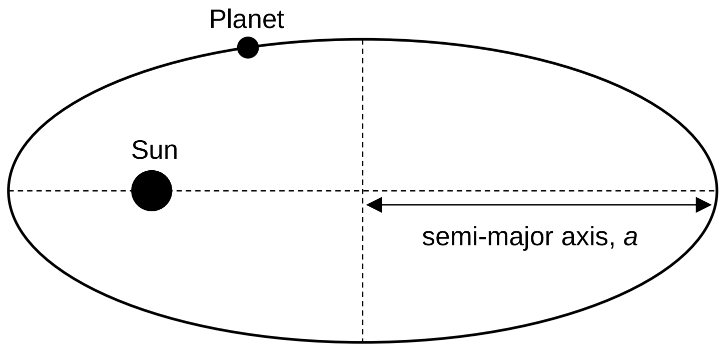
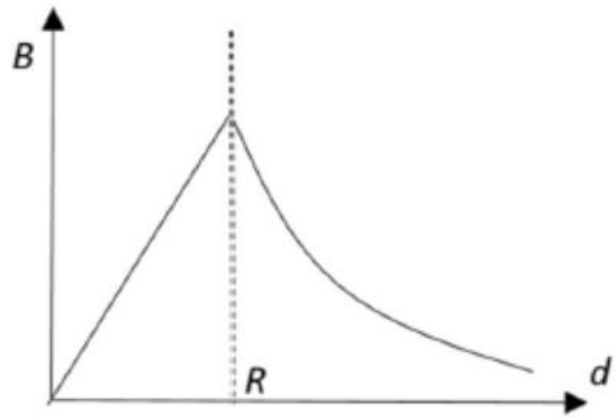
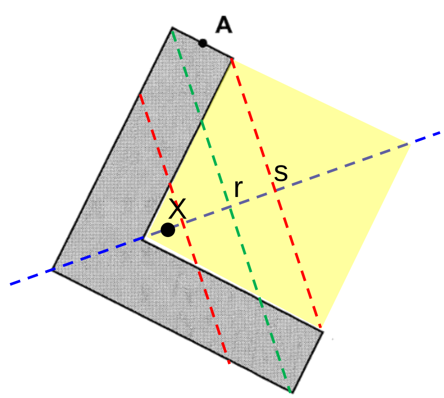
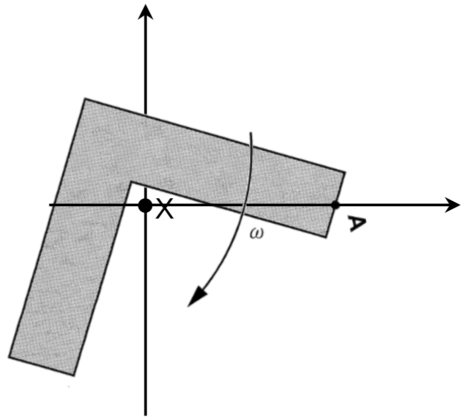
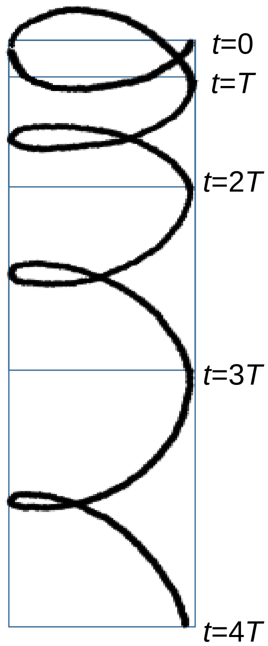
2019 A level paper (9814) suggested solutions

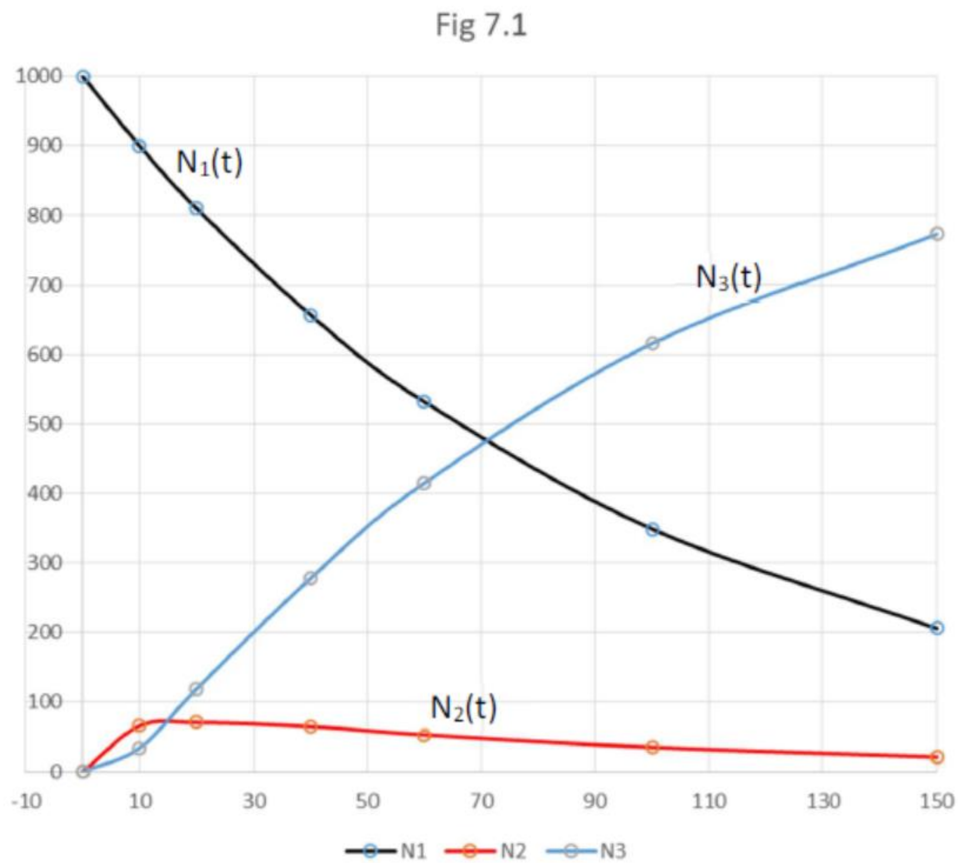
1	**(a)**		 Draw the elliptical shape. Correct labelling of the maximum distance between the centre of ellipse to the point on the orbit.
2	**(a)**		Gauss's Law for magnetic fields states that the total magnetic flux through a closed surface is zero, $\oint \vec{B} \bullet d\vec{A} = 0$
	(b)		A magnetic field line that does not form an open loop will imply the presence of magnetic monopoles at the ends of the field line. As magnetic monopoles do not exist, magnetic field lines will always form a closed loop.
	(c)		If a magnetic monopole exists, the magnetic flux calculated based on a closed surface that enclose the magnetic monopole will be non-zero which is inconsistent with Gauss's Law for magnetic fields.
3	**(a)**		$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$ Ampere's Law states that where $\vec{B}$ is the magnetic field strength, $d\vec{s}$ is a line element along the integration path, is the permeability of free space and I is the current passing through the area enclosed by the integration path.
	(b)	**(i)**	Apply Ampere's Law along a circular path of radius *d* centered at the axis of the wire that encloses the wire, when *d* is larger than the radius of the wire, $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$ $B(2\pi d) = \mu_0 I$ $B = \dfrac{\mu_0 I}{2\pi d}$
		(ii)	When *d* is shorter than the radius of the wire, assuming that the current is uniform across the cross section of the wire,

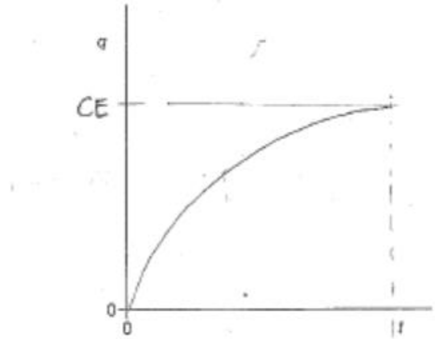
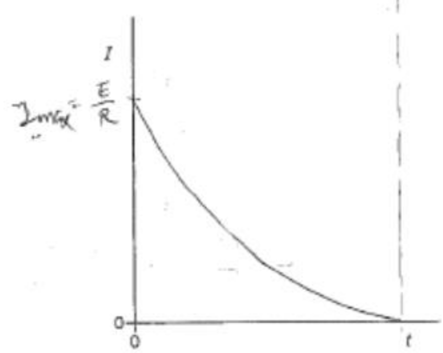
			$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$ $B(2\pi d) = \mu_0 I\left(\frac{\pi d^2}{\pi r^2}\right)$ $B = \frac{\mu_0 I}{2\pi r^2}d$ Hence, we have the following graph 
		(iii)	A magnetic field exists inside the wire as the current flows through all parts of the cross section of the wire, hence when we apply Ampere's Law over a circular path inside the wire, the path will enclose a fraction of the current that flows in the wire resulting in a non-zero magnetic field in the wire.
4	**(a)**	 The centre of mass must lie somewhere along the line of symmetry (blue dashed line). It should also lie to the left of the green dashed line.	
		COMMENT: The C.M. of the completed square is exactly at *r*. The C.M. of the missing square is exactly at *s*, on the right of *r*. So the C.M. of the L-shape must be left of *r*.	

	(b)	**(i)**	 $x = R\cos\omega t$ $y = -R\sin\omega t$ R is the distance between A and X.
		(ii)	
			COMMENT: Sketch should show increasing "pitch".
		(iii)	$y = -R\sin\omega t \Rightarrow v_y = -R\omega\cos\omega t$ Maximum upward velocity due to rotation $= R\omega$ $= (0.50)(25)$ $= 12.5 \text{ m s}^{-1}$ $(v = u + at)$ Velocity of centre of mass $= (9.81)t$ Solve for $12.5 = 9.81t$ $t = 1.274 \text{ s}$

		(iv)	$T = \frac{2\pi}{\omega} = \frac{2\pi}{25} = 0.2513\ \text{s}$ $t = 1.274\ \text{s} = 1.274 \div 0.2513 = 5.07T$ After the 5th rotation, the vertical velocity is always downward. So the trajectory stops intersecting itself. The last intersection occurs during the 4th rotation. From the sketch, it looks like the intersection occurs close to the left extreme horizontal position. Estimated time of intersection $= 4.5T = 4.5 \times 0.2513 = 1.13\ \text{s}$
5	Define U as the energy stored in the inductor at any time. The potential difference across the inductor is $V = L\frac{dI}{dt}$ $\frac{dU}{dt} = VI = L\frac{dI}{dt}I$ $dU = LIdI$ $\int_0^{E_T} dU = L\int_0^{I_f} IdI$ $E_T = \frac{LI_f^2}{2}$		
6	**(a)**	Volume of oil in test-tube, $V = \pi r^2 h = \left(3.0 \times 10^{-4}\right)10^{-6}$ $h = \frac{3.0 \times 10^{-10}}{\pi\left(10^{-2}\right)^2} = 9.55 \times 10^{-7}\ \text{m}$	
	(b)	At air-to-oil boundary, the reflected blue light had a phase change of π rad. At oil-to-water boundary, the reflected blue light had no phase change. Since the blue light interferes constructively at the surface of oil film, the path difference for the two rays of reflected blue light is an odd number of half wavelengths. Path difference = $2h = \left(m + \frac{1}{2}\right)\lambda_{\text{in oil}} = \left(m + \frac{1}{2}\right)\frac{n_{air}\lambda_{\text{in air}}}{n_{oil}}$ where $n_{air} = 1.000$, $n_{oil} = 1.390$, $h = 9.55 \times 10^{-7}$ m and $\lambda_{\text{in air}} = 486$ nm Substituting the values in, it is found that m = 5 (m must be an integer). Rewriting the path difference to find a more accurate value of h, $h = \frac{\left(5 + \frac{1}{2}\right)\left(486 \times 10^{-9}\right)}{2\left(1.390\right)} = 9.62 \times 10^{-7}\ \text{m}$	

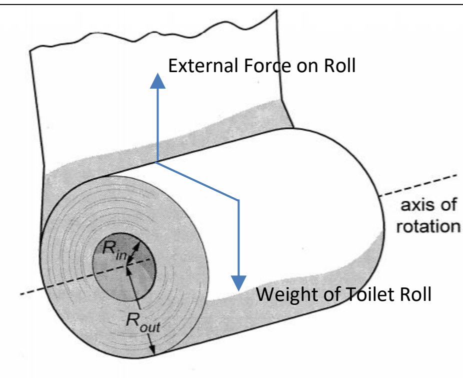
7	**(a)**	$\frac{dN}{dt} = -\lambda N$ $\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$ $\ln N - \ln N_0 = -\lambda t \rightarrow \ln \frac{N}{N_0} = -\lambda t$ $\frac{N}{N_0} = e^{-\lambda t} \rightarrow N = N_0 e^{-\lambda t}$	
	(b)	**(i)**	$\lambda_1 = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{65.9} = 0.0105 \text{ h}^{-1}$ $\lambda_2 = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{6.01} = 0.115 \text{ h}^{-1}$ Using $N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$ $N_2 = 59$ and $N_3 = 1000 - 900 - 59 = 41$
		(ii)	$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$ To find maximum N_2: $\frac{dN_2}{dt} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} \right) = 0$ $\lambda_2 e^{-\lambda_2 t} = \lambda_1 e^{-\lambda_1 t}$ $\frac{\lambda_1}{\lambda_2} = \frac{e^{-\lambda_2 t}}{e^{-\lambda_1 t}} = e^{(\lambda_1 - \lambda_2)\, t}$ Solving for t, t = 22.8 h.

		(iii)	
	(c)	**(i)**	Beta particles are ionizing radiation which can cause tissue damage.
		(ii)	The half life of ^{99}Tc is 211000 years. Number of ^{99}Tc nuclide at 150 h is 773. Therefore, the activity of ^{99}Tc at 150 h, $A = \lambda N = \dfrac{\ln(2)}{211000 \times 365.25 \times 24 \times 3600} \times 773 = 8 \times 10^{-11}\ Bq$
		(iii)	The activity is very small. It corresponds to approximately one decay every 400 years, which is much longer than a human life. $\dfrac{1}{A} = \dfrac{1}{8 \times 10^{-11}} \approx 400\ years$

8	**(a)**	**(i)**	CE
		(ii)	 
	(b)	**(i)**	$\dfrac{CE^2}{2}$
		(ii)	CE^2
		(iii)	$E = V_R + V_C$ $E = \dfrac{dq}{dt}R + \dfrac{q}{C}$
		(iv)	$\dfrac{dq}{dt}R + \dfrac{q}{C} = E$ $\dfrac{dq}{dt} = \dfrac{E}{R} - \dfrac{q}{RC}$ $\int_0^q \dfrac{dq}{q - CE} = \int_0^t -\dfrac{dt}{RC}$ $\ln\left(\dfrac{q - CE}{-CE}\right) = -\dfrac{t}{RC} \rightarrow \dfrac{q - CE}{-CE} = e^{-\frac{t}{RC}}$ $q = CE - CEe^{-\frac{t}{RC}}$ It is given that $q = A + Be^{-t/\tau}$

			$A = CE$ $B = -CE$ $\tau = RC$
		(v)	$I = \dfrac{dq}{dt} = -\dfrac{CE}{-RC} e^{-\frac{t}{RC}} = \dfrac{E}{R} e^{-\frac{t}{RC}}$
		(vi)	$P = I^2 R = \left(\dfrac{E}{R}\right)^2 e^{-\frac{2t}{RC}} R = \dfrac{E^2}{R} e^{-\frac{2t}{RC}}$ Total energy dissipated in the resistor $\text{Energy} = \dfrac{E^2}{R}\int_0^{\infty} e^{-\frac{2t}{RC}} dt = \dfrac{CE^2}{2}$
		(vii)	Yes, answer to **(b)(vi)** (*the energy dissipated in the resistor while fully charging the capacitor*) is the same as **(b)(ii)** (*work done by the battery fully charging the capacitor*) **minus (b)(i)** (*energy stored in the capacitor*)
	(c)	**(i)**	Same Q on each capacitor. $9 = \dfrac{Q}{C_1} + \dfrac{Q}{C_2} = \dfrac{Q}{2.5 \times 10^{-6}} + \dfrac{Q}{1.1 \times 10^{-6}}$ $Q = 6.88 \times 10^{-6}$ C
		(ii)	$V_1 = \dfrac{Q}{C_1} = \dfrac{6.875 \times 10^{-6}}{2.5 \times 10^{-6}} = 2.75$ V $V_2 = 9 - 2.75 = 6.25$ V
		(iii)	$9 = (58 + R) I$ $6.25 = 58 I$ $I = 0.10775862$ A $R = 25.5\ \Omega$

9	**(a)**	Roll A will land first. Since the drop height is the same for both rolls, the loss in GPE is the same. For roll B, loss in GPE is converted to both rotational and translational KE.	
	(b)	Consider the mass of a thin hollow cylinder ρ is the density per unit area. $\rho = \dfrac{M}{\pi\left(R_{out}^2 - R_{in}^2\right)L}$ Moment of inertia of a thin hollow cylinder about axis through its centre of mass $I_r = \left(\delta M\right)r^2$ $= \rho\left(2\pi r dr\right)Lr^2 = 2\pi r^3 \rho L dr$ Moment of inertia of a solid hollow cylinder about an axis through its centre of mass, $I_{cm} = 2\pi\rho L \int_{R_{in}}^{R_{out}} r^3 dr = 2\pi\rho L \left.\dfrac{r^4}{4}\right\|_{R_{in}}^{R_{out}}$ $= \dfrac{2\pi\rho L}{4}\left(R_{out}^4 - R_{in}^4\right) = \dfrac{\pi\rho L}{2}\left(R_{out}^2 - R_{in}^2\right)\left(R_{out}^2 + R_{in}^2\right)$ $= \dfrac{1}{2}M\left(R_{out}^2 + R_{in}^2\right)$	
	(c)	**(i)**	For solid cylinder, R_{in} is approximately zero, R_{out} is R. $I_{cm} = \dfrac{1}{2}M\left(R_{out}^2 + 0\right) = \dfrac{MR^2}{2}$
		(ii)	For thin cylindrical shell, R_{out} is approximately R_{in}. $I_{cm} = \dfrac{1}{2}M\left(R^2 + R^2\right) = MR^2$

	(d)	**(i)**	 **Fig. 9.2**
		(ii)	$Torque = MgR_{out} = I_{roll,left}\alpha = \left(\frac{1}{2}M\left(R_{out}^2 + R_{in}^2\right) + MR_{out}^2\right)\alpha$ $MgR_{out} = \frac{1}{2}M\left(3R_{out}^2 + R_{in}^2\right)\alpha$ $\alpha = \frac{2gR_{out}}{\left(3R_{out}^2 + R_{in}^2\right)}$
		(iii)	By using PCOE, $\Delta GPE + \Delta KE_{rotational} + \Delta KE_{translational} = 0$ As the roll unrolls, its height changes. Let the change in height be h and $h = R_{out}\theta$, where θ is the angular displacement. $Mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv^2$ $Mg\left(R_{out}\theta\right) = \frac{1}{2}\left(\frac{1}{2}M\left(R_{out}^2 + R_{in}^2\right)\omega^2\right) + \frac{1}{2}MR_{out}^2\omega^2$ (pure roll) $Mg\left(R_{out}\theta\right) = \frac{1}{2}\left(\frac{1}{2}M\left(3R_{out}^2 + R_{in}^2\right)\omega^2\right)$ $Mg\left(R_{out}\theta\right) = \frac{1}{2}\left(\frac{1}{2}M\left(3R_{out}^2 + R_{in}^2\right)2\alpha\theta\right)$ (angular kinematics) $\alpha = \frac{2gR_{out}}{\left(3R_{out}^2 + R_{in}^2\right)}$
	(e)	For free falling roll A, it travels in a straight vertical track with uniform acceleration. $H = 0 + \frac{1}{2}gt^2$ For unwinding roll B,	

		$\theta = 0 + \frac{1}{2}\alpha t^2$ $H = \frac{1}{2} g \frac{2\theta}{\alpha} = \frac{g\theta}{\alpha}$ $H = \frac{g\theta}{2gR_{out}}\left(3R_{out}^2 + R_{in}^2\right) = \frac{h / R_{out}}{2R_{out}}\left(3R_{out}^2 + R_{in}^2\right)$ $H = \frac{1.50}{2(5.50)^2}\left(3(5.50)^2 + (1.50)^2\right) = 2.31\text{ m}$	
10	**(a)**	**(i)**	The gravitational force by the Earth provides the centripetal force on the satellite. The mass of the satellite is *m*. $\frac{GMm}{r^2} = m\omega^2 r = m\left(\frac{2\pi}{T}\right)^2 r$ $T^2 = \frac{4\pi^2}{GM} r^3 \rightarrow k = \frac{4\pi^2}{GM}$
		(ii)	$v = \omega r = \frac{2\pi}{T} r$ $= \frac{2\pi}{(90)(60)}\left(\frac{((90)(60))^2\, 6.67\times10^{-11}\left(5.97\times10^{24}\right)}{4\pi^2}\right)^{1/3} = 7737$ $= 7740\text{ m s}^{-1}$
	(b)	**(i)**	Kinetic energies of the satellite and asteroid are the same. $\frac{1}{2}mv^2 = \frac{1}{2}\frac{m}{100}v_{ast}^2 \rightarrow v_{ast} = 10v$ The satellite and asteroid move towards each other.  The velocity of the zero momentum frame (centre of mass frame) is $V_{CM} = \frac{mv - \frac{m}{100}10v}{m + \frac{m}{100}} = \frac{90}{101}v = +0.891v$

			So, the velocity of the **satellite** relative to the zero-momentum frame is, $$v-\frac{90}{101}v=\frac{11}{101}v=0.109v$$ The velocity of the **asteroid** relative to the zero-momentum frame is, $$-10v-\frac{90}{101}v=-\frac{1100}{101}v=-10.9v$$ The total momentum in the zero-momentum frame (centre of mass frame) must be zero. Let's check: $$m\frac{11}{101}v-\frac{m}{100}\frac{1100}{101}v=0$$
		(ii)	In the zero-momentum frame (centre of mass frame), the velocity of particles (in this case, satellite and asteroid) switch **signs** after a perfectly elastic collision. Hence, $v_{\text{satellite,after}}=-0.109v$ $v_{\text{asteroid,after}}=10.9v$ Using Galilean transformation, the velocity of the satellite in the Earth frame of reference becomes $$v_{\text{satellite,after\_Earth}}=v_{\text{satellite,after\_CM}}+v_{\text{CM\_Earth}}=-0.109v+\frac{90v}{101}=0.782v$$ **Note:** You may convince yourself how the velocities switch signs after a collision in zero-momentum frame as follows: The collision is modelled as perfectly elastic. We know that the total momentum in the zero-momentum frame must be zero. By the principle of conservation of momentum, the total momentum after the collision would be $$mv_{sat,after}+\frac{m}{100}v_{ast,after}=0\rightarrow v_{sat,after}=-\frac{v_{ast,after}}{100}$$ We can use RSOA = RSOS for perfectly elastic collisions,

			$u_1 - u_2 = v_2 - v_1$ $\frac{11v}{101} - \left(-\frac{1100v}{101}\right) = v_{ast,after} - v_{sat,after} = v_{ast,after} - \left(-\frac{v_{ast,after}}{100}\right)$ $\frac{1111v}{101} = \frac{101v_{ast,after}}{100}$ $\rightarrow v_{ast,after} = 10.9v$ $\rightarrow v_{sat,after} = -0.109v$
		(iii)	The collision is modelled as perfectly inelastic in this part. In the zero-momentum frame, the total momentum is zero before and after the collision. Both the satellite and asteroid move together with the same speed, which is zero, after a perfectly inelastic collision. Using Galilean transformation, the velocity of the satellite in the Earth frame of reference becomes, $v_{\text{satellite,after\_Earth}} = v_{\text{satellite,after\_CM}} + v_{\text{CM\_Earth}}$ $v_{\text{satellite,after\_Earth}} = 0 + \frac{90v}{101} = \frac{90v}{101} = 0.89v$
		(iv)	The change in momentum for the satellite is much greater for the perfectly elastic collision. Hence, the physical damage is likely greater in this model.
		(v)	The velocity v_A of the satellite at the point of impact is also the speed of satellite at apogee (furthest). This speed relationship can be derived from the following conservation laws: **Angular Momentum** at apogee and perigee: $mv_A r_A = mv_P r_P$ $v_P = v_A \frac{r_A}{r_P}$ **Total Mechanical Energy** at apogee and perigee: $\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}mv_P^2 - \frac{GMm}{r_P}$ Substituting $v_P = v_A \frac{r_A}{r_P}$ into the equation,

			$\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}m\frac{v_A^2 r_A^2}{r_P^2} - \frac{GMm}{r_P}$ $\frac{1}{2}mv_A^2\left(1 - \frac{r_A^2}{r_P^2}\right) = \frac{GMm}{r_A} - \frac{GMm}{r_P}$ $\frac{1}{2}mv_A^2\left(\frac{(r_P - r_A)(r_P + r_A)}{r_P^2}\right) = GMm\left(\frac{1}{r_A} - \frac{1}{r_P}\right)$ where $(r_P + r_A) = 2a$ $\frac{1}{2}mv_A^2\left(\frac{2a}{r_P}\right) = GMm\left(\frac{1}{r_A}\right) \rightarrow \frac{1}{2}mv_A^2 = \frac{GMm}{2a}\left(\frac{r_P}{r_A}\right)$ Hence, the total energy of the satellite is: $\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{GMm}{2a}\left(\frac{r_P}{r_A}\right) - \frac{GMm}{r_A} = -\frac{GMm}{2a}$ *The total energy in terms of semi-major axis is* $-\frac{GMm}{2a}$ *(good to know this.)* Rewriting the total energy: $\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = -\frac{GMm}{(r_P + r_A)}$ $\frac{1}{2}mv_A^2 = \frac{GMm}{r_A} - \frac{GMm}{(r_P + r_A)} = \frac{GMm(r_P + r_A - r_A)}{(r_P + r_A)r_A} = \frac{GMmr_P}{(r_P + r_A)r_A}$ $\frac{1}{2}mv_A^2 r_A r_P + \frac{1}{2}mv_A^2 r_A^2 = GMmr_P$ $r_P = \frac{\frac{1}{2}mv_A^2 r_A^2}{GMm - \frac{1}{2}mv_A^2 r_A} = \frac{v_A^2 r_A^2}{2GM - v_A^2 r_A}$ Given that, $v_A = v_1$, $r_A = r$, $r_P = h_{\min} + R_E \rightarrow h_{\min} = r_P - R_E$ $$h_{\min} = \frac{v_1^2 r^2}{2GM - v_1^2 r} - R_E$$