NAME: ____________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS   4049/01

Paper 1

Date: 21 August 2024   **Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142.

For Examiner's Use

Table of Penalties		**Question Number**		
Presentation	☐ 1 ☐ 2			
Rounding off	☐ 1		Parent's/Guardian's Signature	**90**

Setters: Mrs Lim CC and Ms Thio LH

This question paper consists of 22 printed pages

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

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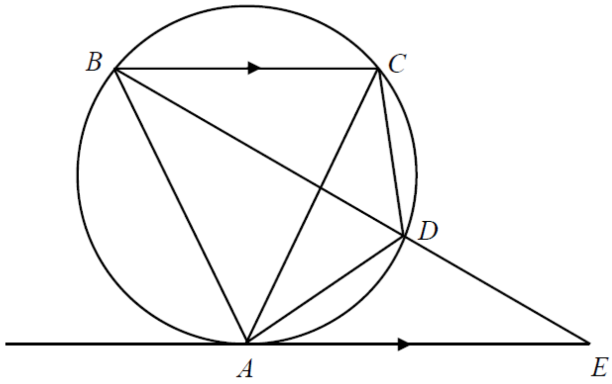
Answer **all** the questions.

1 **(i)** Express $4x^2+8x-5$ in the form $p(x+q)^2+r$, where p, q and r are constants to be found. [3]

(ii) Hence, state the coordinates of the turning point of the curve $y=4x^2+8x-5$. [1]

2 Without using a calculator, find the values of a and b for which the solution of the equation $x\sqrt{24} = x\sqrt{3} + \sqrt{6}$ is $\dfrac{a + \sqrt{b}}{7}$. [5]

3 Points A, B, C and D lie on a circle. The tangent to the circle at A meets BD produced at E. AE is parallel to BC.

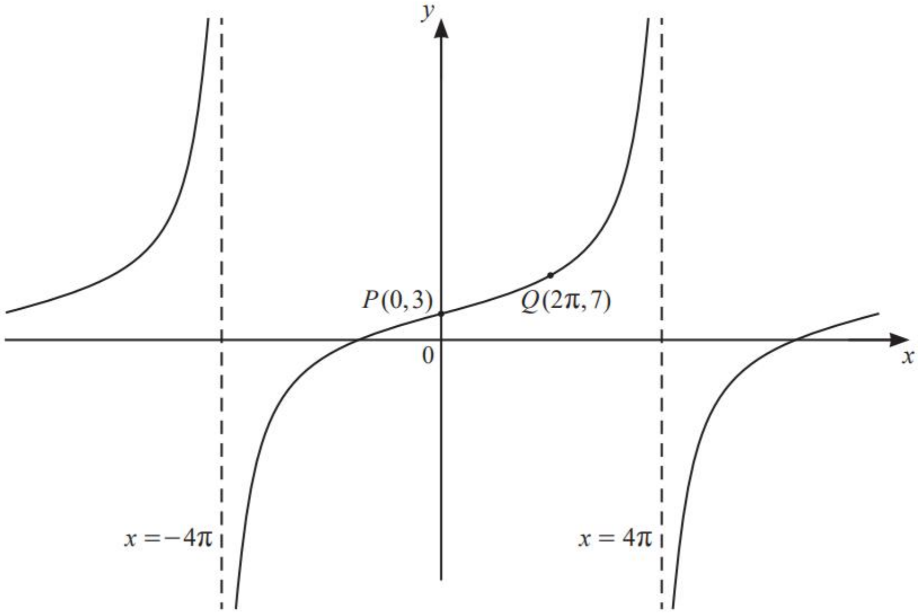


Prove that

(i) $AB = AC$, [3]

(ii) $\angle CDE = 2\angle ABC$. [3]

4 **(a)** The diagram shows part of the graph of $y = a\tan bx + c$. The graph has vertical asymptotes at $x = -4\pi$ and $x = 4\pi$ and passes through the points P and Q.

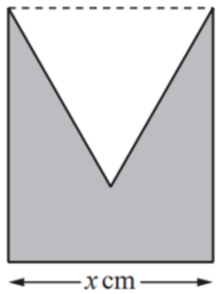


(i) Explain why $b = \dfrac{1}{8}$. [2]

(ii) Hence find the equation of the curve. [2]

(b) The function $f(x)$ is defined by $f(x) = 4 + 3\sin 2x$ for $0° \le x \le 360°$.
Sketch the graph of $y = f(x)$ on the axes below. [2]

5 The diagram shows a shape made by cutting an equilateral triangle out of a rectangle of width x cm.



The perimeter of the shape is 20 cm.

(i) Show that the area, A cm^2, of the shape is given by $A = 10x - \left(\dfrac{6+\sqrt{3}}{4}\right)x^2$. [3]

(ii) Given that x can vary, find the value of x which produces the maximum area and calculate this maximum area. Give your answers to 2 significant figures. [4]

6 **(a)** Express $\dfrac{8x+13}{(1+2x)(2+x)^2}$ in partial fractions. [5]

(b) Hence, evaluate $\int_1^2 \frac{8x+13}{(1+2x)(2+x)^2}\ \mathrm{d}x$. [3]

7 The polynomial $f(x)$ is such that $f(x) = 6x^3 + ax^2 - 50x + b$, where a and b are integers. It is given that $f(x)$ is divisible by $2x - 3$ and that $f'(1) = 6$.

(a) Find the values of a and b. [5]

(b) Using your values of a and b, solve the equation $\mathrm{f}(x) = 0$. [3]

8 **(a)** Show that the solution of the equation $2^{3x+4} \times 5^{2x-1} = 16^{x} \times 5^{3x}$ is $\lg\dfrac{16}{5}$. [3]

(b) Express $2\log_2 x - \log_2(x-4) = 3$ as a quadratic equation in *x* and explain why there are no real solutions. [5]

9 **(a)** The variables x and y increase in such a way that when $x = 3$, the rate of increase of y with respect to time is three times the rate of increase of x with respect to time. Given that $y = k\sqrt{3x+7}$, where k is a constant, find the value of k. [4]

(b) The mass, m grams, of a radioactive sample, present at time t days after being observed, is given by $m = 24e^{-0.02t}$.

Find

(i) the initial mass of the radioactive sample, [1]

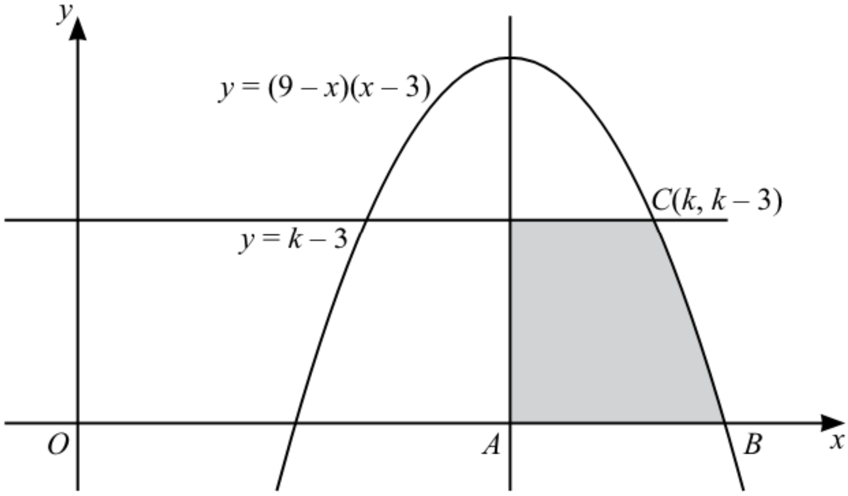
(ii) the time taken for the sample to decrease to half its initial mass, [2]

(iii) the rate at which the mass is decreasing after 12 hours. [2]

10 **(i)** Show that $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=\frac{2}{\sin x}$. [5]

(ii) Hence solve the equation $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=1+3\sin x$ for $0^\circ \leq x \leq 360^\circ$. [4]

11 The diagram shows part of the curve $y=(9-x)(x-3)$ and the line $y=k-3$, where $k>3$. The line through the maximum point of the curve, parallel to the y-axis, meets the x-axis at A. The curve meets the x-axis at B and the line $y=k-3$ meets the curve at the point $C(k,k-3)$.



(i) Show that the value of k is 8. [4]

(ii) Find the area of the shaded region. [5]

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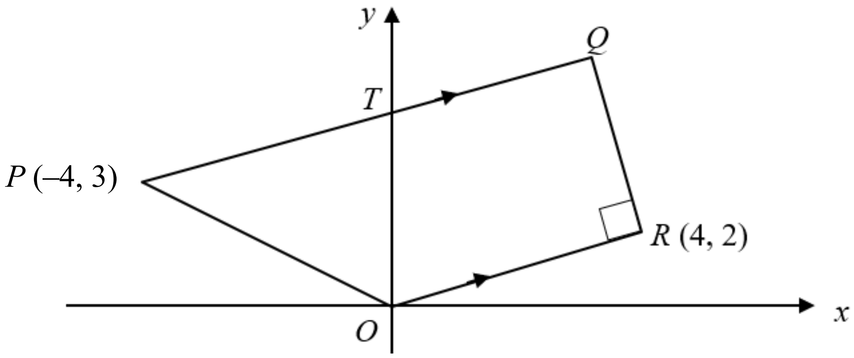
12 **Solutions to this question by accurate drawing will not be accepted.**

The diagram (not drawn to scale) shows a trapezium $OPQR$ in which PQ is parallel to OR and $\angle ORQ = 90°$. The coordinates of P and R are (–4, 3) and (4, 2) respectively and O is the origin.



(i) Find the coordinates of Q. [5]

(ii) PQ meets the y-axis at T. Show that triangle ORT is isosceles. [3]

The point S is such that $ORPS$ forms a parallelogram.

(iii) Find the coordinates of S. [1]

(iv) Find the area of the trapezium $OPQR$. [2]

~ End of Paper ~