FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS   4052/01

Paper 1

Date: 22 August 2023   **Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.
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READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142.

For Examiner's Use

Table of Penalties		Question Number		
Presentation	□ 1			
	□ 2		Parent's / Guardian's Signature	**90**
Rounding off	□ 1			

Setters: Ms Shamsiah and Mr Kua KT

This question paper consists of <u>23</u> printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \dfrac{1}{3}\pi r^2 h$

Volume of a sphere $= \dfrac{4}{3}\pi r^3$

Area of a triangle $ABC = \dfrac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \dfrac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

NAME: _____ (   )   CLASS: _____

Answer **all** the questions.

1 **(a)** Express 540 as a product of its prime factors.

Answer …………………………………… [1]

(b) The number $\dfrac{540m}{n}$ is a perfect cube.

m and n are prime numbers.
Find the value of m and the value of n.

Answer $m =$ …………………………………

$n =$ ………………………………… [1]

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2 **(a)** Calculate $\dfrac{13.4^3}{7.56 - 4.89}$.

Write your answer correct to 5 significant figures.

Answer …………………………………… [1]

(b) Write your answer to **part (a)** in standard form.

Answer …………………………………… [1]

3   The first four terms of a sequence are 13, 17, 21, 25.

    (a)   Write down the 7th term of the sequence.

Answer …………………………………… [1]

    (b)   Write down an expression for the *n*th term of the sequence.

Answer …………………………………… [1]

    (c)   Explain why 318 is not a term of this sequence.

    Answer

…………………………………………………………………………………………………

…………………………………………………………………………………………….. [1]

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4   The pie chart below shows the age groups (in years) of 240 adults who took part in a triathlon.
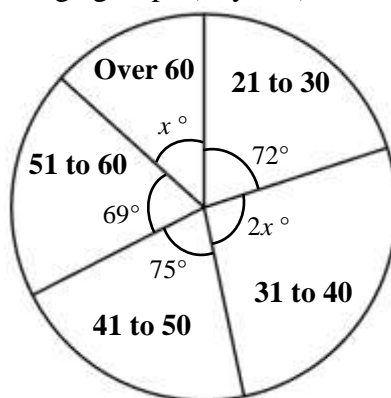
    (a)   Find the value of *x*.

Answer   *x* = …………………………… [1]

    (b)   Calculate the number of adults aged 41 to 50 years old who took part in the triathlon.

Answer ………………………… adults [1]

5   *L* is a line with a negative gradient and it has positive *x*- and *y*-intercepts. The value of *y*-intercept is five times the value of *x*-intercept. Given that the *x*-intercept is $\dfrac{2}{5}$, find the equation of *L*.

Answer ………………………………… [3]

6   An empty fuel tank is filled using a cylindrical pipe with diameter 8 cm. Fuel flows along this pipe at a rate of 2 metres per second. It takes 24 minutes to fill the tank. Calculate the capacity of the tank. Give your answer in litres.

Answer ………………………………… *l* [3]

7 **(a)** Simplify $\left(81x^4\right)^{-\frac{3}{4}}$.

Answer ………………………………… [2]

(b) Solve $32^{\frac{1}{5}} \times 2^x = 8^{\frac{1}{4}}$.

Answer $x =$ ……………………..……… [2]

8 **(a)** $\xi = \{\text{integers } x : 2 \le x < 24\}$

$P = \{\text{multiples of 3}\}$
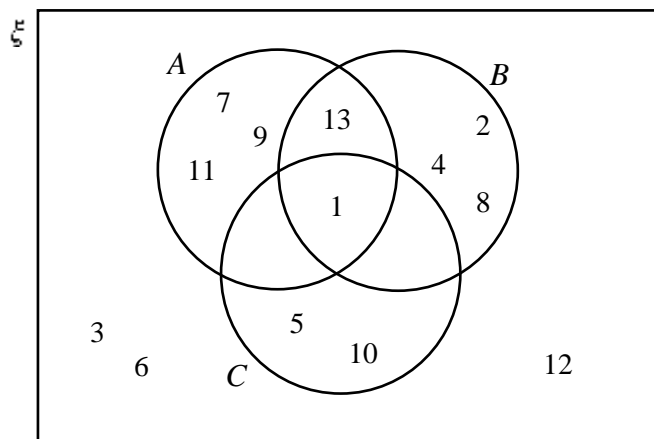
$Q = \{\text{prime numbers}\}$

List the elements in

(i) P,

Answer ………………………………………………………………… [1]

(ii) $(P \cup Q)'$.

Answer ………………………………………………………………… [1]

(b) The Venn diagram below shows the elements of $\xi = \{\text{integers } x: 1 \le x \le 13\}$ and three sets A, B and C.

(i) Circle the correct statement(s) from the list below.

$n(A) = 3$ $A \cup B = \{1, 13\}$ $A' \cap (B \cap C) = \varnothing$

$5 \in A' \cap C$ $B' \subset C$

[2]

(ii) Find the value of $n[B' \cap (A \cup C)]$.

Answer ………………………………………… [1]

9 Simplify $\dfrac{4m^2 - 20mn + 16n^2}{3m - 12n}$.

Answer ………………………………… [3]

10 Ching and Lex each have a savings account.
The ratio   Ching's savings : Lex's savings = 3 : 5.

They each spent \$60 from their savings.
The new ratio   Ching's savings : Lex's savings = 4 : 7.

Find the **total** amount of money Ching and Lex have in their accounts now.

Answer \$ ………………………………… [4]

11   (a)   Solve $\cos x = -\cos 65°$, where $0° \leq x \leq 180°$.

Answer $x = $ …………………………..… $°$ [1]

(b)   The area of a triangle PQR is 15 cm^2, $PQ = 10$ cm and $PR = 6$ cm.

Find the possible values of $\angle QPR$.

Answer $\angle QPR = $ …………$°$ or …………$°$ [3]

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12   Given that $\mathbf{A} = \begin{pmatrix} 4 & 6 \\ 0 & -2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & k \\ 0 & -1 \end{pmatrix}$, find

(a) $\mathbf{A}^2$,

Answer   ………………………………… [1]

(b)   the value of k if $\mathbf{A} = 2\mathbf{B}$.

Answer $k = $ …………..………………… [1]

13 The sales of the IMic and Lovono laptops, in dollars, made by PC Enterprise in the years 2021 and 2022 are summarised below.

Year	Sales	
	IMic	**Lovono**
2021	34 000	20 100
2022	14 500	30 000

(a) Represent the information in a 2×2 matrix **S**.

Answer   **S** = …………………..……………… [1]

(b) Evaluate the matrix **R** = (1   1) **S**.

Answer   ………………………………… [1]

(c) State what each element in matrix **R** represents.

Answer ……………………………………………………………………………………

………………………………………………………………………………………… [1]

14   In the diagram below, the lines *AB* and *CD* are parallel.
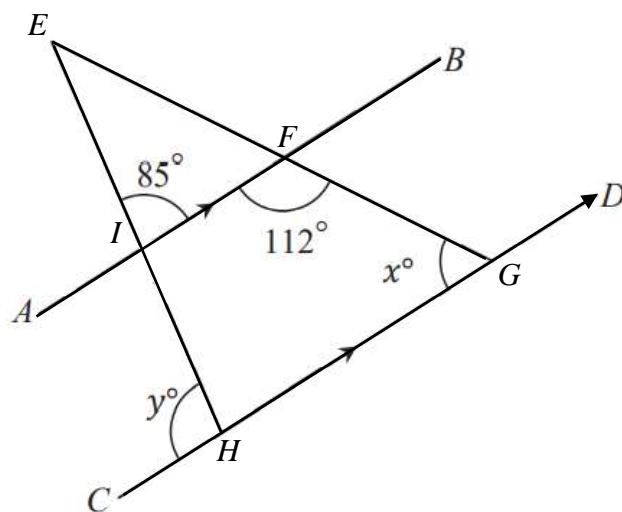
By stating your reasons clearly, find the values of

(a)   *x*,

Answer $x =$ .... [1]
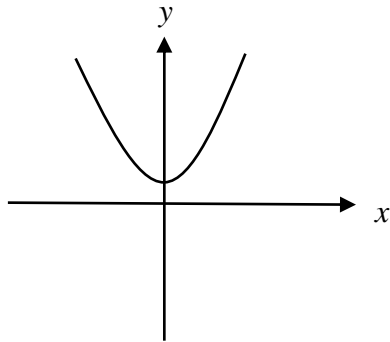
(b)   *y*.

Answer $y =$ .... [2]

_____

15 Write down a possible equation for each of the graph below.

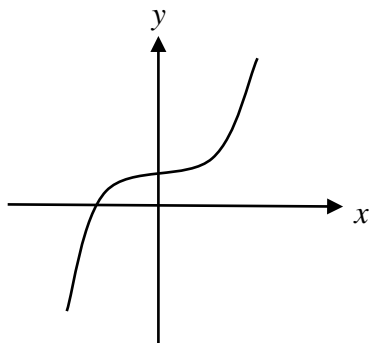In each case, select one of the equations from the table below.

$y = x^3 + 3$	$y = x^2 + 3$	$y = 3x^{-2}$	$y = 3x + 2$
$y = 3 - x^2$	$y = -\dfrac{3}{x^2}$	$y = x^3 - 3$	$y = 3^x + 3$

(a)

Answer ………………………………… [1]

(b)

Answer ………………………………… [1]

(c)
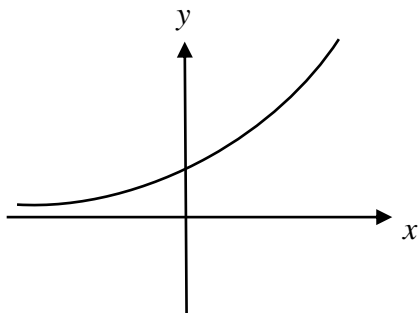
Answer ………………………………… [1]

(d)
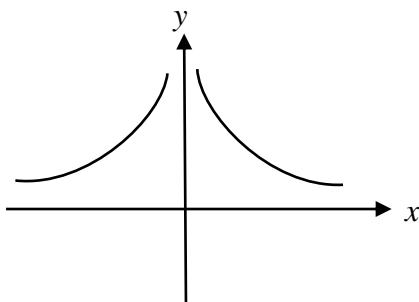
Answer ………………………………… [1]
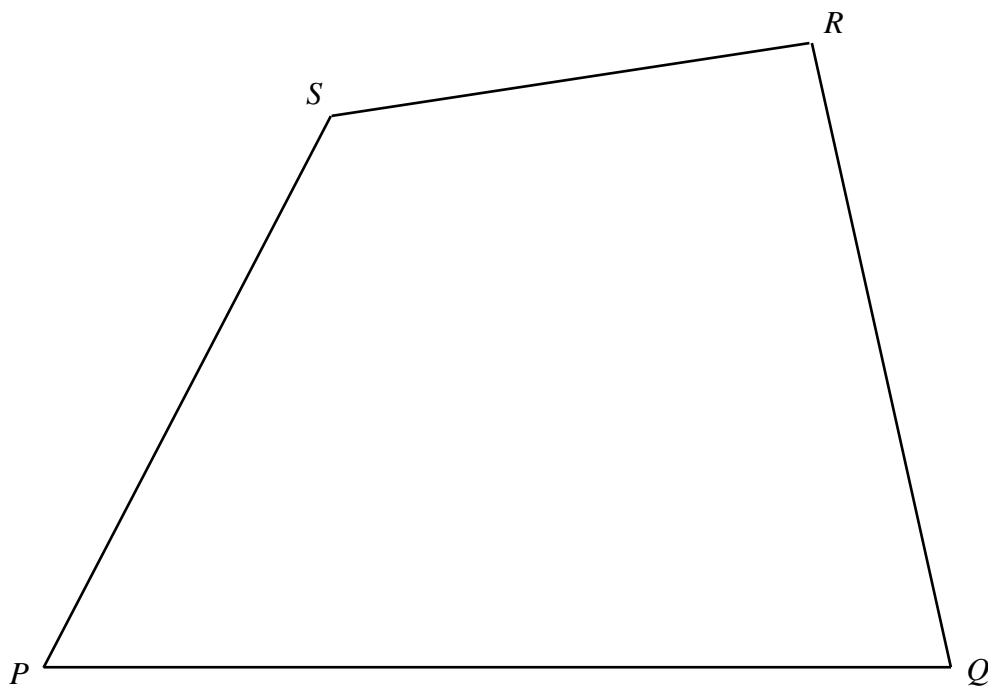
16   The diagram below represents a plot of land, *PQRS*, which is to be used for a park.

(a)   Construct the perpendicular bisector of *PQ*.   [1]

(b)   Construct the bisector of angle *PSR*.   [1]

(c)   A children's playground is to be built in the park. The planned location of the

playground is nearer to *Q* than to *P*, and nearer to *PS* than to *RS*.

Shade the region where the playground can be built.   [1]

17   Factorise completely

(a) $3ax + 16by - 12ay - 4bx$,

Answer ……………………………………………………… [2]

(b) $3mn - 243mn^5$.

Answer ……………………………………………………… [3]

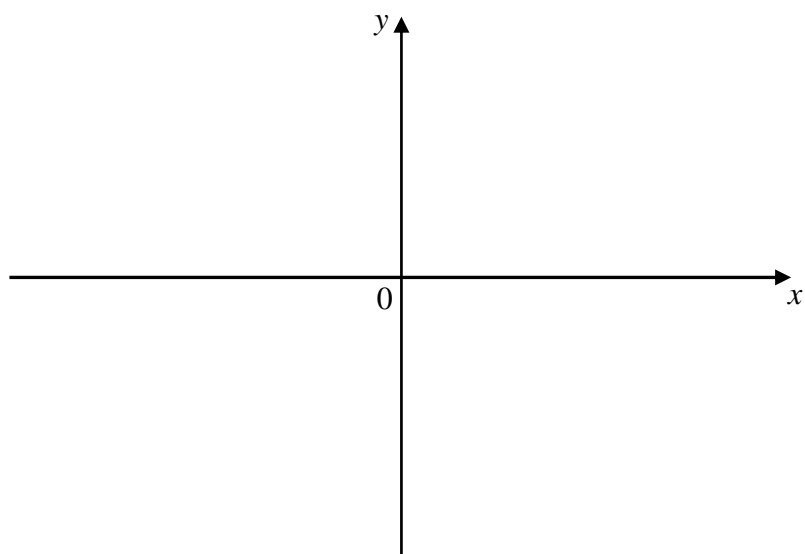**18 (a)** Express $x^2 + 16x - 30$ in the form of $(x + h)^2 - k$.

Answer ………………………………… [1]

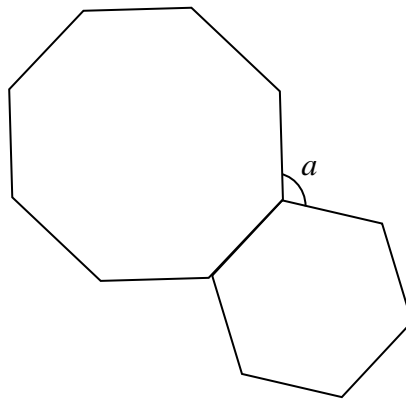**(b)** Hence, solve the equation $x^2 + 16x - 30 = 0$, giving your answers correct to 2 decimal places.

Answer $x =$ …………..…… or ……………..… [2]

(c) Sketch the graph of $y = x^2 + 16x - 30$, showing the turning point and *y*-intercept clearly.

Answer   [2]

19   Ryan joins two tiles together as shown below. One tile is a regular hexagon and the other tile is a regular octagon.

(a)   Show that the angle a is $105°$.

Answer   [3]

(b)   Ryan claims that there is another tile in the shape of a regular polygon with interior angle a. Is Ryan correct? Show your reasoning.

Answer Ryan is correct / incorrect* (*Circle the correct answer*) because
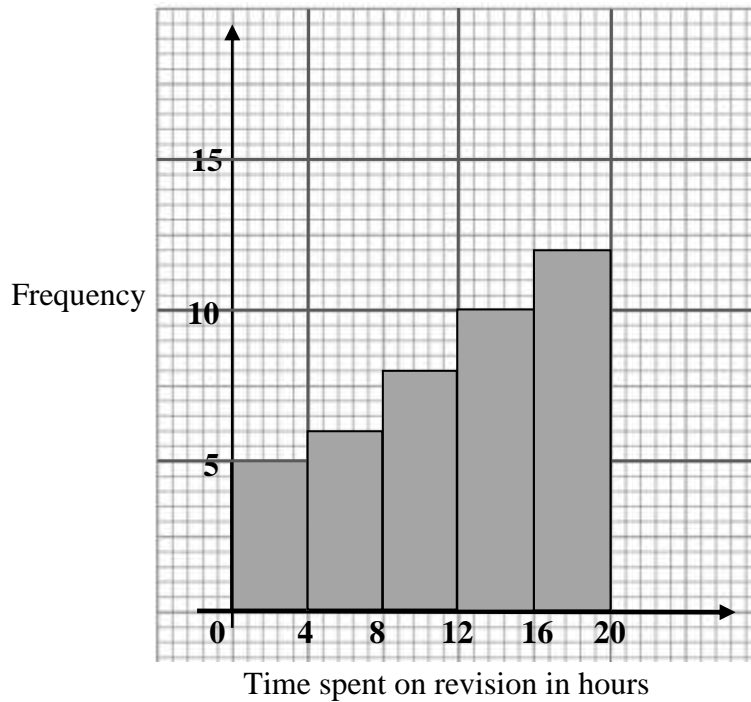
…………………………………………………………………………………………………

…………………………………………………………………………………………………

……………………………………………………………………………………………[3]

20   The histogram below shows the distribution of the time spent in hours by 41 students on revision in a week.

Time spent on revision in hours

(a)   Find the percentage of students who spent more than 12 hours in a week for revision.

Answer   ……………………………… % [1]

(b)   State the class interval which the median lies.

Answer   ……..………………………… h [1]

20   **(c)**   Calculate
          (i)   the estimated mean and

Answer …….………………………… h [1]

       (ii)   the standard deviation of time spent for revision.

Answer …….………………………… h [1]

_____

21 In the diagram below, *AB* and *CD* are two equal chords of the circle with centre *O* and radius 25 cm. The chords are extended and meet at the point *E*.
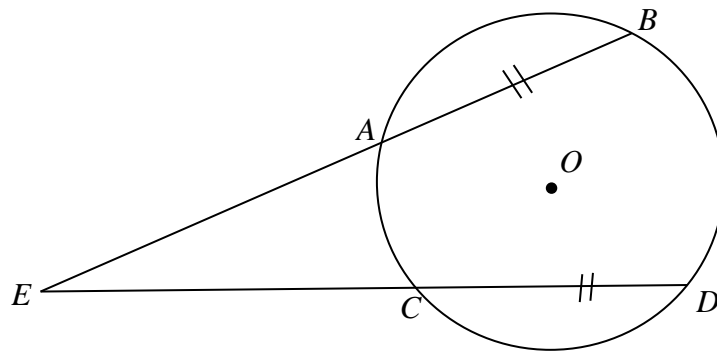
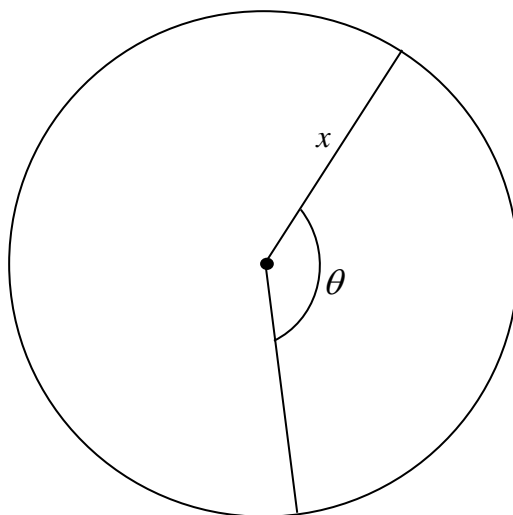**(a)** Prove that *EA* = *EC*.

Answer   [4]

(b) Given that *AB* = 40 cm and angle *BED* = 30°, find the length of *AE*.

Answer ……..………………………… cm [3]

22   The diagram below shows a circle with radius *x* cm. The circle is divided into two sectors. The angle of the minor sector is θ radians.

The perimeter of the major sector is thrice the perimeter of the minor sector.

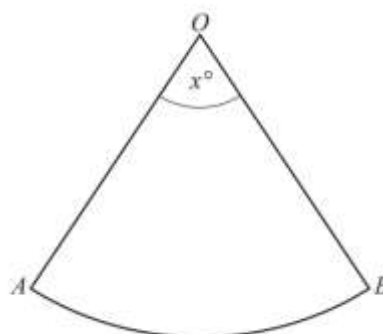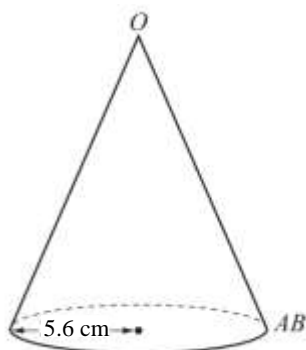Find the value of θ. Give your answer correct to 3 decimal places.

Answer ………………………… radians [4]

23 **(a)** The diagram below is a hollow cone of radius 5.6 cm and its volume is 259.44 cm^3.

The cone is cut along the slant height from O to AB and is opened to form a sector OAB of a circle with centre O. Calculate the sector angle $x°$.
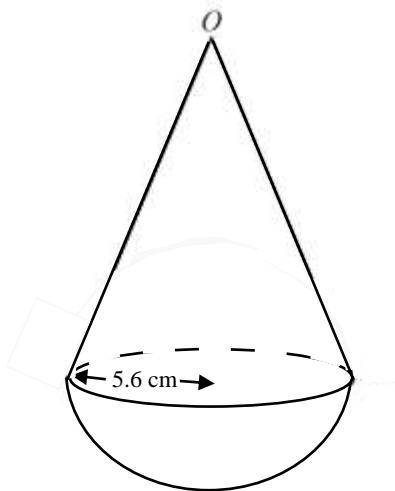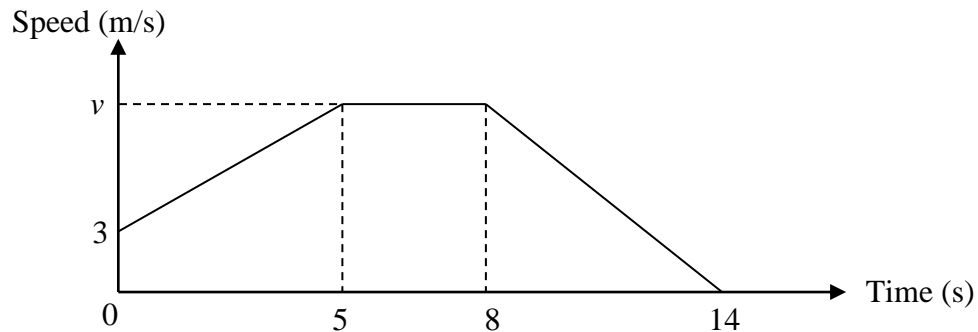

Answer $x =$ ..…………………………… [3]

23   (b)   Another cone in part **(a)** is joined to a solid hemisphere to form an ornament as shown below. Calculate the volume of the ornament.
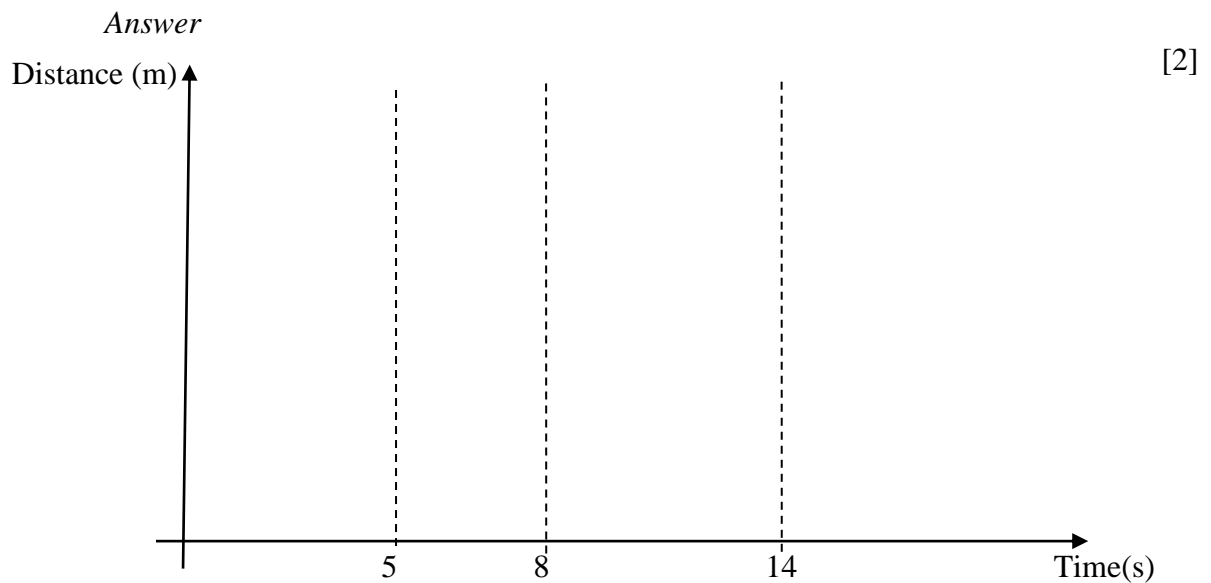

Answer ……………………………. cm^3 [3]

24   A particle starts moving at 3m/s and accelerates uniformly at 2 m/s^2 for the first

5 seconds. It then moves with constant speed for 3 seconds, and takes another 6 seconds

to slow down uniformly to rest. The speed time graph is shown below.
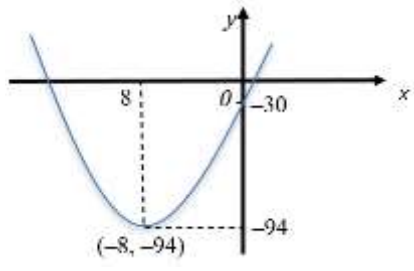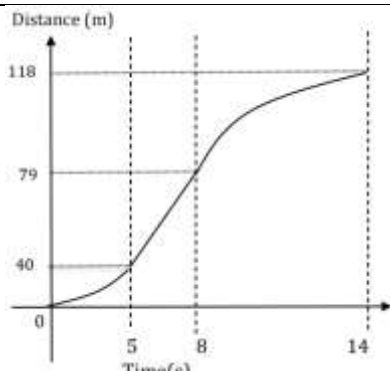

(a)   State the value of *v*.

Answer   *v* = ...…………………………… [1]

(b) Sketch the distance time graph for the motion of the particle.

Answer

[2]

End of paper

Fairfield Methodist School (Secondary)
Secondary 4 Express / 5 Normal (Academic) 2023
Mathematics Preliminary Examination Paper 1
One-page Answer

1(a)	$540 = 2^2 \times 3^3 \times 5$	**15(a)**	$y = x^2 + 3$	
1(b)	$m = 2$, $n = 5$	**15(b)**	$y = x^3 + 3$	
2(a)	901.16	**15(c)**	$y = 3^x + 3$	
2(b)	9.0116×10^2 or 9.01×10^2 (3 s.f.)	**15(d)**	$y = 3x^{-2}$	
3(a)	37	**16**	Refer to next page	
3(b)	$9 + 4n$ or $13 + 4(n-1)$	**17(a)**	$(3a - 4b)(x - 4y)$ or $(4b - 3a)(4y - x)$	
3(c)	$n = 77\frac{1}{4}$ Since n is not a **positive** integer, 318 is not a term of the sequence.	**17(b)**	$3mn(1 + 9n^2)(1 + 3n)(1 - 3n)$	
		18(a)	$(x+8)^2 - 94$	
		18(b)	$x = 1.70$ (2 d.p.) or -17.70 (2 d.p.)	
4(a)	$x = 48$	**18(c)**		
4(b)	No. of adults = 50			
5	$y = -5x + 2$			
6	14500 l (to 3 s.f.)			
7(a)	$\dfrac{1}{27x^3}$			
7(b)	$x = -\dfrac{1}{4}$ or -0.25	**19(b)**	$n = 4.8$ Ryan is incorrect because n is not a positive integer, thus the polygon does not exist.	
8(a)(i)	{3, 6, 9, 12, 15, 18, 21}			
8(a)(ii)	{4, 8, 10, 14, 16, 20, 22}			
8(b)(i)	$A' \cap (B \cap C) = \varnothing$ $5 \in A' \cap C$	**20(a)**	53.7% (3 s.f.)	
8(b)(ii)	n$[B' \cap (A \cup C)] = 5$	**20(b)**	12 hr <time $\le$ 16 hr	
9	$\dfrac{4(m-n)}{3}$ or $\dfrac{4m - 4n}{3}$	**20(ci)**	11.8 h	
10	$1320	**20(cii)**	5.45 h	
11(a)	$x = 115°$	**21(b)**	$AE = 36.0$ cm (3 s.f.)	
11(b)	$\angle QPR = 30°$ or $150°$	**22**	$\theta = 0.571$ radians (3 d.p.)	
12(a)	$\mathbf{A}^2 = \begin{pmatrix} 16 & 12 \\ 0 & 4 \end{pmatrix}$	**23(a)**	$x = 208.2°$ (1 d.p.)	
		23(b)	Volume = 627 cm^3 (3 s.f.)	
12(b)	$k = 3$	**24(a)**	$v = 13$	
13(a)	$\mathbf{S} = \begin{pmatrix} 34000 & 20100 \\ 14500 & 30000 \end{pmatrix}$	**24(b)**		
13(b)	$\mathbf{R} = \begin{pmatrix} 48500 & 50100 \end{pmatrix}$			
13(c)	The elements represent the **total/combined sales in 2021 and 2022** for **IMic and Lovono laptops respectively**.			
14(a)	$x = 68$			
14(b)	$y = 95$			

Q16