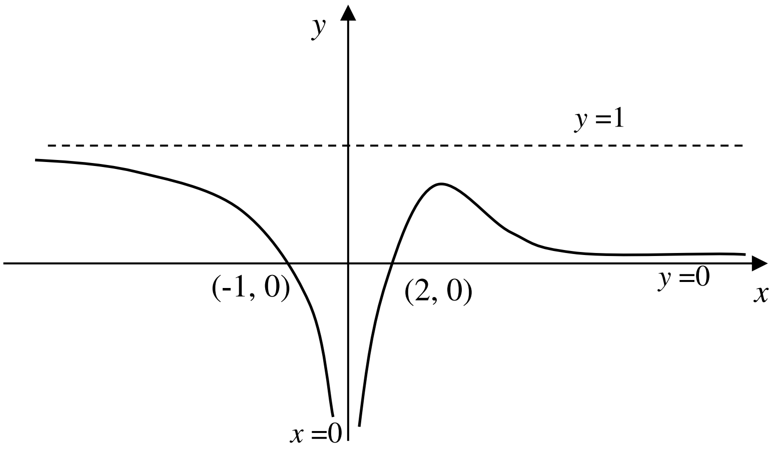
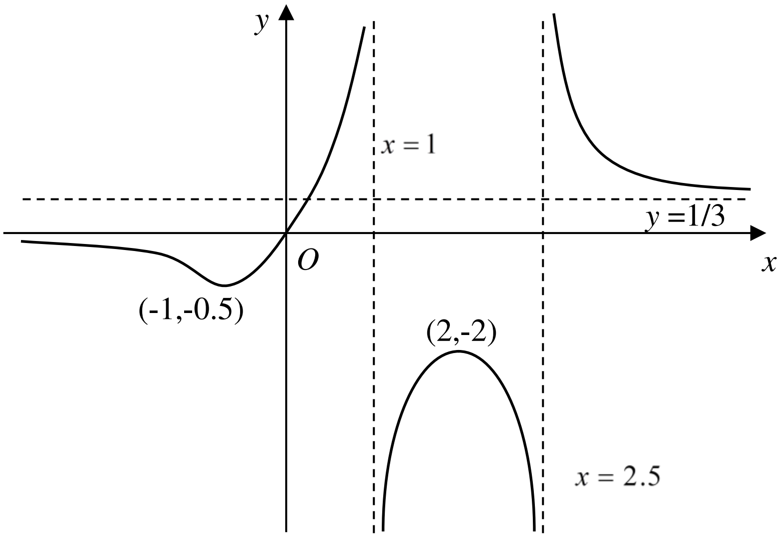
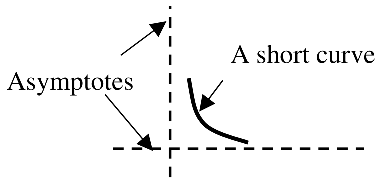
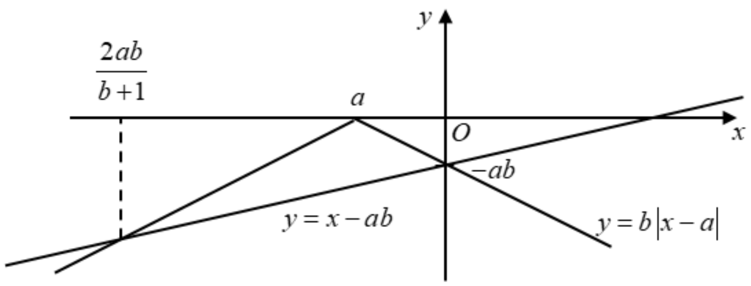
2020 ACJC H2 Math Prelim P1 Marker's Report

Marking Annotations used in scripts:
K:Knowledge Gap; C:Carelessness; R:Read/Interpret question wrongly; P:Presentation issue

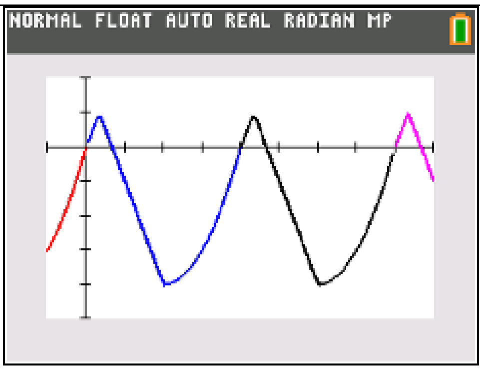
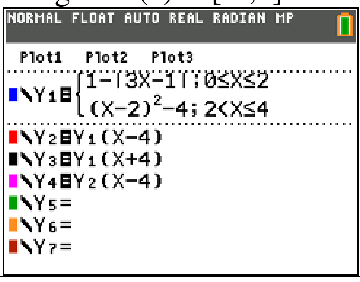
Qn	Solutions	Comments
1(i)	Given $u = 2x - 1$. Then $x = \frac{1}{2}(u+1)$ and $\frac{dx}{du} = \frac{1}{2}$. $\int \frac{x}{\sqrt{1-(2x-1)^2}}\,dx$ $= \int \frac{\frac{1}{2}(u+1)}{\sqrt{1-u^2}}\frac{1}{2}\,du$ $= \frac{1}{4}\left[\int \frac{u}{\sqrt{1-u^2}}\,du + \int \frac{1}{\sqrt{1-u^2}}\,du\right]$ $= \frac{1}{4}\int \left(-\frac{1}{2}\right)(-2u)\left(1-u^2\right)^{-\frac{1}{2}}\,du + \sin^{-1}u + C$ $= \frac{1}{4}\left[-\frac{1}{2}\frac{\left(1-u^2\right)^{\frac{1}{2}}}{\frac{1}{2}} + \sin^{-1}u\right] + C$ $= \frac{1}{4}\left[\sin^{-1}u - \sqrt{1-u^2}\right] + +C$ $= \frac{1}{4}\sin^{-1}(2x-1) - \frac{1}{4}\sqrt{1-(2x-1)^2} + C$ where C is an arbitrary constant.	The common mistakes are • Changing 'dx' to '$2du$' • When applying of the formula $\int f'(u)[f(u)]^n\,du = \frac{[f(u)]^{n+1}}{n+1} + C$, for the 1st integral, the negative sign or ½ in the denominator is left out. • The result of the 1st integral $\int \frac{u}{\sqrt{1-u^2}}\,du$ becomes $\frac{(1-u^2)^{3/2}}{3/2}$ or $\ln\sqrt{1-u^2}$ Some students are able to integrate correctly but do not substitute u as $2x - 1$ and add the arbitrary constant C in the final answer.
1(ii)	$\int \sin^{-1}(2x-1)\,dx$ $u = \sin^{-1}(2x-1)$ $\quad \frac{dv}{dx} = 1$ $\frac{du}{dx} = \frac{2}{\sqrt{1-(2x-1)^2}}$ $\quad v = x$ $\int \sin^{-1}(2x-1)\,dx$ $= x\sin^{-1}(2x-1) - \int \frac{2x}{\sqrt{1-(2x-1)^2}}\,dx$	The part is badly done. Not many students are able to see this is a question on 'Integration by Parts'. Some students are able to choose 'u' as $\sin^{-1}(2x - 1)$, but fail to differentiate it correctly.

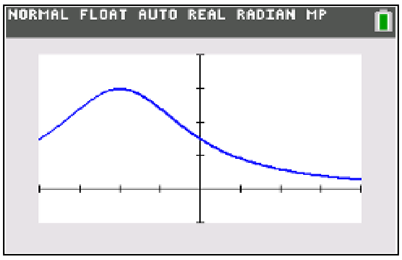
	$= x\sin^{-1}(2x-1) - \frac{1}{2}\sin^{-1}(2x-1) + \frac{1}{2}\sqrt{1-(2x-1)^2} + C$ $= \left(x - \frac{1}{2}\right)\sin^{-1}(2x-1) + \frac{1}{2}\sqrt{1-(2x-1)^2} + C$ where C is an arbitrary constant.	
2(i)	 Note: Do indicate coordinates of points when question requires it.	Many students do not indicate the coordinates of the x-intercepts and the equation of the three asymptotes as required in the question. The common mistakes are • The asymptote $y = 3$ still remains in the graph • The maximum turning point at $x = 2.5$. • The asymptote $y = 1$ is missing
2(ii)	 Note: Do indicate coordinates of points when question requires it.	Generally well done. However, the parts of the curve when $x \to \pm\infty$ are not properly drawn. The curve should get closer and closer to the asymptote $y = 1/3$ or the asymptote $y = 0$ when $x \to \pm\infty$. The common mistake is that the curve is either too short, turns away from asymptote or seems going to cut the asymptote. For example,  Another common mistake can be seen when $x \to -\infty$. The curve cuts the negative x-axis and approaches to the asymptote $y = 1/3$ instead of $y = 0$.
3(i)	$f(x) = e^{3x}\sec 2x$ $f'(x) = 3e^{3x}\sec 2x + 2e^{3x}\sec 2x\tan 2x$	Some students do not differentiate sec $2x$ directly.

	$= 3y + 2y\tan 2x$ $= y\left(3 + 2\tan 2x\right)$	They differentiate $\dfrac{1}{\cos 2x}$ by quotient rule which required more working steps.
3(ii)	$\mathrm{f}''(x) = \dfrac{\mathrm{d}y}{\mathrm{d}x}(3+2\tan 2x) + y\left(4\sec^2 2x\right)$ When $x = 0$, $\mathrm{f}(0)=1$, $\mathrm{f}'(0)=3$, $\mathrm{f}''(0)=13$ $\mathrm{e}^{3x}\sec 2x = 1+3x+\dfrac{13}{2}x^2+\ldots$	Some students differentiate $\mathrm{f}'(x)=\mathrm{e}^{3x}\sec 2x(3+2\tan 2x)$ by using product rule instead of doing implicit differentiation, the working becomes very tedious and often ends up with mistakes.
3(iii)	$\left(\mathrm{e}^{3x}\sec 2x\right)(3+2\tan 2x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}\sec 2x\right)$ $= \dfrac{\mathrm{d}}{\mathrm{d}x}\left(1+3x+\dfrac{13}{2}x^2+\ldots\right)$ $= 3+13x+\ldots$ **OR** $\left(\mathrm{e}^{3x}\sec 2x\right)(3+2\tan 2x) = (1+3x+\ldots)(3+4x+\ldots)$ $= 3(1+3x+\ldots) + 4x(1+3x+\ldots)$ $= 3+13x+\ldots$	Some students only give the coefficients of the two terms, that are 3 and 13 instead of the terms 3 + 13x.
3(iv)	$\mathrm{e}^{3x}\sec 2x = \mathrm{e}^{3x}(\cos 2x)^{-1}$ $=\left(1+3x+\dfrac{(3x)^2}{2}+\ldots\right)\left(1-\dfrac{4x^2}{2}+\ldots\right)^{-1}$ $=\left(1+3x+\dfrac{9x^2}{2}+\ldots\right)\left(1+2x^2+\ldots\right)$ $= 1+3x+\dfrac{9x^2}{2}+2x^2+\ldots$ $= 1+3x+\dfrac{13}{2}x^2$ (verified)	Many students did not read the question carefully. They verify part (iii) instead of (ii). Common mistakes are • $\cos 2x = 1-\dfrac{x^2}{2}+\ldots$ • $\dfrac{1}{\cos 2x}=\dfrac{1}{1-2x^2}=\dfrac{1}{2}-\dfrac{1}{2x^2}$
4(i)	$x-a=\dfrac{x}{b}-a$ or $a-x=\dfrac{x}{b}-a$ $x-\dfrac{x}{b}=0$ or $\dfrac{x}{b}+x=2a$ $x=0$ or $x=\dfrac{2ab}{b+1}$	Most students know that to find the roots they need to consider $\lvert x-a\rvert=(x-a)$ or $\lvert x-a\rvert=-(x-a)$ OR square both sides. For method 1 some students used 'AND' instead of 'OR' For method 2 they prefer to expand rather than factorise using A^2-B^2 = (A – B)(A + B) thus arriving at a more complicated equation. <u>Other issues:</u> Did not simplify the expression for one of the root example:

		$x=\dfrac{2ab(b-1)}{b^2-1}$ Some thought that b is a root . Thus when $x(b-1)=0$ they tend to conclude that $x = 0$ or $b = 1$. Some even went on further to simplify the other root $x=\dfrac{2ab}{b+1}=\dfrac{2a(1)}{1+1}=2a$. Some were able to point out that the solution exist only under certain conditions example $b\neq 1$.
4(ii)(a)	To have a negative root, $x=\dfrac{2ab}{b+1}<0$. Since $ab>0$, $b+1<0 \Rightarrow b<-1$	Many gave the answer as $b<-1$ with no OR long explanation But some gave possible values of x for the equation to have negative roots example $x < ab$. Some even consider discriminant < 0.
4(ii)(b)	 For $b\|x-a\|\geq x-ab$, from graph, $\dfrac{2ab}{b+1}\leq x\leq 0$. For $-b\|e^x+a\|\leq e^x+ab$, $b\|e^x+a\|\geq -e^x-ab$ $b\|-(e^x+a)\|\geq -e^x-ab$ $b\|-e^x-a\|\geq -e^x-ab$ From above, replace x with $-e^x$. $\dfrac{2ab}{b+1}\leq -e^x<0$ $0<e^x\leq -\dfrac{2ab}{b+1}$ $x\leq \ln\left(-\dfrac{2ab}{b+1}\right)$	Not well done. Many gave a V shape curve . Some gave a graph with the correct shape but gave the x-intercept as $(-a, 0)$. Some gave the x-intercept as $(a, 0)$ but marked the point on the right side of the y axis thus assuming that $a>0$. Quite a number of candidates did not show both the graph and the line intersecting at $(0, -ab)$ Not well done. Many students did not use the above graph to solve this part Some common mistakes: $\dfrac{2ab}{b+1}<x<0$ or $0\leq x\leq\dfrac{2ab}{b+1}$ Not well done. Students tend to replace x with $-e^x$ or b with $-b$. Even with the correct substitution some common mistake: $0<x\leq \ln\left(-\dfrac{2ab}{b+1}\right)$ or $x\leq\ln\left(\dfrac{2ab}{b+1}\right)$ $x\geq\ln\left(-\dfrac{2ab}{b+1}\right)$

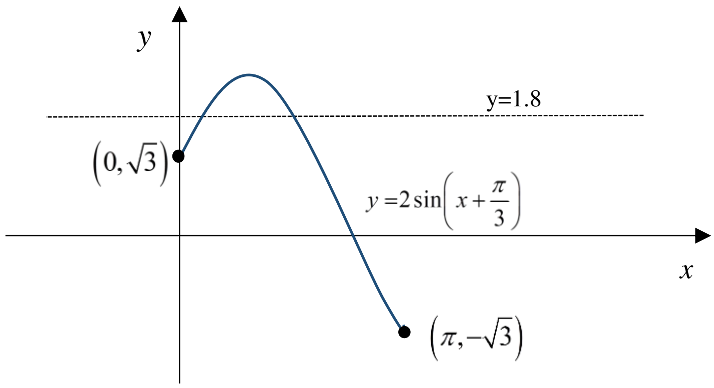
5(i)	$\mathrm{f}(n)=\dfrac{3^n}{n}$ $\mathrm{f}(n+1)-\mathrm{f}(n)$ $=\dfrac{3^{n+1}}{n+1}-\dfrac{3^n}{n}$ $=\dfrac{3^{n+1}n-3^n(n+1)}{n(n+1)}$ $=\dfrac{3^n\left[3n-(n+1)\right]}{n(n+1)}$ $=\dfrac{3^n(2n-1)}{n(n+1)}$	A familiar question and done well by many students. Some errors made were due to weak foundation in indices. Others did not show their working, jump straight fr 3rd to last step, hence no marks awarded.
5(ii)	$\displaystyle\sum_{n=1}^{N}\left(\frac{3^n}{n+1}\left(1-\frac{1}{2n}\right)\right)$ $\displaystyle=\sum_{n=1}^{N}\frac{3^n(2n-1)}{2n(n+1)}$ $\displaystyle=\sum_{n=1}^{N}\frac{1}{2}\left[\mathrm{f}(n+1)-\mathrm{f}(n)\right]$ $=\dfrac{1}{2}\left[\begin{array}{lll} & \cancel{\mathrm{f}(2)} - & \mathrm{f}(1) \\ + & \cancel{\mathrm{f}(3)} - & \cancel{\mathrm{f}(2)} \\ + & \cancel{\mathrm{f}(4)} - & \cancel{\mathrm{f}(3)} \\ \vdots & & \vdots \\ + & \mathrm{f}(N+1) - & \cancel{\mathrm{f}(N)} \end{array}\right]$ $=\dfrac{1}{2}\left[\mathrm{f}(N+1)-\mathrm{f}(1)\right]=\dfrac{1}{2}\left(\dfrac{3^{N+1}}{N+1}-3\right)$	Students are able to express general term as a single sum.. Regards to evidence of using (i), some students were unable to link accurately to (i) by taking out the factor ½. Method of difference were still not done correctly by some students, even in wrongly listing the terms. And finally, many left their answers as $\mathrm{f}(N+1)-\mathrm{f}(1)$. The final answer should be expressed in terms of *N*.
5(iii)	$\displaystyle\sum_{n=1}^{N}\frac{3^n(2n+1)}{(n+1)(n+2)}$ $\displaystyle=\sum_{n-1=1}^{n-1=N}\frac{3^{n-1}(2n-1)}{n(n+1)}$ $\quad$ (Replace n by $n-1$) $\displaystyle=\sum_{n=2}^{n=N+1}\frac{3^n(2n-1)}{3n(n+1)}$ $\displaystyle=\frac{2}{3}\sum_{n=2}^{n=N+1}\frac{3^n(2n-1)}{2n(n+1)}$ $\displaystyle=\frac{2}{3}\left[\sum_{n=1}^{n=N+1}\frac{3^n(2n-1)}{2n(n+1)}-\frac{3(2-1)}{2\times 2}\right]$	Attempt at changing index, whether from (ii) to (iii) or vice versa are accepted if done correctly. However, not all students uses this approach. They found their answer by using method of difference (or result of) which is easier. $\displaystyle\frac{1}{3}\sum_{n=1}^{N}\frac{3^{n+1}(2n+1)}{n(n+1)}$ $\displaystyle=\frac{1}{3}\sum_{n=1}^{N}\left[\mathrm{f}(n+2)-\mathrm{f}(n+1)\right]$ $=\dfrac{1}{3}\left[\mathrm{f}(N+2)-\mathrm{f}(2)\right]$ Finally, not writing the values to be determined, namely *a* and *b*.

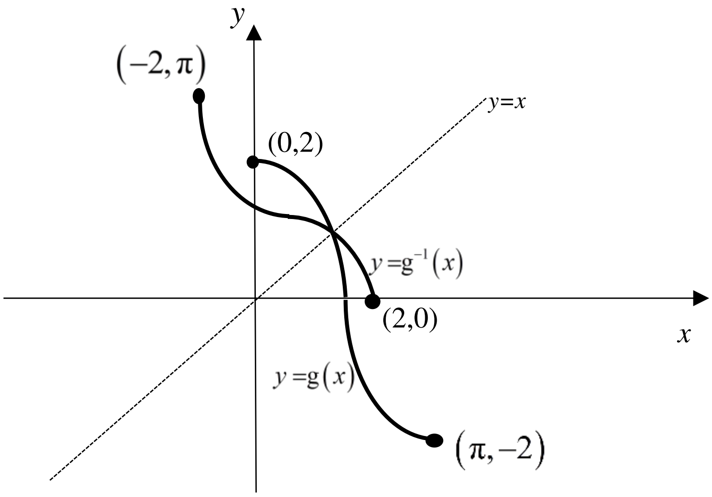
	$=\frac{2}{3}\left[\frac{1}{2}\left(\frac{3^{N+2}}{N+2}-3\right)-\frac{3}{4}\right]$ $=\frac{1}{3}\left(\frac{3^{N+2}}{N+2}-\frac{9}{2}\right)$ Hence, $a=\frac{1}{3},\quad b=\frac{9}{2}$	
6(i)	 f(9)= $\mathrm{f}(5+4)=\mathrm{f}(5)=\mathrm{f}(1+4)=\mathrm{f}(1)=1-\lvert 3-1\rvert=-1$ f(-1)= $\mathrm{f}(-1+4)=\mathrm{f}(3)=(3-2)^2-4=-3$ Coordinates of end-points (-1,-3) and (9,-1) Range of f(x) is [-4,1]  	A relatively easy question as GC can be used to sketch the graph and to find the coordinates of endpoints. Please read the question well and check that you have given all parts required in the question, for example, coordinates of end points, sketching the graph for the specified values of x and the range of f. Copy graph from GC with accuracy for example the correct number of repetitions of the cyclic pattern and having consistent amplitude. Some students are very careless about drawing graphs clearly. Be careful when writing down range as quite a number of students wrongly stated [1,-4] as the range which does not make any sense.
6(ii)	A: Reflection in the y-axis $y=\mathrm{g}(-x)=\frac{1}{1+(-x-1)^2} \qquad -2\le -x\le 2$ $=\frac{1}{1+(x+1)^2} \qquad -2\le x\le 2$ B: Scaling parallel to the x-axis by a factor of 2 $y=\mathrm{g}\left(-\frac{x}{2}\right)=\frac{1}{1+\left(\frac{x}{2}+1\right)^2} \qquad -2\le\frac{x}{2}\le 2$ $=\frac{4}{4+(x+2)^2} \qquad -4\le x\le 4$ C: Scaling parallel to the y-axis by a factor of 3 $\mathrm{h}(x)=3\mathrm{g}\left(-\frac{x}{2}\right)=\frac{12}{4+(x+2)^2} \qquad -4\le x\le 4$	Some confusion in replacing variables for all the three transformations. For A: Replace x with $-x$ For B: Replace x with $\frac{x}{2}$ For C: Replace y with $\frac{y}{3}$ The same replacements are needed to find the corresponding domain as well.

6(iii)	Range of f(x) is [-4,1] Domain of h(x) is [-4,4] Since $R_{\mathrm{f}} \subset D_{\mathrm{h}}$, hf(x) exists. Range of hf(x) = $\left[\frac{12}{13},3\right]$ 	It is not sufficient to state $R_{\mathrm{f}} \subset D_{\mathrm{h}}$. R_f and D_h should be stated when showing that one is a subset of the other. Range of composite function is still a problem with many students either not attempting this part or getting it wrong hf(x)=h(f(x)), h is now acting on f(x) ie what comes out of f(x), which is the range of f(x) ie [-4,1]. So sketch graph of h(x) and mark the portion of this curve where x is in the region [-4,1] Read the highest and lowest y-value for this portion. Highest y-value is at the max point i.e y=3 and the lowest is at x =1 ie. $y=\frac{12}{13}$.
7(i)	$x = 2\cos\theta + \sin 2\theta, \quad y = 2 + 2\sin\theta, \quad (-\pi < \theta \le \pi)$ Area of the sticker $= 2\int_0^4 x\ \mathrm{d}y$ $= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\cos\theta + \sin 2\theta)(2\cos\theta)\ \mathrm{d}\theta$ $= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4\cos^2\theta + 4\sin\theta\cos^2\theta)\ \mathrm{d}\theta$ $= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\cos 2\theta + 2 + 4\sin\theta\cos^2\theta)\ \mathrm{d}\theta$ $= 2\left[\sin 2\theta + 2\theta - \frac{4\cos^3\theta}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ $= 4\pi$ Alternatively, method for second integral (using factor formula) $\int 4\sin\theta\cos^2\theta\ \mathrm{d}\theta = \int 2\sin\theta(\cos 2\theta + 1)\ \mathrm{d}\theta$ $= \int 2\sin\theta\cos 2\theta + 2\sin\theta\ \mathrm{d}\theta$ $= \int \sin 3\theta - \sin\theta + 2\sin\theta\ \mathrm{d}\theta$	This should be considered a straightforward question. Area should be written out correctly in Cartesian format first with the correct limits. Many students made several mistakes at this first stage. Since the area in this case is enclosed with the y-axis, the integral should be with respect to y. Also the limits are given very clearly as y=0 to y=4. Many errors with this basic formula were made. Many students also forgot to multiply by 2. In this case the equation of the curve is given in parametric form, hence x, dy and the limits must be now be substituted in terms of the parameter θ before the integration can be done. Many students who managed to get the integral correct, failed to do the integration correctly or accurately. $\int (4\cos^2\theta)\ \mathrm{d}\theta$ is evaluated using double angle formula $\int (4\sin\theta\cos^2\theta)\ \mathrm{d}\theta$ is a standard form $\int f'(x)[f(x)]^n\ \mathrm{d}\theta = \frac{[f(x)]^{n+1}}{n+1} + c$
7(ii)	$x = 2\cos\theta + \sin 2\theta, \quad y = 2 + 2\sin\theta, \quad (0 \le \theta \le 2\pi)$	This proof was quite easily done.

	$x = 2\cos\theta + 2\sin\theta\cos\theta$ $x = \cos\theta(2 + 2\sin\theta)$ $x = y\cos\theta \quad \cdots(1)$ From $y = 2 + 2\sin\theta$ $\Rightarrow \sin\theta = \dfrac{y-2}{2}$ Since $\cos^2\theta = 1 - \sin^2\theta$ $\cos^2\theta = 1 - \left(\dfrac{y-2}{2}\right)^2 \quad \cdots(2)$ Subst (2) into (1) $x = y\cos\theta$ $x^2 = y^2\cos^2\theta$ $x^2 = y^2\left[1 - \left(\dfrac{y-2}{2}\right)^2\right]$ $x^2 = y^2\left[\dfrac{4-(y-2)^2}{4}\right]$ $4x^2 = y^2\left[4 - (y^2 - 4y + 4)\right]$ $4x^2 = 4y^3 - y^4$	Many tedious working done without clear intention for the next part. Using triangle method to get $\cos\theta$ from $\sin\theta = \dfrac{y-2}{2}$ is not recommended since the quadrant in which θ lies is unknown.
7(iii)	Volume of balloon $= \pi\int_0^4 x^2 \, dy$ $= \pi\int_0^4 \left(y^3 - \frac{1}{4}y^4\right) dy$ $= \pi\left[\dfrac{y^4}{4} - \dfrac{y^5}{20}\right]_0^4$ $= \pi\left[\dfrac{256}{4} - \dfrac{1024}{20}\right]$ $= \dfrac{64\pi}{5}$ or 40.2	Some had the wrong formula and others failed to realise that the Cartesian equation of the curve is now known from part(ii) above. So x can be directly expressed in terms of y.
8(i)	Length of film in the 1st layer = 42π Length of film in the 2nd layer = $(42 + 2x)\pi$ Length of film in the 3rd layer = $(42 + 4x)\pi$ $\vdots \qquad \vdots$ Length of film in the nth layer = $[42 + 2x(n-1)]\pi = 124\pi$ $2x(n-1) = 82$ $\therefore \quad x(n-1) = 41$ (shown)	This AP/GP question was generally quite well done. However, a few students were not able to quote u_n and S_n of AP and GP. Some students missed the fact that $d = 2\pi x$.

8(ii)	$S_n = 16766\pi$ mm, $a = 42\pi$ mm, $l = 124\pi$ mm Using $S_n = \frac{n}{2}(a+l)$ $n = \frac{2S_n}{a+l} = \frac{2(16766\pi)}{42\pi+124\pi} = 202$ Using (i), $x(n-1) = 41$, $x = \frac{41}{n-1} = 0.20398$ $x = 0.204$ (to 3 s.f.)	Some wrong answers for n and x because "π" was missing for S_n or a or l in some scripts. For those who used $S_n = ½\, n[2a + (n-1)d]$ $= ½\, n[2(42\pi) + (n-1)(2\pi x)]$ $= 16766\pi$, some were not able to see that the result of (i) $(n-1)x = 41$ can be used here. Thus, n can be easily found with $n(42 + 41) = 16766$.
8(iii)	The sum of the first 10 terms = k (the sum of the first 5 terms) $\frac{a(1-r^{10})}{1-r} = k \times \frac{a(1-r^5)}{1-r}$ $1-r^{10} = k(1-r^5)$ $(1-r^5)(1+r^5) = k(1-r^5)$ Since $r \neq 1$, $1-r^5 \neq 0$ Therefore, $k = 1+r^5$ (shown)	Wrong formula for S_n was quoted, resulting in wrong expressions like $1 + r^{10}$, $(1-r)^{10}$, $1-r^9$, $¼(r^5-1)$. Factorising $(1-r^{10})$ into $(1+r^5)(1-r^5)$ or using long division is required to complete the proof.
8(iv)	Since k =33 and a = 50, $1+r^5 = 33$ $r^5 = 32$ $r = 2$ Suppose the roll of film is cut into n pieces. The sum of the first n terms ≤ total length of the film $\frac{50(2^n-1)}{2-1} \leq 52580$ $2^n - 1 \leq 1051.6$ $2^n \leq 1052.6$ $n \ln 2 \leq \ln 1052.6$ $n \leq 10.03974$ $\Rightarrow$ the largest possible value of n is 10. Hence the largest number of pieces is 10 pieces.	There was no evaluation of $\sqrt[5]{32}$ in a few scripts. A good number did not consider inequality, so the answer 10 was given as the nearest whole number inaccurately.

9(i)	$\sin x + \sqrt{3}\cos x = R\sin\left(x + \frac{\pi}{3}\right)$ $= R\sin x\cos\frac{\pi}{3} + R\cos x\sin\frac{\pi}{3}$ $= \frac{R}{2}\sin x + \frac{R}{2}\sqrt{3}\cos x$ $\therefore R = 2$	Generally well done by most students, using either the correct trigonometric identity or their knowledge of the R-formula. The most common wrong answer is $\frac{1}{2}$, from careless comparison of terms.
9(ii)	$\mathrm{f} : x \mapsto \sin x + \sqrt{3}\cos x, \quad x \in \mathbb{R},\ 0 \le x \le \pi.$ $\mathrm{f}(x) = 2\sin\left(x + \frac{\pi}{3}\right)$  The line y=1.8 shown in diagram cuts the graph of function twice. Therefore, f is not a one-to-one function. Hence f^{-1} does not exist.	It seemed quite evident from this question that many students still rely heavily on the GC to sketch graphs, with little or no consideration for the algebra of the functions. Most common errors for the graph in this question include: • Not considering the domain of the function • Not indicating the coordinates of the end points (or at least the domain) • Wrong values for the end points • Accuracy of the shape of the graph (e.g. there should only be one turning point at $\left(\frac{\pi}{6}, 2\right)$) To explain why f does not have an inverse, many students are still using the arbitrary horizontal line "$y = k,\ k \in \mathbb{R}$", with some students not even sketching the line in the graph, or sketching it at a level that only cuts the graph of $y = \mathrm{f}(x)$ once. There are also those who would state a range of values of k such that the line cuts the graph more than once. Students should be reminded that to show f is **not** 1-1, the most concrete justification is to state one particular value of k such that $y = k$ cuts the graph more than once. For those who did state values for k, a common error is to give a rounded off value of $\sqrt{3}$ that is actually less than $\sqrt{3}$, for example 1.732. This is an evidence of students relying too much on the GC for answers. A small fraction of students did not sketch any graphs, despite the question requiring a "graphical method" to show the result.
9(iii)	$\mathrm{g} : x \mapsto 2\sin\left(x + \frac{\pi}{3} + a\right), \quad x \in \mathbb{R},\ 0 \le x \le \pi$ where a >0. Translation of a units in the negative x-direction.	Responses to the first part of **(iii)** were varied. Students who managed to identify the coordinates of the maximum point and recognise the transformation as a translation in the

	$\therefore a=\dfrac{\pi}{6}$
	
	
	$gg^{-1}(x)=g^{-1}g(x)=x$
	$D_{gg^{-1}}=D_{g^{-1}}=R_g=[-2,2]$
	$D_{g^{-1}g}=D_g=[0,\pi]$
	$\therefore gg^{-1}(x)=g^{-1}g(x)$ for common domain $0\le x\le 2$
		negative x-direction were usually successful. It was also clear that there were students who did not understand the significance of $f(x+a)$, giving non-angle related answers.
		
		The graph sketching part was poorly done. Students were either careless or did not know the correct way to handle the translation of a units of the graph of $f(x)$. Common errors included:
		• Wrong domains, such as $\left[\frac{\pi}{6},\pi\right]$, $\left[0,\frac{5\pi}{6}\right]$ and $\left[0,\frac{7\pi}{6}\right]$
		• Not labelling coordinates of end points, or wrong coordinates, the most common one being $(\pi,-\sqrt{3})$ copied from part **(ii)**
		• Wrong reflection in the line $y=x$ to obtain $y=g^{-1}(x)$
		• Sketching $y=\dfrac{1}{g(x)}$ instead of $y=g^{-1}(x)$
		• Accuracy in positioning of the graphs and endpoints, e.g.
		  o the x-intercept of $y=g(x)$, $\left(\frac{\pi}{2},0\right)$, should lie to the left of the end-point $(2,0)$ of $y=g^{-1}(x)$
		  o the end-points $(0,2)$ and $(-2,\pi)$ of the two graphs should not lie at the same distance from the x-axis
		• Accuracy in shape of $y=g^{-1}(x)$: the graph should also be 1-1, so the curve around the point $\left(0,\frac{\pi}{2}\right)$ should be drawn carefully.
		• Despite the question's instructions, some students omitted the labelling of the line of symmetry $y=x$.
		
		The last part of **(iii)** was poorly done. Students were aware that $gg^{-1}(x)$ and $g^{-1}g(x)$ were both equal to x, but stopped there and did not go on to consider the domains of

		the two composite functions. A majority of students either misinterpreted or assumed the question was related to the concept of the graphs of $y=\mathrm{g}(x)$ and $y=\mathrm{g}^{-1}(x)$ intersecting at the line $y=x$, and used their GC to find this single value of *x*.
9(iv)	$\mathrm{h}: x \mapsto xe^{x^2}, \quad x \in \mathbb{R}$. $\mathrm{h}'(x)=x\left(2xe^{x^2}\right)+e^{x^2}=e^{x^2}\left(2x^2+1\right)>0$ for all $x \in \mathbb{R}$ Therefore, h is an increasing function. $\mathrm{h}^{-1}\mathrm{g}\left(x\right)\le 0.5$ Since h is an increasing function $\mathrm{h}(\mathrm{h}^{-1}\mathrm{g}\left(x\right))\le \mathrm{h}(0.5)$ $g(x)\le 0.5e^{(0.5)^2}$ $2\sin\left(x+\frac{\pi}{2}\right)\le 0.5e^{0.25}$ $\sin\left(x+\frac{\pi}{2}\right)\le 0.25e^{0.25} \quad x\in D_{\mathrm{g}}=[0,\pi]$ Using GC: $1.24\le x\le \pi$	The first part of **(iv)** was related to differentiation, which helped students who were weaker in the chapter on functions. Common errors seen here included: • Incorrect application of product rule in differentiation • Incorrect justification of why h was increasing, e.g. ○ "As $x\to\infty$, $e^{x^2}\to\infty$" ○ Finding and showing $\mathrm{h}''(x)>0$ • Saying $x^2>0$ instead of $x^2\ge 0$ Students should also note that for the A Levels, we take the definition of "h is increasing" to be $\mathrm{h}'(x)>0$ rather than the weaker definition of $\mathrm{h}'(x)\ge 0$. Students struggled with the 2nd part of **(iv)**. Most students tried to find an expression for $\mathrm{h}^{-1}(x)$ and got stuck when they couldn't make *x* the subject. These students were probably unaware of the significance of proving h is increasing, that they can apply h to both sides of the inequality without affecting the inequality sign. For those who did manage to use this concept, they were stuck at solving the inequality and did not realise they could simply sketch graphs in the GC to get the numerical answer. Some students who managed to get $x\ge 1.24$ omitted the upper bound of π, which is due to the domain of g. BONUS: One brave student successfully found the exact solution of the inequality.

10(i)	$L_1: r=\begin{pmatrix}1\\2\\0\end{pmatrix}+s\begin{pmatrix}-2\\1\\2\end{pmatrix}$ $L_2: r=\begin{pmatrix}0\\5\\-9/2\end{pmatrix}+t\begin{pmatrix}2\\-2\\3\end{pmatrix}$ $\begin{pmatrix}-2\\1\\2\end{pmatrix}\neq k\begin{pmatrix}2\\-2\\3\end{pmatrix}$ L_1 is not parallel to L_2. Equate the two lines: $2t+2s=1$ $2t+s=3$ $3t-2s=\frac{9}{2}$ No solution from GC OR Solution for first two equation $s=-2$, $t=\frac{5}{2}$ do not satisfy third equation. Hence they do not interect. They are skew lines.	Quite a significant minority wrote the direction vector of L_1 as $\begin{pmatrix}5\\0\\-9/2\end{pmatrix}$ instead of $\begin{pmatrix}0\\5\\-9/2\end{pmatrix}$, or did not show that the 2 lines are not parallel. Since this is a 'show' question, no credit will be given for simply quoting that there are no consistent values of *s* and *t* when the two equations are equated. Students must either state the use of GC or show clearly that the two values of *s* and *t* obtained from 2 equations do not satisfy the 3rd equation.
10(ii)	Let *F* be foot of perpendicular from *P* to *M*. Equation of line through *P* and *F*: $r=\begin{pmatrix}1\\5\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\-1\end{pmatrix}=\begin{pmatrix}1\\5+\lambda\\-\lambda\end{pmatrix}$ Substitute into equation of plane *M* $\begin{pmatrix}1\\5+\lambda\\-\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}=2$ $5+2\lambda=2\Rightarrow\ \lambda=-\frac{3}{2}\quad\therefore\overrightarrow{OF}=\begin{pmatrix}1\\7/2\\3/2\end{pmatrix}$ $\overrightarrow{OP'}=2\overrightarrow{OF}-\overrightarrow{OP}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ Coordinates of P' are $(1,2,3)$. **Alternative method to find foot of perpendicular**: *A*(0,2,0) is a point on plane M	Common mistakes: • Writing $\overrightarrow{PF}=\begin{pmatrix}1\\5\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\-1\end{pmatrix}$ instead of $\overrightarrow{OF}=\begin{pmatrix}1\\5\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\-1\end{pmatrix}$ • Writing $\overrightarrow{PF}=\begin{pmatrix}0\\1\\-1\end{pmatrix}$ instead of $\overrightarrow{PF}=\lambda\begin{pmatrix}0\\1\\-1\end{pmatrix}$ • Finding any point *A* on the plane *M* and stating $\overrightarrow{PA}=\overrightarrow{AP'}$, not realising that this is only true if *A* is the foot of perpendicular from *P* to the plane *M* • Writing point *P'* in column vector form instead of coordinates • For the Alternative Method of using projected vector component, some students wrote $\overrightarrow{PF}=(\overrightarrow{AP}\,.\,\hat{\mathbf{n}})\hat{\mathbf{n}}$ instead of $\overrightarrow{PF}=(\overrightarrow{PA}\,.\,\hat{\mathbf{n}})\hat{\mathbf{n}}$.

	$\overrightarrow{PF}=\left[\overrightarrow{PA}\bullet\frac{1}{\sqrt{1+1}}\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]\frac{1}{\sqrt{1+1}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $=\frac{1}{2}\left[\begin{pmatrix}-1\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]\begin{pmatrix}0\\1\\-1\end{pmatrix}=\frac{-3}{2}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $\overrightarrow{OF}=\frac{-3}{2}\begin{pmatrix}0\\1\\-1\end{pmatrix}+\begin{pmatrix}1\\5\\-0\end{pmatrix}=\begin{pmatrix}1\\7/2\\3/2\end{pmatrix}$ $\overrightarrow{OP'}=2\overrightarrow{OF}-\overrightarrow{OP}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ Coordinates of P' are $(1,2,3)$.	
10(iii)	Normal of plane $\prod$: $\underset{\sim}{n}=\begin{pmatrix}-2\\1\\2\end{pmatrix}\times\begin{pmatrix}0\\0\\3\end{pmatrix}=3\begin{pmatrix}1\\2\\0\end{pmatrix}$ Eq of plane containing P' and L_1 : $\underset{\sim}{r}.\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}1\\2\\0\end{pmatrix}.\begin{pmatrix}1\\2\\0\end{pmatrix}=5$ Cartesian equation of $\prod$ is $x+2y=5$	This part is mostly well-done. Some mistakes: • Using position vector to find the normal vector to plane, instead of finding a displacement vector from point *P* to any point on the plane *M* • Incorrect cross-product of vectors to obtain the normal vector, often due to wrong sign in the **j** component [*students should always check that cross product is correct by checking that dot product of normal vector and original vector is 0*] • Writing equation of plane in scalar product form or parametric form instead of Cartesian form
10(iv)	$L_2: r=\begin{pmatrix}0\\5\\-9/2\end{pmatrix}+t\begin{pmatrix}2\\-2\\3\end{pmatrix}=\begin{pmatrix}2t\\5-2t\\-9/2+3t\end{pmatrix}$ $\prod: x+2y=5$ $2t+2(5-2t)=5\Rightarrow t=\frac{5}{2}$ Point of intersection is (5,0,3)	This part is well-done.
10(v)	Let *Q* be the point (5,0,3) *Q* is on L_2. *P'Q* and L_1 are coplanar and are not parallel, so they will intersect. Line *P'Q* is the reflected ray that intersects the two skew lines.	Very few students completed this part. Common mistakes: • Writing the direction vector of L_3 as $\overrightarrow{P'A}$ where *A* is the point (1,2,0) on Line L_1

	Vector equation of $P'Q$: $r=\begin{pmatrix}5\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\0\end{pmatrix},\ \lambda\in\mathbb{R}$ Cartesian equation: $x=5+2\lambda$ $y=-\lambda$ $z=3$ $\frac{5-x}{2}=y,\ z=3$	• Writing the direction vector of L_3 as cross product of the 2 direction vectors of L_1 and L_2 • Writing equation of line in vector form instead of Cartesian form
10(vi)	Angle of incidence is the angle between L_3 and mirror $\sin\theta=\frac{\left\|\begin{pmatrix}2\\-1\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}\right\|}{\sqrt{4+1}\sqrt{1+1}}=\frac{1}{\sqrt{10}}$ $\theta=\sin^{-1}\left(\frac{1}{\sqrt{10}}\right)=18.4^{\circ}$ or 0.322 radians	Answer is usually incorrect due to wrong vector obtained earlier, or using cosine instead of sine.
11(i)	At point P, $y=16\tan^{-1}t^3-4t+16$ and $y=4(5-\pi)$ $4\tan^{-1}t^3-t+4=5-\pi$ By observation, when $t=-1$, LHS = $4\left(-\frac{\pi}{4}\right)-(-1)+4=5-\pi=$ RHS OR by G.C, $t=-1$. $x=3(-1)^2-10(-1)-1=12$ $x=3t^2-10t-1$ $y=16\tan^{-1}t^3-4t+16$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{\frac{48t^2}{1+t^6}-4}{6t-10}$ When $t=-1$, $x=12$, $y=4(5-\pi)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{24-4}{-6-10}=\frac{5}{-4}$ Equation of tangent: $y-4(5-\pi)=-\frac{5}{4}(x-12)$ $y+4\pi-20=-\frac{5}{4}x+15$ $5x+4y+16\pi-140=0$ (shown)	Many start by assuming P is a turning point and hence proceed to solve $\frac{dy}{dx}=0$. However quite a number realise that at P $y=4(5-\pi)$ and manage to derive the equation $4\tan^{-1}t^3-t+4=5-\pi$. However some have problem solving the equation using a GC. Common mistakes in finding $\frac{dy}{dt}$ are $\frac{dy}{dt}=\frac{16}{1+t^6}$ or $\frac{dy}{dt}=\frac{16(2t^3)}{1+t^6}$ $\frac{d}{dt}(\tan^{-1}t^3)=\frac{d}{dt}(\tan t^3)^{-1}$

11(ii)	To find m, find coordinates of B. i.e solve the equations simultaneously. Eqn of curve: $x = 3t^2 - 10t - 1$, $y = 16\tan^{-1} t^3 - 4t + 16$ Eqn of tangent: $5x + 4y + 16\pi - 140 = 0$ $5\left(3t^2 - 10t - 1\right) + 4\left(16\tan^{-1} t^3 - 4t + 16\right) + 16\pi - 140 = 0$ Using G.C, $t = -0.568281$ or $t = -1$ (reject since at P) Hence, $m = 16\tan^{-1}\left(\left(-0.568281\right)^3\right) - 4\left(-0.568281\right) + 16 =$ 15.369 (3 d.p.) (or 15.372 if 0.568 was used)	Not well done . Some can form the equation in terms of t but unable to proceed after that or gave the wrong t values.
11(last part)	At $t = -1$, Equation of normal: $y - 4(5-\pi) = \frac{4}{5}(x-12)$ $y + 4\pi - 20 = \frac{4}{5}x - \frac{48}{5}$ $y = \frac{4}{5}x + \frac{52}{5} - 4\pi$ At point E, $y = 0$. $\frac{4}{5}x = -\frac{52}{5} + 4\pi$ $x = 5\pi - 13$ Length of EP $= \sqrt{(5\pi - 13 - 12)^2 + (0 + 4\pi - 20)^2}$ $= \sqrt{(5\pi - 25)^2 + (4\pi - 20)^2}$ $= \sqrt{25(\pi-5)^2 + 16(\pi-5)^2}$ $= \sqrt{41}(5-\pi)$	Most of the candidates used the equation to find E and hence l. Some thought that EP is shortest when E is at F and so the answer is $20-4\pi$. They did not realise that E is a fixed point Very few students using trigonometry which is a much shorter method since $l = \dfrac{PF}{\cos \hat{FPE}} = \dfrac{20-4\pi}{\frac{4}{\sqrt{41}}}$ Since $\hat{FPE} = \hat{PCF} = \tan^{-1}(\frac{5}{4})$ Among those who tried using trigonometry, there were hardly any correct answers . One common mistake is to assume $\hat{PEF} = \hat{PCF}$