CHIJ ST. THERESA'S CONVENT

PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/1

Paper 1 **29 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the gradient of the normal to the curve $y = x^{\frac{3}{2}} + 4x$ at $x = 5$, leaving your answers in the form $\left(a\sqrt{5} + b\right)$, where *a* and *b* are constants. [4]

2 Integrate $\dfrac{5}{3x+1} - \dfrac{4}{(1-x)^2}$ with respect to *x*. [4]

3 **(a)** Express $1-16x-16x^2$ in the form $a\left(x+b\right)^2+c$ and hence state the coordinates of the turning point of the curve $y=1-16x-16x^2$. [4]

(b) Hence, explain why the curve $y=1-16x-16x^2$ will never intersect $y = 7$ for all values of x. [2]

4 The equation of a curve is $y = a\cos bx + c$, where $a > 0$.

(a) Given that the maximum and minimum values of y are 1 and – 9 respectively, state the values of a and c. [2]

(b) Given that the period of y is π, find the value of b. [1]

(c) Hence, sketch the curve $y = a\cos bx + c$ for $0 \le x \le 2\pi$ radians, labelling the maximum and minimum points clearly. [3]

5 The function $\mathrm{f}(x) = ax^3 + 5x^2 + bx + 1$ has a factor $(2x-1)$ and leaves a remainder of 9 when divided by $(x+1)$.

(a) Show that $b = -5$ and find the value of a. [4]

(b) Given that the quadratic expression $x^2 + px - 1$ is also a factor of $\mathrm{f}(x)$, find the value of the constant p. [2]

6 Express $\dfrac{3x^3 - 4x^2 + 4x - 17}{3x^2 - x - 4}$ in partial fractions. [7]

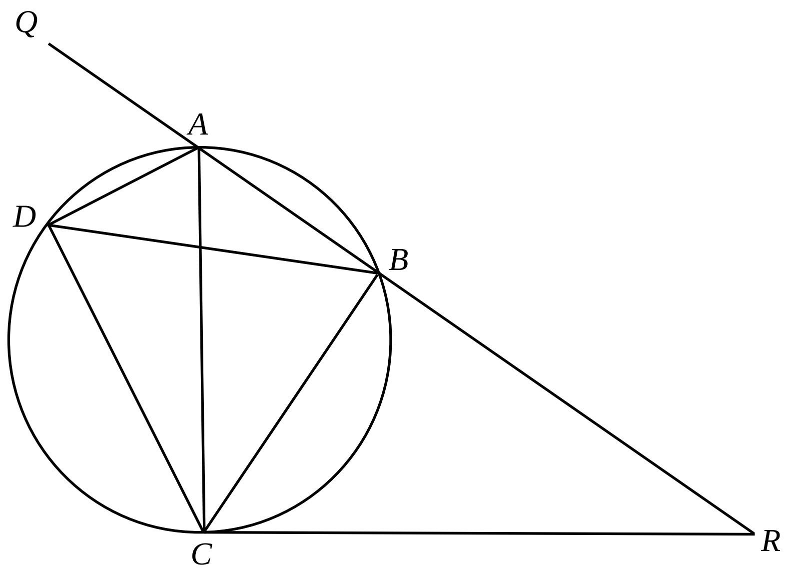
7 The number of bacteria cells, N, in millions, in a circular patch after t hours is given by $N = ke^{2t}$, where k is a constant. The initial number of bacteria cells is 10,000,000.

(a) Explain why $k = 10$. [1]

(b) Find the rate at which the number of bacteria cells is increasing after 30 minutes. [2]

(c) As the number of bacteria cells increases, the area, A units^2, of the circular patch also increases. Given that $4A = N^2$, find the rate of change of A after 5 hours, leaving your answer in exact form. [4]

8 In the diagram, CR is a tangent to the circle at C, and line QR cuts the circle at points A and B. D lies on the circumference of the circle.



(a) Prove that triangle CBR and triangle ACR are similar. [2]

(b) Given that $\angle ADB = \angle BRC$ **,** show that AC is the diameter. [4]

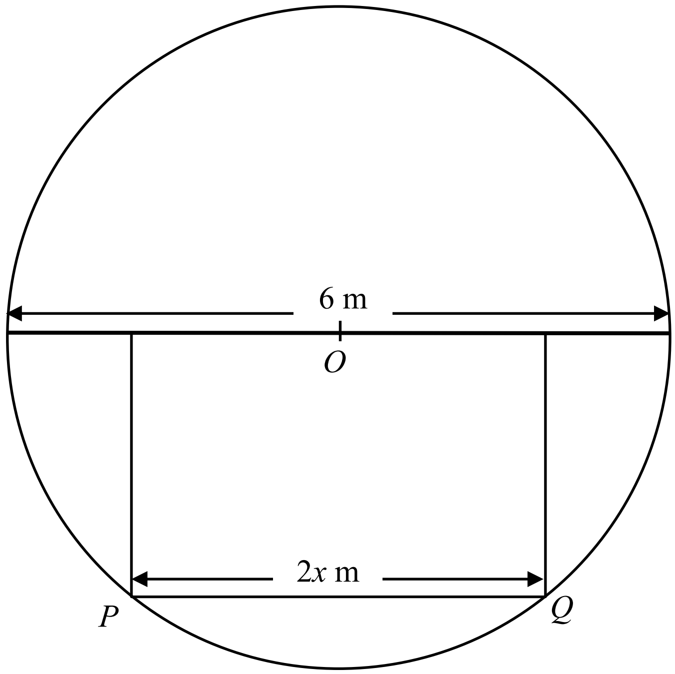
9 **(a)** Prove the identity $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{1+\sin\theta}{\cos\theta}=2\sec\theta$. [4]

(b) Hence solve the equation $\dfrac{\cos\frac{\theta}{2}}{1+\sin\frac{\theta}{2}}+\dfrac{1+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=5$ for $-180^\circ \le \theta \le 180^\circ$. [4]

10 **(a)** Given that $\log_8 x + \log_2 (1 - y) = 1$, express y in terms of x. [4]

(b) Solve the equation $\dfrac{5^{2x-3}}{3^x} = 2$. [4]

11 The diagram represents the circular cross-section area of a commercial plane, with center O. Cargo is stored in a rectangular container in the lower half of the plane, touching the fuselage at points P and Q. The plane is 6 m wide and the width of the rectangular container is $2x$ m.

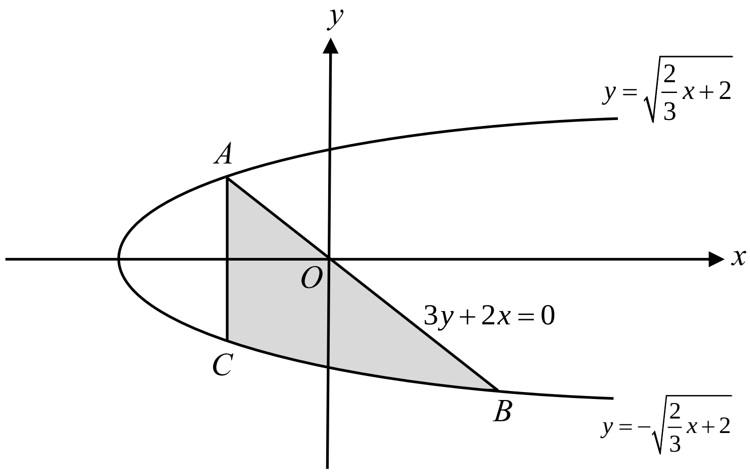


(a) Show that the area, A m^2, of the cargo container, is given by $A = 2x\sqrt{9-x^2}$ m^2. [3]

(b) Given that x can vary, find the stationary value of A and determine its nature. [6]

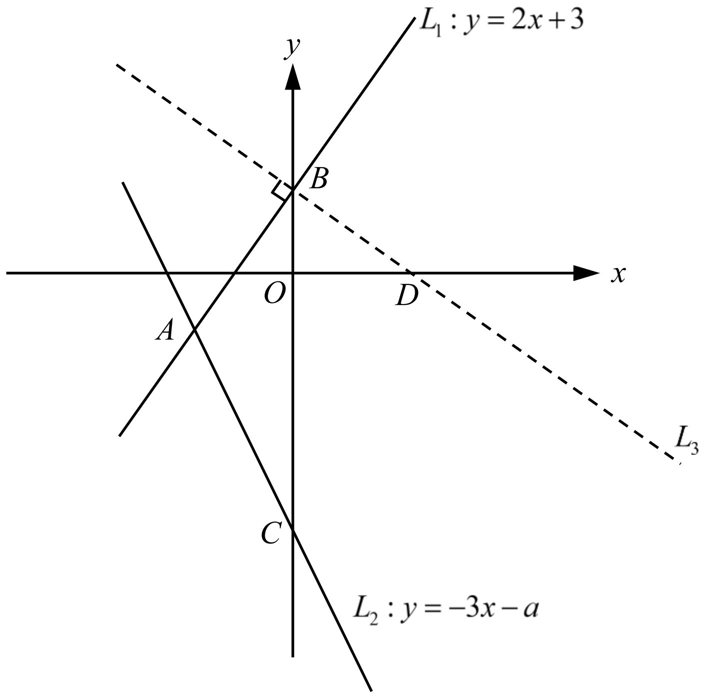
12 **(a)** The curve $y^2 = \frac{2}{3}x + 2$ and the line $3y + 2x = 0$ intersect at the points A and B. Find the coordinates of A and B. [4]

(b) The curve of $y^2 = \frac{2}{3}x + 2$ is made of two parts; the part above the x-axis is given by $y = \sqrt{\frac{2}{3}x + 2}$ and the curve below the x-axis is given by $y = -\sqrt{\frac{2}{3}x + 2}$. The diagram shows the curve $y^2 = \frac{2}{3}x + 2$ and the line $3y + 2x = 0$.



Find the area of the shaded region bounded by the curve, the line $3y + 2x = 0$, and the vertical line AC. [5]

13 The diagram, which is not drawn to scale, shows the line $L_1 : y = 2x + 3$ and the line $L_2 : y = -3x - a$. Lines L_1 and L_2 intersect at A. Points B and C lie on the y-axis.



(a) Given that the area of triangle ABC is 10 units^2, show that $a = 7$. [3]

(b) Line L_3 passes through *B* and is perpendicular to L_1. Given that it intersects the *x*-axis at *D*, find the coordinates of *D*. [3]

(c) Hence, find the area of quadrilateral *ABCD*. [2]

(d) Is *ABCD* a trapezium? Justify your answer. [2]

~~~ End of Part B ~~~
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