.05$ Since p-value > level of significance, we do not reject H_0. Hence there is insufficient evidence at 5% level of significance to conclude that the mean height of the plants has increased under the genetic engineering.
(iii)	No, it is not necessary. The sample mean height of the plants can be approximated to a Normal distribution by Central Limit Theorem since the sample size of 100 plants is large enough.
(iv)	Level of Significance: 10% Under H_0, since n = 100 is large, by Central Limit Theorem, $\bar{X}\sim \text{N}\left(31.4, \dfrac{\frac{100}{99}\times 55}{100}\right)$ approximately and $Z=\dfrac{\bar{X}-31.4}{\sqrt{\dfrac{\frac{100}{99}\times 55}{100}}}\sim \text{N}(0,1)$ approximately. Using a one-tailed test, reject H_0 when $z_{\text{cal}}\geq 1.28155$ Since H_0 is rejected,

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	$z_{\text{cal}} = \dfrac{h-31.4}{\sqrt{\dfrac{\frac{100}{99}\times 55}{100}}} \geq 1.28155$ $h \geq 31.4 + 1.28155\sqrt{\dfrac{55}{99}}$ $h \geq 32.4$

Qn 12	Solution
(i)	Let X and Y be the duration of a patient's consultation in minutes with a GP and a specialist respectively. $X\sim N(6.5, 1.7^2)$, $Y\sim N(\mu, \sigma^2)$ $P(Y<11.8)=P(Y>12.8)$ By symmetry, $\mu=\frac{11.8+12.8}{2}=12.3$ $P(Y<11.8)=0.406$ $P\left(Z<\frac{11.8-12.3}{\sigma}\right)=0.406$ $\frac{-0.5}{\sigma}=-0.23785 \Rightarrow \sigma=2.1$
(ii)	$Y\sim N(10.5, 2.9^2)$ $X_1+X_2+X_3-2Y \sim N(3\times6.5-2\times10.5,\ 3\times1.7^2+2^2\times2.9^2)$ $X_1+X_2+X_3-2Y \sim N(-1.5,\ 42.31)$ $P(X_1+X_2+X_3-2Y<0)=0.591$
(iii)	$P(X>a)=0.1$ Using GC, $a=8.68$ minutes
(iv)	Let G and S be the consultation fees for a randomly chosen patient visiting GP and specialist respectively. $G=10+X \sim N(16.5, 1.7^2)$ $S=25+2Y \sim N(46, 33.64)$ $S_1+S_2-(G_1+....+G_5) \sim N(2\times46-5\times16.5, 2\times33.64+5\times1.7^2)$ $S_1+S_2-(G_1+....+G_5) \sim N(9.5, 81.73)$ $P(-5<S_1+S_2-(G_1+....+G_5)<5)=0.255$
(v)	Let W be the duration of a patient's consultation in minutes with a dentist. Given $E(W)=15.2$ $Var(W)=3.1^2$ Let $\overline{W}=\frac{W_1+....+W_{80}}{80}$ Since n is large, by Central Limit Theorem, $\overline{W}\sim N(15.2, \frac{3.1^2}{80})$ approximately $P(16<\overline{W}<18)=0.0105$