Tampines Meridian Junior College
2024 H2 Mathematics (9758)
Chapter 4 Equations and Inequalities
Learning Package

Resources

- ☐ Core Concept Notes
- ☐ Discussion Questions
- ☐ Extra Practice Questions

SLS Resources

- ☐ Recordings on Core Concepts
- ☐ Quick Concept Check

Reflection or Summary Page

H2 Mathematics (9758)
Chapter 4 Equations and Inequalities
Core Concept Notes

Success Criteria:

SURFACE	DEEP	TRANSFER
<u>Solving an equation</u> ☐ Solve polynomial equations by factorisation and/or quadratic formula ☐ Solve polynomial equations using graphing calculator ☐ Use of graphing calculator to solve a non-polynomial equation by sketching graphs <u>System of linear equations</u> ☐ Solve the system of linear equations using the graphing calculator <u>Inequalities</u> ☐ Represent inequalities on a real number line (e.g. $x>3$) using open/closed circle ☐ Interpret the mathematical language "and" and "or" on a real number line ☐ Solve linear or quadratic inequalities by hand or graphical methods ☐ Solve simple inequalities involving modulus by hand or graphical methods ☐ Use of graphing calculator to solve inequalities (graphical methods)	<u>System of linear equations</u> ☐ Interpret the solutions found using a graphing calculator <u>Inequalities</u> ☐ Solve inequalities of the form $\frac{\mathrm{f}(x)}{\mathrm{g}(x)}>0$ ($<, \geq, \leq$) by hand or graphical methods, where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are linear or quadratic expressions. ☐ Solve inequalities of the form $a<\mathrm{f}(x)<b$ ($\leq$) by hand or graphical methods	<u>System of linear equations</u> ☐ Formulate a contextual problem into possible mathematical equations and provide possible solutions, and interpret/reflect the solutions in the real world context <u>Inequalities</u> ☐ Formulate a contextual problem into possible mathematical inequalities and provide possible solutions, and interpret/reflect the solutions in the real world context (Examples can be in subsequent topics such as Arithemtic and Geometric Series, and Binomial / Normal Distributions.)

§1 Finding the Numerical Solution of an Equation

Recap: The solution (roots/zeros) to an equation is/are the value(s) which satisfies (satisfy) the equation.

To find the solution (roots/zeros) of an equation, we can use one of the following methods :

(a) Algebraic Factorisation/ Formula

(b) Using Application: **PlySmlt2** on a Graphing Calculator (GC)

(c) Sketching graphs using a Graphing Calculator (GC)

(a) Algebraic Factorisation/Formula *(When a question needs an **EXACT** solution)*

Example 1 Find the values of *x* satisfying the equation $x^2 + x - 2 = 0$.

Algebraic Factorisation	**Formula**
$x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $\therefore x = -2$ or $x = 1$ Factorise & solve	$x^2 + x - 2 = 0$ $x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)}$ $= \dfrac{-1 \pm \sqrt{9}}{2}$ $x = -2$ or $x = 1$

(b) Using Application: PlySmlt2 on a Graphing Calculator (GC)

This application can only be used to solve polynomials up to **degree 10**.

Example 2 Solve the equation $x^3 - 2x^2 - 5x = -6$.

$x^3 - 2x^2 - 5x = -6$ Bring everything to one side

$x^3 - 2x^2 - 5x + 6 = 0$

Using GC, $x = -2$ or $x = 1$ or $x = 3$

Using PlySmlt2 (POLY ROOT FINDER) to find roots of an equation with ONE variable

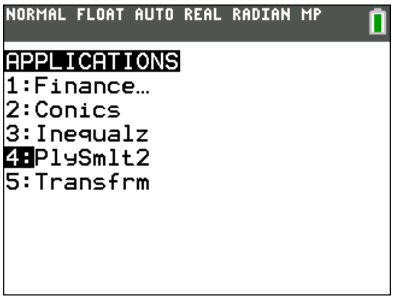
This APPS can be used only if the equation to be solved is a polynomial equation i.e. $a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0 = 0$.

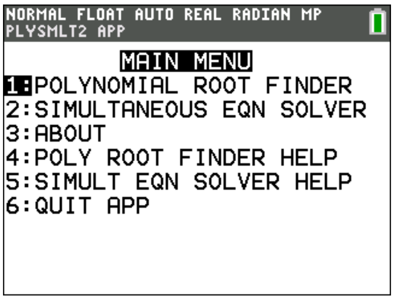
Step 1:

(i) Press $\boxed{\text{APPS}}$ and select **4:PlySmlt2**.

(ii) Press $\boxed{\text{ENTER}}$ to enter the main menu.

(iii) Select **1: POLYNOMIAL ROOT FINDER**

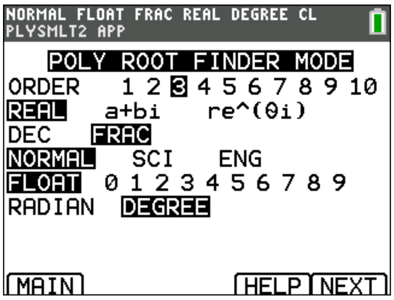




Step 2:

(i) Select the required parameters for the equation $x^3-2x^2-5x+6=0$. (Using **Example 2** on pg. 2)

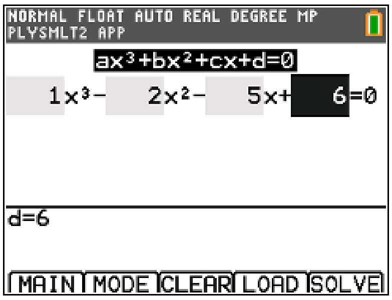
(ii) Press $\boxed{\text{GRAPH}}$ to go **NEXT**.

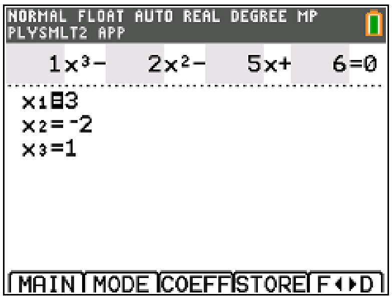


Step 3:

(i) Key in the coefficients of the equation $x^3-2x^2-5x+6=0$.

(ii) Press $\boxed{\text{GRAPH}}$ to go **SOLVE**.





(c) Sketching graphs using a Graphing Calculator (GC)

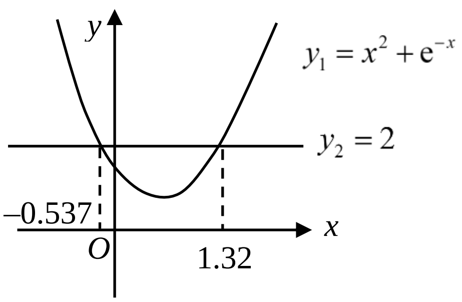
Example 3 Solve the equation $x^2 + e^{-x} = 2$.

Note: The equation involves a **non-polynomial** expression i.e. e^{-x}. Hence, **<u>PlySmlt2</u>** application **cannot be used**.

<u>Method (a): Using “**intersection**” method</u>

Step 1: Sketch the LHS and RHS of the given equation as two graphs on the same diagram.
Step 2: Find the points of intersections between these two graphs.
Step 3: The *x*-coordinates are the roots to the given equation.

Interpretation: By sketching the LHS and RHS as two graphs $y_1 = x^2 + e^{-x}$ and $y_2 = 2$, solving the equation $x^2 + e^{-x} = 2$ means finding values of x where $y_1 = y_2$ (that is, the *x*-values of the points of intersection)



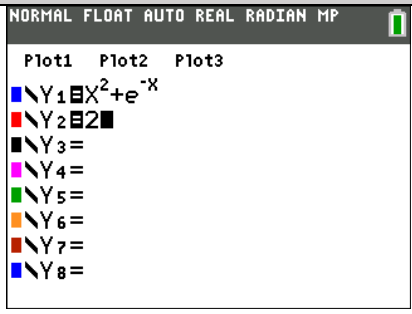
$\therefore$ the roots of the equation $x^2 + e^{-x} = 2$ are -0.537 and 1.32 (to 3 s.f.).

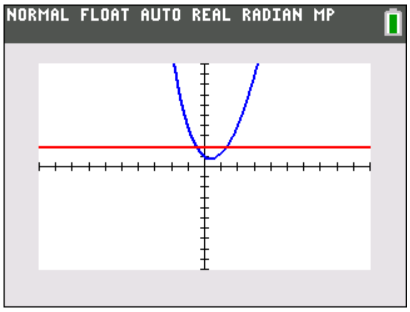
This method can be used to find the solution for any type of equation with **<u>ONE</u>** variable.
<u>The “intersection” method</u> *(Finding the intersection points of two graphs)*

<u>Step 1:</u>

(i) Press $\boxed{\text{Y=}}$.
(ii) Enter the equation $y = x^2 + e^{-x}$ in **Y1**
(iii) Press $\boxed{\text{ENTER}}$.
(iv) Enter the equation $y = 2$ in **Y2**
(v) Press $\boxed{\text{GRAPH}}$.

(Press $\boxed{\text{ZOOM}}$ followed by **2: Zoom In** to change the zoom settings)

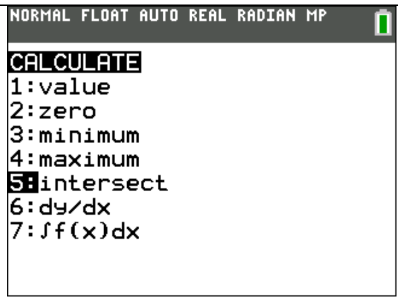




Step 2:

Press $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ and select **5: intersect**

(*Keystrokes:* $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ opens the **CALCULATE MENU** for graphs)

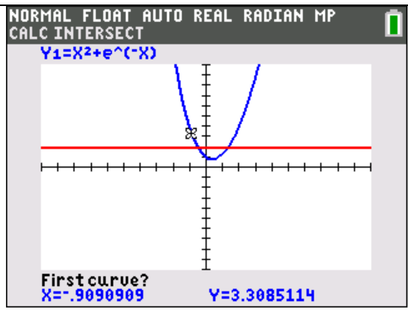


Step 3:

Press $\boxed{\text{ENTER}}$ once.

The calculator will prompt you: First curve?

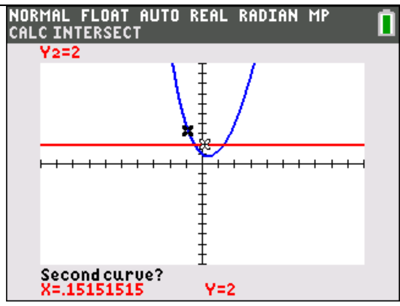
Check that it is the selected equation is **Y1**



Press $\boxed{\text{ENTER}}$ again.

The calculator will prompt you: Second curve?

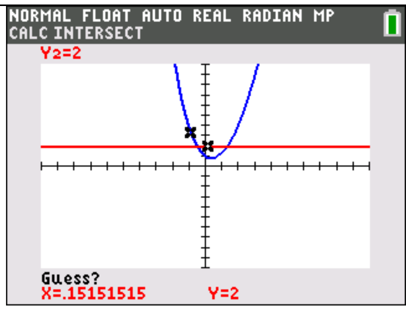
Check that it is the selected equation is **Y2**



Press $\boxed{\text{ENTER}}$ again.

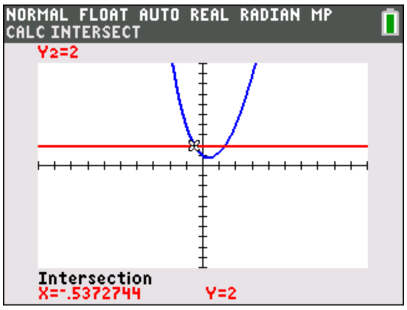
The calculator will prompt you: Guess?

Using $\boxed{\blacktriangleleft}$ and $\boxed{\blacktriangleright}$, move the blinking cursor nearer to the intersection that you want to find.



Press $\boxed{\text{ENTER}}$ one last time and the *x*-value and y-value of the intersection will be shown.

(Repeat the process to find the other intersection)



Method (b): Using "**zeros**" method

Step 1: Rewrite the given equation such that the RHS is now equals to zero.

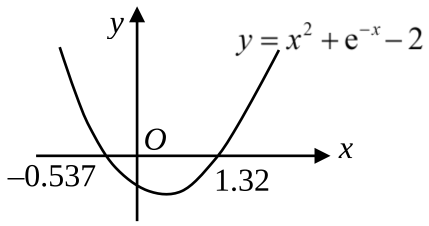
i.e. $x^2 + e^{-x} = 2 \quad \Rightarrow \quad x^2 + e^{-x} - 2 = 0$

Step 2: Sketch the LHS of the equation as one single graph.

Step 3: Find the x-intercepts of the graph. The x-intercepts are the roots to the equation $x^2 + e^{-x} - 2 = 0$.

Interpretation: Sketching the LHS as one graph $y_1 = x^2 + e^{-x} - 2$ and finding the x-intercept is equivalent to drawing two graphs $y_1 = x^2 + e^{-x} - 2$ and $y_2 = 0$ and hence solving the equation $x^2 + e^{-x} - 2 = 0$ means finding values of x where $y_1 = y_2$ (that is, the x-values of the x-intercepts).

From the GC, the graph of $y = x^2 + e^{-x} - 2$ is



$\therefore$ the roots of the equation $x^2 + e^{-x} = 2$ are –0.537 and 1.32 (to 3 s.f.).

The "zeros" method *(Finding the **x-intercepts** of a graph)*

Step 1:

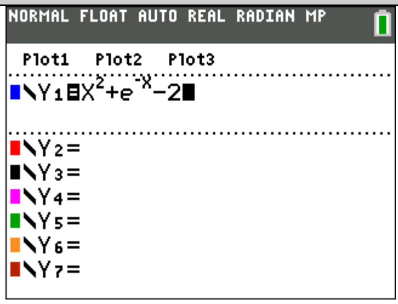
(i) Press $\boxed{\text{Y=}}$.

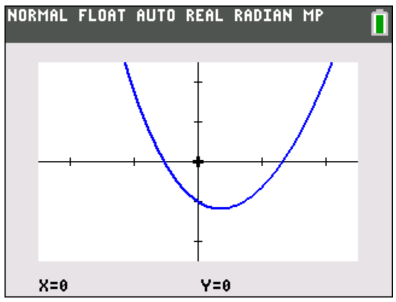
(ii) Enter the equation $y = x^2 + e^{-x} - 2$ in **Y1**

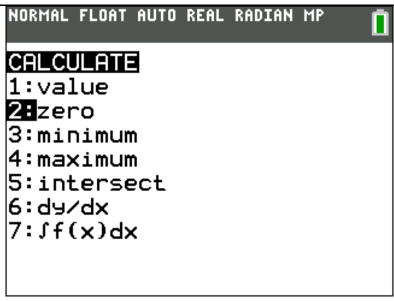
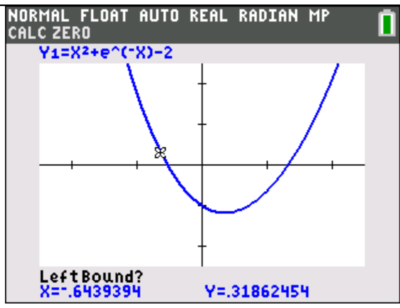
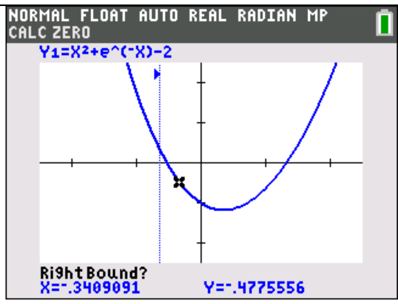
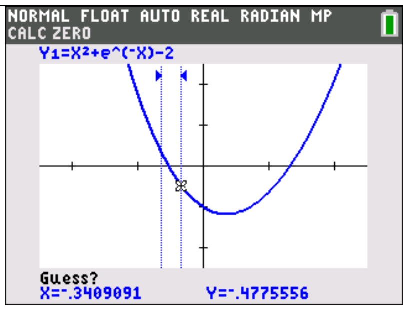
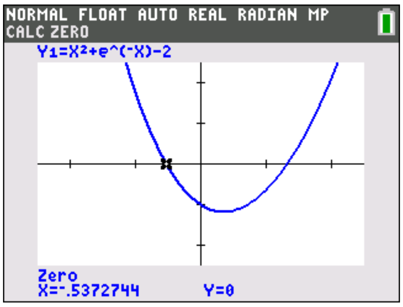
Press $\boxed{\text{ENTER}}$.

(iii) Press $\boxed{\text{GRAPH}}$.

(Press $\boxed{\text{ZOOM}}$ followed by **2: Zoom In** to change the zoom settings)





<u>Step 2:</u> Press $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ and select **2: zero** (*Keystrokes:* $\boxed{\text{2nd}}\boxed{\text{TRACE}}$ opens the **CALCULATE MENU** for graphs)	
<u>Step 3:</u> Press $\boxed{\text{ENTER}}$ once. The calculator will prompt you: Left Bound? Using $\boxed{\blacktriangleleft}$ and $\boxed{\blacktriangleright}$, move the blinking cursor to the left of the *x*-intercept that you want to find.	
Press $\boxed{\text{ENTER}}$ again. The calculator will prompt you: Right Bound? Using $\boxed{\blacktriangleleft}$ and $\boxed{\blacktriangleright}$, move the blinking cursor to the right of the *x*-intercept that you want to find.	
Press $\boxed{\text{ENTER}}$ again. The calculator will prompt you: Guess? *Do not move the cursor!* Press $\boxed{\text{ENTER}}$ one last time and the value of the *x*-intercept will be shown. (Check that the *y*-value is equal to zero) (Repeat the process to find the other two intersections)	 

§2 System of Linear Equations

A line in the *x*-*y* plane can be represented algebraically by an equation of the form

$$a_1x + a_2y = b.$$

This is known as a linear equation in the variables *x* and *y*.

More generally, a linear equation in *n* variables $x_1, x_2, \ldots, x_n$ can be expressed in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b,$$

where $a_1, a_2, \ldots, a_n$, and *b* are real constants. The variables in a linear equation are sometimes called the unknowns.

Examples of linear equations:

- $x + 3y = 7$
- $x_1 - 2x_2 - 3x_3 + x_4 = 7$
- $y = \frac{1}{2}x + 3z + 1$
- $x_1 + x_2 + \cdots + x_n = 1$

The following are **not** linear equations:

- $x + 3x^2 = 7$
- $3x + 2y - z + xz = 4$
- $x - \sin x = 0$
- $\sqrt{x_1} + 2x_1 + x_2 = 1$

A solution of a linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ is a sequence of *n* numbers $s_1, s_2, \ldots, s_n$ such that the equation is satisfied when we substitute

$$x_1 = s_1, \quad x_2 = s_2, \quad \ldots, \quad x_n = s_n.$$

The following is an example of a system of 2 linear equations in 2 variables (x_1 and x_2):

$$x_1 + x_2 = 5$$
$$2x_1 + 5x_2 = 19$$

Solving by substitution or elimination, we obtain $x_1 = 2, x_2 = 3$.

In general, given a system of *m* linear equations in *n* variables $x_1, x_2, \ldots, x_n$,

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\qquad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

Solving the system will yield either

(i) ***no*** *solution* or

(ii) ***a unique*** *solution* or

(iii) ***infinitely many*** *solutions*.

Definition: A system of equations that possesses a unique solution or infinitely many solutions is said to be ***consistent.***
If no solution exists, the system is said to be ***inconsistent.***

We shall use the GC to solve a system of linear equations**.**

Example 4 Solve the system of equations

$$x-2y+2z=2$$
$$4x-7y+az=5$$
$$3x+ay-7z=3$$

for the cases when **(i)** $a=6$, **(ii)** $a=7$ and **(iii)** $a=-5$.

(i) When $a=6$,

$$x-2y+2z=2$$
$$4x-7y+6z=5$$
$$3x+6y-7z=3$$

Using GC, $x=2,\ y=3$ and $z=3$.

There is a unique solution.

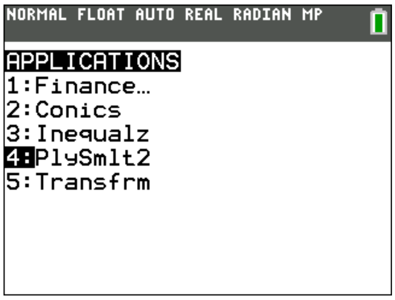
Steps in using GC App “**PlySmlt2**” to solve a system of linear equations:

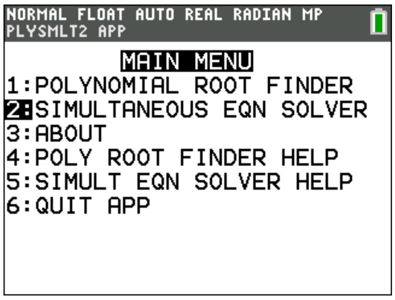
Step 1:

(i) Press [APPS] and select **4:PlySmlt2**.

(ii) Press [ENTER] to enter the main menu.

(iii) Select **2: SIMULTANEOUS EQN SOLVER**

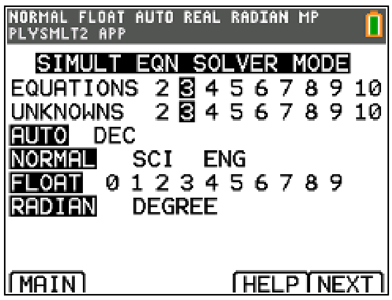




Step 2:

(i) Select the required parameters.

(ii) Press [GRAPH] to go **NEXT**.

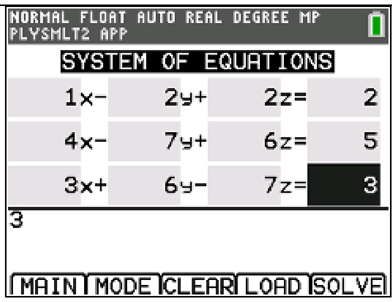


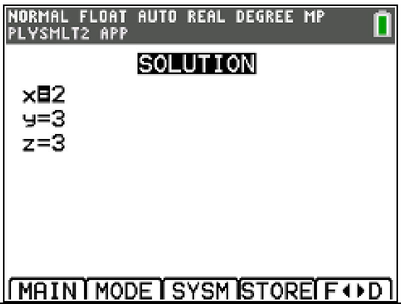
Step 3:

(i) Key in the 3 linear equations accordingly.

(ii) **Remember to press ENTER after keying in the last value.**

(iii) Press GRAPH to **SOLVE**.

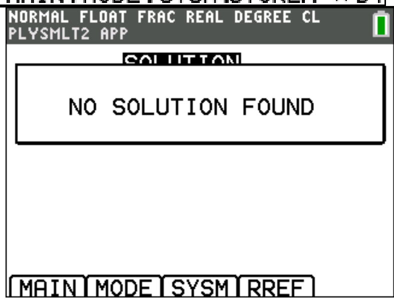




(ii) When $a = 7$,

$$x - 2y + 2z = 2$$
$$4x - 7y + 7z = 5$$
$$3x + 7y - 7z = 3$$

Using GC, there is no solution. The system is inconsistent.



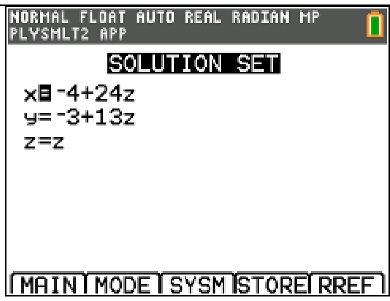
(iii) When $a = -5$,

$$x - 2y + 2z = 2$$
$$4x - 7y - 5z = 5$$
$$3x - 5y - 7z = 3$$

Using GC,

$$x = -4 + 24t$$
$$y = -3 + 13t$$
$$z = t\ ,\ t \in \mathbb{R}$$

There are infinitely many solutions.



The parameter t is used as an arbitrary variable to represent the relationship between x, y and z.

$\in$ **: is an element of**

$\mathbb{R}$ **: the set of all real numbers**

Example 5 [2012(9740)/I/1]

A cinema sells tickets at three different prices, depending on the age of the customer. The age categories are under 16 years, between 16 and 65 years, and over 65 years. Three groups of people, *A*, *B* and *C*, go to the cinema on the same day. The numbers in each age category for each group, together with the total cost of the tickets for each group, are given in the following table.

Group	Under 16 years	Between 16 and 65 years	Over 65 years	Total cost
A	9	6	4	\$162.03
B	7	5	3	\$128.36
C	10	4	5	\$158.50

Write down and solve equations to find the cost of a ticket for each of the age categories.

Let x, y and z be the cost of the tickets for the groups under 16 years, between 16 and 65 years, and over 65 years respectively (in \$).

Step 1: Define variables

$$9x+6y+4z=162.03 \quad ...(1)$$
$$7x+5y+3z=128.36 \quad ...(2)$$
$$10x+4y+5z=158.50 \quad ...(3)$$

Step 2: Form equations

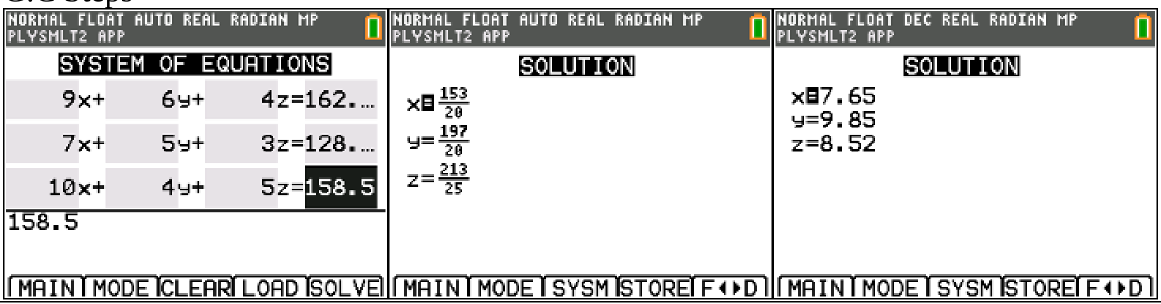
Using GC,

$x=7.65,\qquad y=9.85,\qquad z=8.52$

Step 3: Use GC to solve

Therefore, the cost of a ticket for under 16 years is \$7.65, between 16 and 65 years is \$9.85 and over 65 years is \$8.52.

G.C Steps



To convert fraction to decimal

Example 6

The curve with equation $y = ax^3 + bx^2 + cx + d$ passes through the points (0, 0), (10, 8), (20, 8) and (40, 32). Find the equation of the curve.

The curve passes through the points (0, 0), (10, 8), (20, 8) and (40, 32).

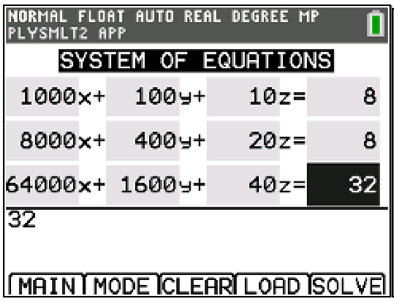
At (0, 0), $d = 0$

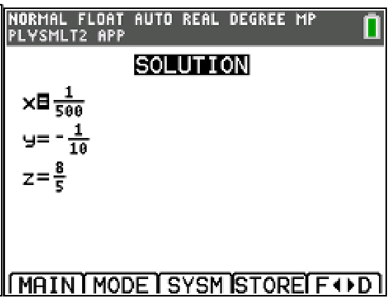
At (10, 8), $1000a + 100b + 10c = 8$
At (20, 8), $8000a + 400b + 20c = 8$
At (40, 32), $64000a + 1600b + 40c = 32$

Using GC, $a = \frac{1}{500}, b = -\frac{1}{10}, c = \frac{8}{5}, d = 0.$

Therefore, the equation of the curve is $y = \frac{1}{500}x^3 - \frac{1}{10}x^2 + \frac{8}{5}x.$

G.C Steps





§3 Inequalities

In solving equations involving unknown x, we are finding the values of x which satisfy the equation. This can also be interpreted graphically as finding the values of x where two graphs intersect. What happens when we change the equation into an inequality?

3.1 Properties of Inequalities

Let $a,\ b,\ c \in \mathbb{R}$.

(i) If $a < b$ and $b < c$, then $a < c$.

(ii) If $a < b$, then $a + c < b + c$.

Example: Given $4 < 7$, then $4 + 2 < 7 + 2$; $4 + (-3) < 7 + (-3)$.

(iii) If $a < b$ and $c > 0$, then $ac < bc$; $\dfrac{a}{c} < \dfrac{b}{c}$.

Example: Given $3 < 5$, then $3 \times 2 < 5 \times 2$; $\dfrac{3}{4} < \dfrac{5}{4}$.

(iv) If $a < b$ and $c < 0$, then $ac > bc$; $\dfrac{a}{c} > \dfrac{b}{c}$.

Example: Given $3 < 9$, then $3 \times (-2) > 9 \times (-2)$; $\dfrac{3}{-3} > \dfrac{9}{-3}$.

(v) If $0 \le a < b$, then $a^2 < b^2$.

Example: Given $2 < 3$, then $2^2 < 3^2$;

Note: Given $a < b$, this **does not necessarily imply** that $a^2 < b^2$.

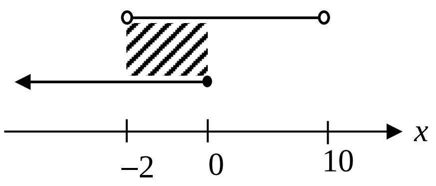
Example: Given $-2 < 1$ but $(-2)^2 > 1^2$.

Example 7 By drawing number lines, simplify the following ranges of values of x:

(a) $x \le 0$ and $-2 < x < 10$, **(b)** $-5 < x < 12$ or $x \le 10$.

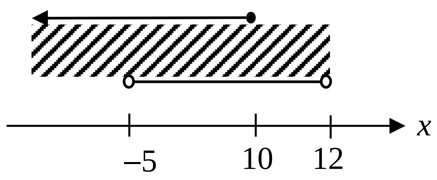
Note: "**and**" – to find the <u>intersection</u> of the ranges;
"**or**" – to find the <u>union</u> of the ranges.
Open circle ○ – to indicate points to be <u>excluded</u>.

(a) $x \le 0$ and $-2 < x < 10$



$\therefore -2 < x \le 0$

(b) $-5 < x < 12$ or $x \le 10$



$\therefore x < 12$

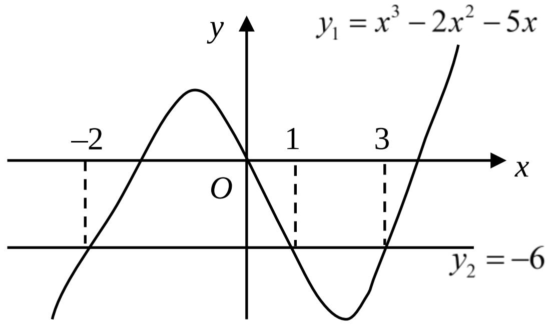
3.2 Solving Inequalities (Graphical Methods)

Example 8 Solve the inequality $x^3 - 2x^2 - 5x < -6$. (Revisit **Example 2**)

Method (a): Using "**intersection**" method

This method involves sketching the LHS and RHS as two graphs.

Interpretation: By sketching the LHS and RHS as two graphs $y_1 = x^3 - 2x^2 - 5x$ and $y_2 = -6$, solving the inequality $x^3 - 2x^2 - 5x < -6$ means finding values of x where $y_1 < y_2$.



The 2 graphs $y_1 = x^3 - 2x^2 - 5x$ and $y_2 = -6$ intersect at $x = -2,\ x = 1,\ x = 3$.

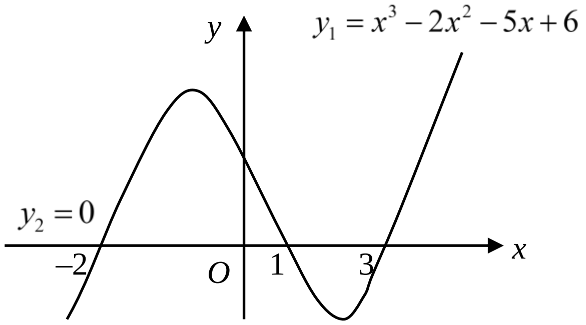
$\therefore x < -2 \quad \text{or} \quad 1 < x < 3$.

Method (b): Using "**zeros**" method

This method involves rewriting the equation such that RHS is equal to zero, sketching the LHS as one graph.

$x^3 - 2x^2 - 5x < -6 \ \Rightarrow x^3 - 2x^2 - 5x + 6 < 0$

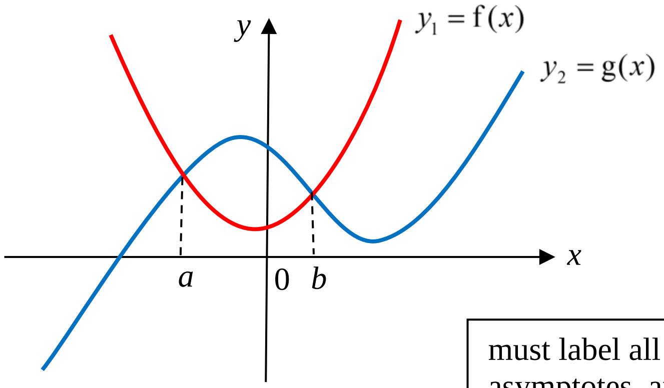
Interpretation: Sketching the LHS as one graph $y_1 = x^3 - 2x^2 - 5x + 6$ and finding the x-intercept is equivalent to drawing two graphs $y_1 = x^3 - 2x^2 - 5x + 6$ and $y_2 = 0$ and hence solving the inequality $x^3 - 2x^2 - 5x + 6 < 0$ means finding values of x where $y_1 < y_2$.



The x- intercepts are $x = -2,\ x = 1,\ x = 3$.

$\therefore x < -2 \quad \text{or} \quad 1 < x < 3$.

Conclusion: In general, solving an inequality means to find the values of x which satisfy the inequality. Given 2 graphs $y_1 = \mathrm{f}(x)$ and $y_2 = \mathrm{g}(x)$,



must label all details of graph i.e. asymptotes, axial intercept etc.

For $\mathrm{f}(x) > \mathrm{g}(x)$, $x < a$ or $x > b$.

For $\mathrm{f}(x) < \mathrm{g}(x)$, $a < x < b$.

3.3 Solving Inequalities (Algebraic method)

In this section, we will learn about various algebraic methods available to solve different types of inequalities.

3.3.1 Inequalities involving Polynomial

Example 9 Find the solution set of the following inequalities:

(a) $x^3 - 2x^2 - 5x < -6$, **(b)** $x^2 - 4x \le -1$, **(c)** $x^2 - 4x + 5 > 0$.

Solution:

(a) Method (1) Factorisation

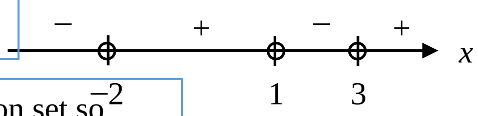
(Algebraic presentation to graphing method (b) in Example 8)

$x^3 - 2x^2 - 5x < -6$ Bring everything to one side

$x^3 - 2x^2 - 5x + 6 < 0$

$(x-1)(x+2)(x-3) < 0$ Factorise

$\therefore\ x < -2$ or $1 < x < 3$ Solve using number line



$\{ x \in \mathbb{R} : x < -2 \text{ or } 1 < x < 3 \}$ Qn asks for solution set so give answer in the set notation

Observe sign changes between critical values by substituting in values (i.e. multiplication of the **sign** of respective factor that results in the final sign)

<u>Extension:</u> Try if the sign changes to $>$, $\le$, $\ge$.

(b) Method (2): Formula

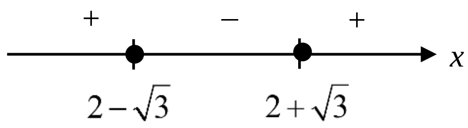
$$x^2 - 4x \le -1$$
$$x^2 - 4x + 1 \le 0$$
$$\left(x-\left(2-\sqrt{3}\right)\right)\left(x-\left(2+\sqrt{3}\right)\right) \le 0$$

Bring everything to one side

Factorise

Consider $x^2 - 4x + 1 = 0$

$$x = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$



Solve using number line

$$\therefore 2-\sqrt{3} \le x \le 2+\sqrt{3}$$
$$\left\{ x \in \mathbb{R} : 2-\sqrt{3} \le x \le 2+\sqrt{3} \right\}$$

Method (3): Complete the Square

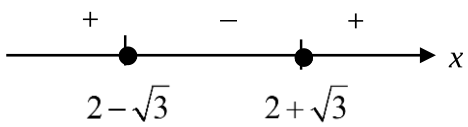
Bring everything to one side

$$x^2 - 4x + 1 \le 0$$
$$x^2 - 4x + 2^2 - 2^2 + 1 \le 0$$
$$(x-2)^2 - 3 \le 0$$
$$(x-2)^2 - \left(\sqrt{3}\right)^2 \le 0$$
$$\left(x-2+\sqrt{3}\right)\left(x-2-\sqrt{3}\right) \le 0$$
$$\left(x-\left(2-\sqrt{3}\right)\right)\left(x-\left(2+\sqrt{3}\right)\right) \le 0$$

Factorise

Recall: $a^2 - b^2 = (a+b)(a-b)$

Solve using number line



$$\therefore 2-\sqrt{3} \le x \le 2+\sqrt{3}$$
$$\left\{ x \in \mathbb{R} : 2-\sqrt{3} \le x \le 2+\sqrt{3} \right\}$$

(c) $x^2 - 4x + 5 > 0$

Method (1): Complete the square

$x^2 - 4x + 5 = (x-2)^2 + 1 > 0$ for all real values of x.

The solution set = $\mathbb{R}$

Method (2): Discriminant<0 AND coefficient of x^2

Discriminant $(-4)^2 - 4(1)(5) = -4 < 0$ **and** coefficient of $x^2 > 0$

Hence, $x^2 - 4x + 5 > 0$ for all real values of x.

The solution set = $\mathbb{R}$

<u>Extension:</u>

$x^2 - 4x + 5 < 0$

The solution set = $\varnothing$

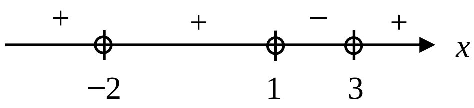
Note:
Observe that in Example 9, if all the factors are linear with no repeated roots, the sign alternates

Question: What do you do if there are repeated linear factors?

Example 10 Solve the inequality $(x-1)^3(x+2)^2(x-3)<0$.

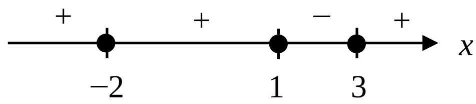
Solution:

$\therefore\ 1<x<3$



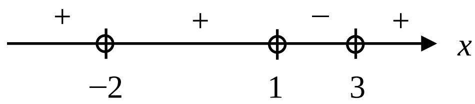
<u>Extension</u>

• $(x-1)^3(x+2)^2(x-3)\le 0 \Rightarrow 1\le x\le 3,$ or $x=-2$

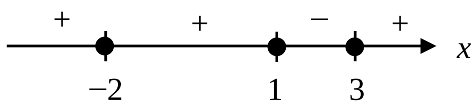


• $(x-1)^3(x+2)^2(x-3)>0 \Rightarrow x<1,\ x\ne -2$ or $x>3$

[Can also be written as $x<-2$ or $-2<x<1$ or $x>3$]



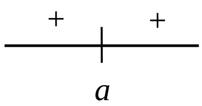
• $(x-1)^3(x+2)^2(x-3)\ge 0 \Rightarrow x\le 1$ or $x\ge 3$



NOTE:

Observe from *Example 9* that if $(x-a)^n$ is a factor and

(i) n is an **even** integer, then sign remains the same.



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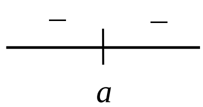
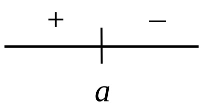


(ii) n is an **odd** integer, then sign changes.



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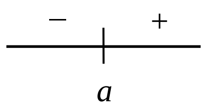


(iii) Coefficients of x are positive for all the factors, then rightmost region is positive.

3.3.2 Inequalities involving Algebraic Fractions

Investigation:

(i) Solve the inequality $\frac{5x-1}{x^2+1} \geq 2$.

$$\frac{5x-1}{x^2+1} \geq 2$$
$$5x-1 \geq 2x^2+2$$
$$2x^2+2-5x+1 \leq 0$$
$$2x^2-5x+3 \leq 0$$
$$(2x-3)(x-1) \leq 0$$
$$\therefore 1 \leq x \leq 1.5$$

Is this correct? Why?

Correct. Since the denominator x^2+1 is always positive for all values of *x*, it will not affect the inequality sign when multiplying both sides by the denominator x^2+1.

(ii) Solve the inequality $\frac{1}{x+2} \geq 1$ algebraically.

$$\frac{1}{x+2} \geq 1$$
$$1 \geq x+2$$
$$x+2 \leq 1$$
$$x \leq -1$$

Is this correct? Hint: does $x=-3$ satisfies the inequality?

Incorrect. Since the denominator $x+2$ can be positive or negative depending on the actual value of *x*, it will affect the inequality sign differently when multiplying both side by denominator $x+2$. Hence **<u>DO NOT</u>** multiply both sides by denominator when unsure of whether denominator is positive or negative.

The correct way to solve this inequality should be

$$\frac{1}{x+2} \geq 1$$

Step 1: Bring everything to one side

$$\frac{1}{x+2} - 1 \geq 0$$

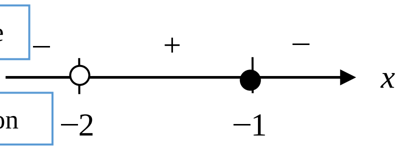
Step 2: Combine into one fraction

$$\frac{1-(x+2)}{x+2} \geq 0$$

Step 3: Simplify/ factorise /completing

$$\frac{-x-1}{x+2} \geq 0$$

Step 4: Solve using number line



$$-2 < x \leq -1$$

NOTE:

1. Multiply both sides by denominator **ONLY** when the denominator is **ALWAYS POSITIVE** for all real values of x
2. Avoid squaring both sides unless you are certain both sides are positive (in which case the inequality sign remains unchanged) or both sides are negative (in which case the inequality sign will be reversed)

For teachers:
To give numerical counterexample and link to property (v) of inequality

In general, we should

(1) Bring all terms to one side of the inequalities,

(2) Combine into 1 single fraction,

(3) Factorise as completely as possible (Example 9a), otherwise use completing the square to check the nature of the quadratic expression (Example 9b)

(4) Solve the inequality with the help of a number line. Include "open"/"close" circle to indicate if the value is included/not included.

Example 11 Solve the following inequality $\dfrac{1}{x} > \dfrac{x-15}{(x-3)^2}$.

$$\frac{1}{x} > \frac{x-15}{(x-3)^2} \quad , \quad x \neq 0, \;\; x \neq 3$$

Step 1: Bring everything to one side

$$\frac{1}{x} - \frac{x-15}{(x-3)^2} > 0$$

Step 2: Combine into one fraction

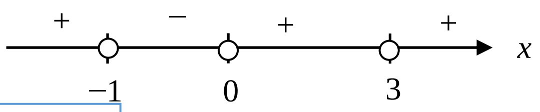
$$\frac{(x-3)^2 - x(x-15)}{x(x-3)^2} > 0$$

$$\frac{x^2 - 6x + 9 - (x^2 - 15x)}{x(x-3)^2} > 0$$

Step 3: Simplify/ factorise /completing

$$\frac{9(x+1)}{x(x-3)^2} > 0$$

$$\frac{x+1}{x(x-3)^2} > 0$$



Step 4: Solve using number line

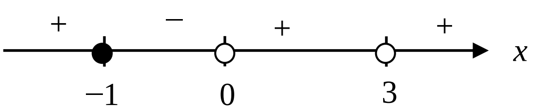
Observe sign changes between critical values by substituting in values

$x < -1$ or $x > 0, \; x \neq 3$

<u>Extension:</u>

Solve the following inequality $\dfrac{1}{x} \geq \dfrac{x-15}{(x-3)^2}$.

$x \leq -1$ or $x > 0, \; x \neq 3$



Example 12 [2011(9740)/I/1]

Without using a calculator, solve the inequality $\dfrac{x^2+x+1}{x^2+x-2}<0$.

$$\frac{x^2+x+1}{x^2+x-2}<0, \quad x\neq -2, \; x\neq 1$$

Simplify/ factorise /completing the square

$$\frac{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}{(x+2)(x-1)}<0$$

$$\frac{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}{(x+2)(x-1)}<0$$

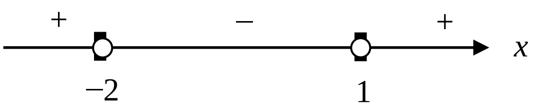
we can divide both sides by $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$ *without affecting its inequalities sign:*

$$\frac{1}{(x+2)(x-1)}<\frac{0}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\Rightarrow \frac{1}{(x+2)(x-1)}<0$$

Since $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}>0$ for all real values of x,

$$\therefore \frac{1}{(x+2)(x-1)}<0$$



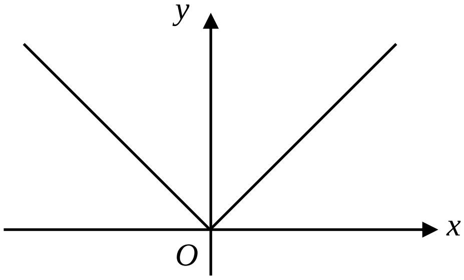
$\therefore -2<x<1$

<u>Extension:</u> What would the solution be if the sign changes to $\leq$?

3.3.3 <u>**Inequalities involving Modulus Functions**</u>

The <u>**modulus**</u> of x, denoted by $|x|$, is the **positive** numerical value of x.

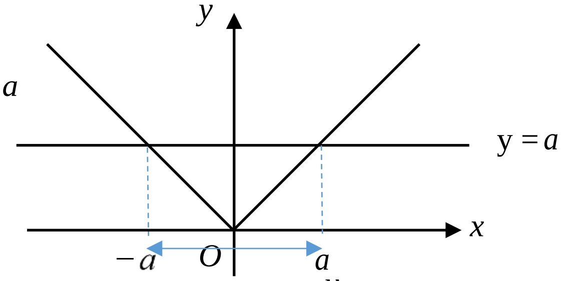
i.e. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



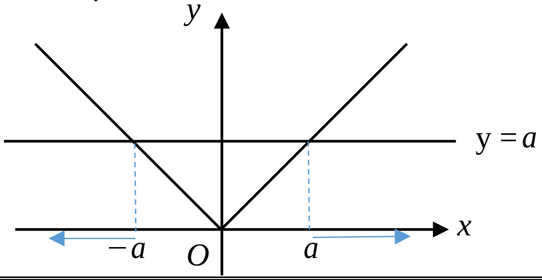
<u>Key Results:</u>

1. $|x|^2 = x^2$

2. For $a > 0$, $|x| < a \Leftrightarrow -a < x < a$



For $a > 0$, $|x| > a \Leftrightarrow x < -a$ or $x > a$



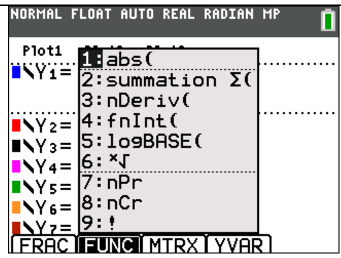
Steps in using GC to obtain the graph of $y = |x|$:

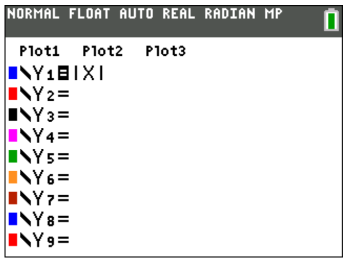
(i) Press $\boxed{\text{Y=}}$.

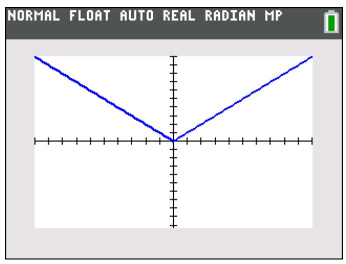
(ii) To enter the equation $y = |x|$,

Press: $\boxed{\text{ALPHA}}\boxed{\text{WINDOW}}\boxed{1}\boxed{\text{X,T,}\Theta\text{,}n}$.
Press $\boxed{\text{ENTER}}$.

(iii) Press $\boxed{\text{GRAPH}}$.





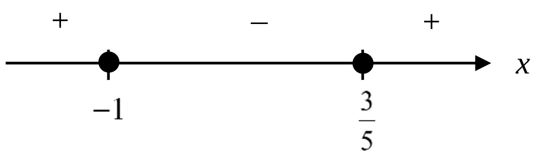


Example 13(a) Solve the inequality $4|x| \le |x-3|$.

Solution:

$$4|x| \le |x-3|$$
$$(4x)^2 \le (x-3)^2$$
$$(4x)^2 - (x-3)^2 \le 0$$
$$\big(4x-(x-3)\big)\big(4x+(x-3)\big) \le 0$$
$$(3x+3)(5x-3) \le 0$$

Why can we take square on both sides?

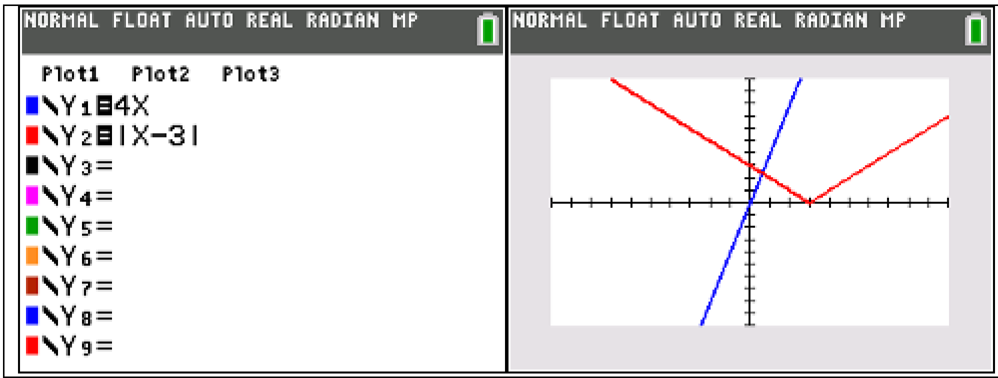


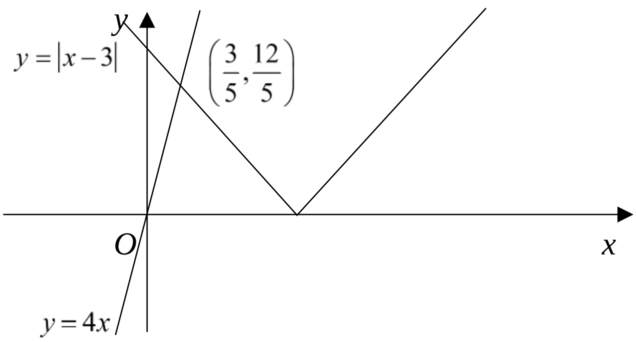
$\therefore \quad -1 \le x \le \frac{3}{5}$

Question: Can we take square on both sides when solving inequalities such as $4x < |x-3|$? Why?

No, we cannot take square on both sides. We do not know if the LHS of the inequality is always non-negative.

Example 13(b) Solve the inequality $4x < |x-3|$.



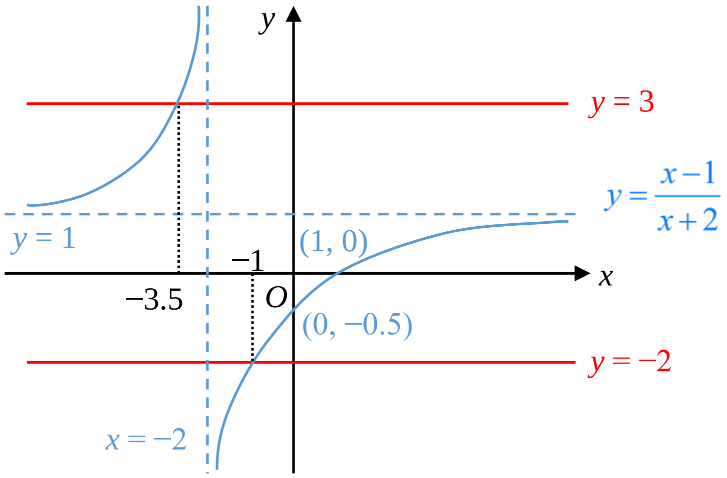


Solving $x < 4|x-3|$ graphically, $x < \frac{3}{5}$.

3.4 Miscellaneous Examples

Example 14 Find the solution set for the inequality $-2 \le \dfrac{x-1}{x+2} < 3$.

Method 1 (Graphical)



For graphical approach, ensure that all key features of the graph are labelled. Please include equations of asymptotes and axial intercepts.

Using the GC, the x-coordinates of intersection points are $x = -3.5$ or $x = -1$.

$\therefore$ the solution set $= \{x \in \mathbb{R} : x < -3.5 \text{ or } x \ge -1\}$

Qn asks for solution set so do remember to give your answer in the set notation

Method 2 (Algebraic)

$$-2 \le \frac{x-1}{x+2} < 3, \quad x \ne -2$$

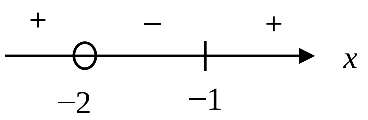
Both inequalities must be satisfied so use "and"

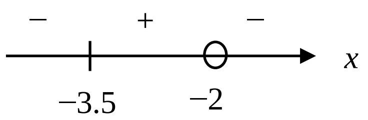
$-2 \le \dfrac{x-1}{x+2}, \quad x \ne -2$ and $\dfrac{x-1}{x+2} < 3, \quad x \ne -2$

$\dfrac{x-1}{x+2} + 2 \ge 0$ and $\dfrac{x-1}{x+2} - 3 < 0$

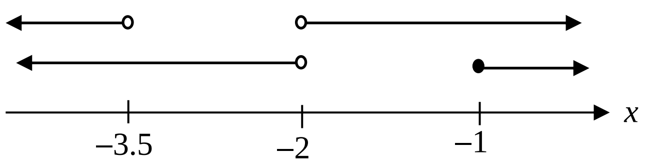
$\dfrac{x-1+2(x+2)}{x+2} \ge 0$ and $\dfrac{x-1-3(x+2)}{x+2} < 0$

$\dfrac{3x+3}{x+2} \ge 0$ and $\dfrac{-2x-7}{x+2} < 0$





$x < -2$ or $x \ge -1$ and $x < -3.5$ or $x > -2$



$\therefore$ the solution set $= \{x \in \mathbb{R} : x < -3.5 \text{ or } x \ge -1\}$.

Example 15

Solve the inequality $\dfrac{x+1}{x-1} > \dfrac{6}{x}$.

Hence solve the inequality **(i)** $\dfrac{e^x+1}{e^x-1} > \dfrac{6}{e^x}$, **(ii)** $\dfrac{|x|+1}{|x|-1} > \dfrac{6}{|x|}$, **(iii)** $\dfrac{-x+1}{x+1} < \dfrac{6}{x}$.

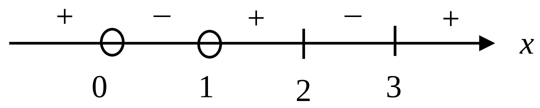
$$\frac{x+1}{x-1} > \frac{6}{x}, \quad x \neq 0, x \neq 1$$

$$\frac{x+1}{x-1} - \frac{6}{x} > 0$$

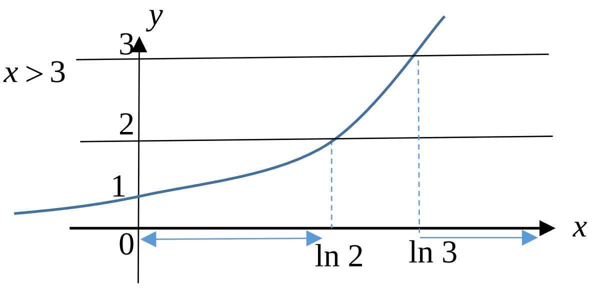
$$\frac{x(x+1)-6(x-1)}{(x-1)x} > 0$$

$$\frac{x^2-5x+6}{x(x-1)} > 0$$

$$\frac{(x-2)(x-3)}{x(x-1)} > 0$$



$\therefore x < 0 \quad \text{or} \quad 1 < x < 2 \quad \text{or} \quad x > 3$



(i) $\dfrac{e^x+1}{e^x-1} > \dfrac{6}{e^x}$

Replace x with e^x,

$e^x < 0$ or $1 < e^x < 2$ or $e^x > 3$

(N.A. $\because e^x > 0 \quad \forall x \in \mathbb{R}$) $0 < x < \ln 2$ or $x > \ln 3$

Note the notation "for all" here.

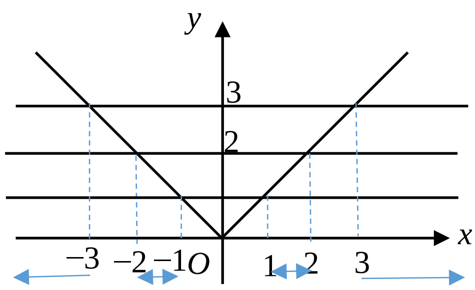
Refer to the graph of $y = e^x$ and observe that it is above the x-axis thus e^x is always positive

(ii) $\dfrac{|x|+1}{|x|-1} > \dfrac{6}{|x|}$

Replace x with $|x|$,

$|x| < 0$ or $1 < |x| < 2$ or $|x| > 3$

(N.A. $\because |x| \geq 0 \ \ \forall x \in \mathbb{R}$) $-2 < x < -1$ or $1 < x < 2$ or $x < -3$ or $x > 3$



(iii) $\dfrac{-x+1}{x+1} < \dfrac{6}{x}$

$-\dfrac{-x+1}{x+1} > -\dfrac{6}{x}$

$\dfrac{-x+1}{-x-1} > \dfrac{6}{-x}$

Replace x with $-x$,

$-x < 0$ or $1 < -x < 2$ or $-x > 3$

$x > 0$ or $-2 < x < -1$ or $x < -3$

Note that when multiply/divide by -1, the inequality sign changes

H2 Mathematics (9758)
Chapter 4 Equations and Inequalities
Discussion Questions

Level 1 Questions

1 Solve the equation $21x^2 - 11x - 2 = 0$ algebraically.

2 Using a graphing calculator, solve the following equations

(a) $2x^3 - 23x^2 + 41x + 115 = 0$,

(b) $\dfrac{1}{x} - 2 + 2\ln x = 0$.

3 Solve the system of linear equations:

$$\begin{array}{ccccccccc} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 10 \\ x_1 & + & x_2 & - & x_3 & - & x_4 & = & 4 \\ x_1 & - & x_2 & + & x_3 & - & x_4 & = & 2 \\ x_1 & - & x_2 & - & x_3 & + & x_4 & = & 0 \end{array}$$

4 By drawing number lines, simplify the following ranges of values of x:

(a) $-3 \le x \le 8$ and $x > 4$,

(b) $3 < x \le 8$ or $-2 < x \le 5$.

5 Without the use of a calculator, solve the following inequalities

(a) $2x \ge 5 + 4x$,

(b) $9x^2 - 6x - 8 > 0$,

(c) $|x+2| < 7$,

(d) $|2x-1| > 3$.

6 By sketching appropriate graphs, solve the following inequalities

(a) $\dfrac{1}{x} < \ln x$,

(b) $|x^2 - 4x + 1| - 1 \le 0$.

Level 2 Questions

7 A VCR manufacturing company produces three models: model *A*, model *B* and model *C*. The time taken for the production, assembly and testing of each model is given in the table below. To minimize costs, the company decides that 385 hours should be allocated for production, 557 hours for assembly, and 128 hours for testing. How many of each model can the company produce if it wants to use up all the allocated time for each phase of the process?

Model	Production time in hrs	Assembly time in hrs	Testing time in hrs
A	1.8	3.0	0.5
B	2.2	3.2	0.8
C	3.0	3.5	1.0

8 **2008/HCI prelim/I/1**

A student has been saving 10 cent, 20 cent and 50 cent coins in a moneybox. When she opened the box after one month, she found the amount saved is $15 and the number of 10 cent coins equals the total number of 20 cent and 50 cent coins. She also found that only half as many coins is needed to save the same amount using just 50 cent coins. Find the number of 10 cent, 20 cent and 50 cent coins in the moneybox. [4]

9 **2015(9740)/I/1**

A curve *C* has equation

$$y = \frac{a}{x^2} + bx + c$$

where *a*, *b* and *c* are constants. It is given that *C* passes through the points with coordinates $(1.6, -2.4)$ and $(-0.7, 3.6)$, and that the gradient of *C* is 2 at the point where $x = 1$.

(i) Find the values of *a*, *b* and *c*, giving your answers correct to 3 decimal places. [4]

(ii) Find the *x*-coordinate of the point where *C* crosses the *x*-axis, giving your answer correct to 3 decimal places. [2]

(iii) One asymptote of *C* is the line with equation $x = 0$. Write down the equation of the other asymptote of *C*. [1]

10 Solve the following inequalities

(a) $(1-3x)(3-x)(1-x) \geq 0$,

(b) $(x+4)(x-1)^2(x-2) \geq 0$,

(c) $(x^2-4x+5)(x-1) \leq 0$.

11 Solve the following inequalities algebraically

(a) $\dfrac{x-1}{3x+4}<0$,

(b) $x+2\le\dfrac{30}{x+1}$.

12 **2010/TPJC/I/4**

Given that x is real, prove that $4x^2+2x+1$ is always positive.

Hence, without using a calculator, solve the inequality $2x+\dfrac{1}{1+2x}>0$. [4]

13 **2002(9740)/I/4 (modified)**

Without using a calculator, find the solution set of the following inequalities.

(a) $x-\dfrac{1}{x}\le 1$, **(b)** $-1<\dfrac{2x+3}{x-1}<1$.

14 **2006(9740)/II/1**

Solve the inequality $\dfrac{x-9}{x^2-9}\le 1$.

Hence, solve the inequalities **(i)** $\dfrac{|x|-9}{x^2-9}\le 1$, **(ii)** $\dfrac{x+9}{x^2-9}\ge -1$.

15 **2019(9758)/I/4**

(i) Sketch the graph of $y=\left|2^x-10\right|$, giving the exact values of any points where the curve meets the axes. [3]

(ii) Without using a calculator, and showing all your working, find the exact interval, or intervals, for which $\left|2^x-10\right|\le 6$. Give your answer in its simplest form. [3]

16 **2018(9758)/I/4**

(i) Find the exact roots of the equation $\left|2x^2+3x-2\right|=2-x$. [4]

(ii) On the same axes, sketch the curves with equations $y=\left|2x^2+3x-2\right|$ and $y=2-x$.

Hence solve exactly the inequality

$$\left|2x^2+3x-2\right|<2-x.$$ [4]

Level 3 Questions

17 2009(9740)/I/1

(i) The first three terms of a sequence are given by $u_1 = 10$, $u_2 = 6$, $u_3 = 5$. Given that u_n is a quadratic polynomial in n, find u_n in terms of n. [4]

(ii) Find the set of values of n for which u_n is greater than 100. [2]

18 2017(9758)/I/2

(i) On the same axes, sketch the graphs of $y = \dfrac{1}{x-a}$ and $y = b|x-a|$, where a and b are positive constants. [2]

(ii) Hence, or otherwise, solve the inequality $\dfrac{1}{x-a} < b|x-a|$. [4]

19 2018/AJC Promo/Q1

A dietitian wishes to plan a meal using three types of ingredients. The meal is to include 8800 microgram (μg) of vitamin *A*, 3380 μg of vitamin *C* and 1020 μg of calcium. The amount of the vitamins and calcium in each unit of the ingredients is summarised in the following table:

	Ingredient I	**Ingredient II**	**Ingredient III**
Vitamin *A*	400	1200	800
Vitamin *C*	110	570	340
Calcium	90	30	60

Assuming that all the 3 ingredients were used, find the three possible combinations of the number of integer units of each ingredient the dietitian should include in the meal in order to meet the vitamins and calcium requirements. [5]

Answer Key	
Level 1	
1 $x=-\frac{1}{7}$ or $x=\frac{2}{3}$	**4 (a)** $4<x\le 8$, **(b)** $-2<x\le 8$
2 (a) $x=-1.46$ or $x=4.87$ or $x=8.08$ **(b)** $x = 0.187$ or $x = 2.16$	**5 (a)** $x\le -\frac{5}{2}$, **(b)** $x<-\frac{2}{3}$ or $x>\frac{4}{3}$ **(c)** $-9<x<5$ **(d)** $x<-1$ or $x>2$
3 $x_1 = 4$, $x_2 = 3$, $x_3 = 2$, $x_4 = 1$	**6 (a)** $x>1.76$ **(b)** $0\le x\le 0.586$ or $3.41\le x\le 4$
Level 2	
7 60 Model *A*, 85 Model *B* and 30 Model *C*	**12** $x>-\frac{1}{2}$
8 30 10-cent coins, 10 20-cent coins, 20 50-cent coins	**13 (a)** $\left\{x\in\mathbb{R}: x\le \frac{1}{2}-\frac{\sqrt{5}}{2} \text{ or } 0<x\le \frac{1}{2}+\frac{\sqrt{5}}{2}\right\}$ **(b)** $\left\{x\in\mathbb{R}: -4<x<-\frac{2}{3}\right\}$
9 (i) $\left.\begin{array}{l} a=-3.59345\approx -3.593 \\ b=-5.18691\approx -5.187 \\ c=7.30274\approx 7.303 \end{array}\right\}$ **(ii)** $x=-0.589$ **(iii)** $y=-5.19x+7.30$	**14** $x<-3$ or $0\le x\le 1$ or $x>3$, **(i)** $-1\le x\le 1$ or $x<-3$ or $x>3$ **(ii)** $x>3$ or $-1\le x\le 0$ or $x<-3$
10 (a) $x\le\frac{1}{3}$ or $1\le x\le 3$ **(b)** $x\le -4$ or $x=1$ or $x\ge 2$ **(c)** $x\le 1$	**15 (ii)** $2\le x\le 4$
11 (a) $-\frac{4}{3}<x<1$ **(b)** $x\le -7$ or $-1<x\le 4$	**16 (i)** $x=-1+\sqrt{3}$ or $x=-1-\sqrt{3}$ or $x=0$ or $x=-1$ **(ii)** $-1-\sqrt{3}<x<-1$ or $0<x<-1+\sqrt{3}$
Level 3	
17 (i) $u_n=\frac{3}{2}n^2-\frac{17}{2}n+17$ **(ii)** $\{n\in\mathbb{Z}: n\ge 11\}$	**19** When $z = 2$, $x = 9$, $y = 3$. When $z = 4$, $x = 8$, $y = 2$. When $z = 6$, $x = 7$, $y =1$.
18 $x<a$ or $x>a+\frac{\sqrt{b}}{b}$	

H2 Mathematics (9758)
Chapter 4 Equations & Inequalities
Extra Practice Questions

1 **2009/VJC Prelim/I/2**

Mr Spongebob went to the supermarket on 3 separate occasions to buy crabs, lobsters and bamboo clams. He observed that while the price of crabs and bamboo clams remained constant, the price of lobsters consistently increased by 10% compared to the immediate previous visit. The amount of crabs, lobsters and bamboo clams that he bought by weight for each visit as well as the total amount spent is shown in the table below.

	1st visit	2nd visit	3rd visit
Crab (kg)	3.20	5.60	4.50
Lobster (kg)	1.50	1.20	2
Bamboo Clam (kg)	7	6.50	6.50
Total amount paid in $	277.50	347.00	395.18

What is the price per kilogram for crab, lobster and bamboo clam during Mr Spongebob's third visit to the supermarket? [5]

2 **2010/SAJC Prelim/I/1**

The first 4 terms of a sequence are given by $u_1 = 63$, $u_2 = 116$, $u_3 = 171$, $u_4 = 234$.

Given that u_n is a cubic polynomial in n, find u_n in terms of n. [3]

Hence write down the value of u_{50}. [1]

3 **2010/NJC Prelim/I/1**

The sum of the digits in a three-digit-number is 15. Reversing the digits in that number decreases its value by 594. Also, the sum of the tenth digit and four times the unit digit is five more than the hundredth digit. Find the number. [4]

4 **2018/RI Promo/Q1**

It is given that $\mathrm{f}(x) = x^3 + ax^2 + bx + c$. When $\mathrm{f}(x)$ is divided by $(x+1)$ and $(x+2)$, the remainders are 24 and 36 respectively. Given also that $(x-1)$ is a factor of $\mathrm{f}(x)$, find the values of a, b and c. [3]

5 **2015/I/2**

(i) Sketch the curve with equation $y = \left|\frac{x+1}{1-x}\right|$, stating the equation of the asymptotes.

On the same diagram, sketch the line with equation $y = x + 2$. [3]

(ii) Solve the inequality $\left|\frac{x+1}{1-x}\right| < x + 2$. [3]

6 **2008/II/1 Modified**

Given that $\mathrm{f}(x) = \mathrm{e}^x \sin x$ and $\mathrm{g}(x) = x + x^2 + \frac{1}{3}x^3$, find, for $-3 \le x \le 3$, the set of values of x for which the value of $\mathrm{g}(x)$ is within 0.5 of the value of $\mathrm{f}(x)$.

7 2007/H1 A Level/I/5(part)

Solve the inequality $2x^2+3x+2 \geq 2x+3$.

Hence, using a sketch of the graph of $x=\cos\theta$, solve the inequality

$$2\cos^2\theta+3\cos\theta+2 \geq 2\cos\theta+3$$

for $0° \leq \theta \leq 540°$.

8 2018/SAJC Promo/Q2

(i) Show that x^2-2x+5 is always positive for all real values of x. [2]

(ii) Hence solve the inequality $\dfrac{4x^2-x+1}{x^2-2x+5} \leq 1$. [3]

(iii) Deduce the solution of the inequality $\dfrac{4x^2-|x|+1}{x^2-2|x|+5} \leq 1$. [2]

9 2018/YJC Promo/Q3

Without using a calculator, solve the inequality $\dfrac{2-x}{x+1} \geq 2x$. [4]

Hence find the exact range of values of x for which $\dfrac{2+\ln x}{1-\ln x} \geq -2\ln x$. [2]

10 2010/VJC Prelim/I/Q4

(i) Given that $x \geq -\dfrac{1}{2}$, using the substitution $y=\sqrt{2x+1}$, show algebraically that $22-8\sqrt{2x+1}+2x$ is always positive. [2]

(ii) Hence solve the inequality

$$\frac{22x-8x\sqrt{2x+1}+2x^2}{(x-k)(x-m)^2} \geq 0,$$

where k and m are real and $0 < k < m$. [4]

<u>Answers</u>

1	\$36.20; \$96.80; \$5.95	**6**	$\{x \in \mathbb{R} : -1.96 \leq x \leq 1.56\}$
2	$u_n = n^3-5n^2+61n+6$; 115556	**7**	$x \leq -1$ or $x \geq \frac{1}{2}$; $\theta = 180°$ or $\theta = 540°$ or $0° \leq \theta \leq 60°$ or $300° \leq \theta \leq 420°$
3	852	**8**	**(ii)** $-\frac{4}{3} \leq x \leq 1$ (iii) $-1 \leq x \leq 1$
4	$f(x) = x^3+2x^2-13x+10$	**9**	$x \leq -2$ or $-1 < x \leq \frac{1}{2}$; $x \geq e^2$ or $e^{-\frac{1}{2}} \leq x < e$
5	**(ii)** $-1.73 < x < 0.414$ or $x > 1.73$	**10**	**(ii)** $-\frac{1}{2} \leq x \leq 0$ or $x > k$, $x \neq m$