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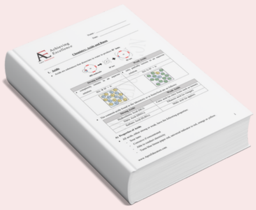

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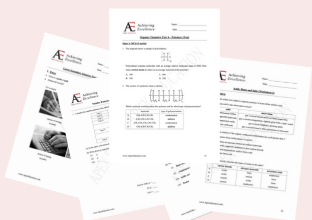

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Secondary 3 Additional Mathematics: Binomial Theorem

1. **Binomial Theorem** for a positive integral index:

   If n is a **positive integer,** the expansion of $(x+y)^n$ is given by the following formula:

   $(x+y)^n$

   $= x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + \binom{n}{n-1}xy^{n-1} + y^n$

   $= x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^{n-r}y^r + \cdots + nxy^{n-1} + y^n$

   (a) By replacing y with $-y$, we obtain the expansion of $(x-y)^n$

   $(x-y)^n$

   $= x^n + \binom{n}{1}x^{n-1}(-y) + \binom{n}{2}x^{n-2}(-y)^2 + \cdots + \binom{n}{r}x^{n-r}(-y)^r + \cdots$

   $+ \binom{n}{n-1}x(-y)^{n-1} + (-y)^n$

   (b) In a similar way, we get the following expansions

   $(1+x)^n = 1 + nx + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + \binom{n}{r}x^r + \cdots + nx^{n-1} + x^n$

   $(1-x)^n = 1 - nx + \binom{n}{2}x^2 - \binom{n}{3}x^3 + \cdots + \binom{n}{r}(-1)^r x^r + \cdots + n(-1)^{n-1}x^{n-1} + (-1)^n x^n$

2. In general, $\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{(1)(2)(3)\ldots(r)} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} = {}^nC_r$

   For example:

   (a) $\binom{n}{1} = \frac{n}{1}$ or $= {}^nC_1 = n$

   (b) $\binom{n}{2} = \frac{n(n-1)}{(1)(2)}$ or $\frac{n(n-1)}{2!}$ or $= {}^nC_2$

   (c) $\binom{n}{3} = \frac{n(n-1)(n-2)}{(1)(2)(3)}$ or $\frac{n(n-1)(n-2)}{3!}$ or $= {}^nC_3$

3. In the expansion of $(x+y)^n$

   (a) there are $n + 1$ terms,

   (b) the powers of x are in descending order while the powers of y are in ascending order.

   (c) the powers of x and y add up to n.

   (d) the $(r+1)^{\text{th}}$ term is $\binom{n}{r}x^{n-r}y^r$

Example:

(a) $(2+5x)^4$

Number of terms $= n+1 = 4+1 = 5$

$$(2+5x)^4 = 2^4 + \binom{4}{1}(2)^3(5x) + \binom{4}{2}(2)^2(5x)^2 + \binom{4}{3}(2)(5x)^3 + (5x)^4$$

$$= 16 + 4(8)(5x) + (6)(4)(25x^2) + (4)(2)(125x^3) + 625x^4$$

$$= \mathbf{16 + 160x + 600x^2 + 1000x^3 + 625x^4}$$

OR

$$(2+5x)^4 = 2^4 + {}^4C_1(2)^3(5x) + {}^4C_2(2)^2(5x)^2 + {}^4C_3(2)(5x)^3 + (5x)^4$$

$$= 16 + 4(8)(5x) + (6)(4)(25x^2) + (4)(2)(125x^3) + 625x^4$$

$$= \mathbf{16 + 160x + 600x^2 + 1000x^3 + 625x^4}$$

(b) $\left(2-\frac{1}{x}\right)^3$

Number of terms $= n+1 = 3+1 = 4$

$$\left(2-\frac{1}{x}\right)^3 = \left[2+\left(-\frac{1}{x}\right)\right]^3$$

$$= 2^3 + \binom{3}{1}(2)^2(-\tfrac{1}{x}) + \binom{3}{2}(2)^1\left(-\frac{1}{x}\right)^2 + \left(-\frac{1}{x}\right)^3$$

$$= 8 + (3)(4)(-\tfrac{1}{x}) + (3)(2)\left(\frac{1}{x^2}\right) + \left(-\frac{1}{x^3}\right)$$

$$= 8 - \frac{12}{x} + \frac{6}{x^2} - \frac{1}{x^3}$$

OR

$$\left(2-\frac{1}{x}\right)^3 = \left[2+\left(-\frac{1}{x}\right)\right]^3$$

$$= 2^3 + {}^3C_1(2)^2(-\tfrac{1}{x}) + {}^3C_2(2)^1\left(-\frac{1}{x}\right)^2 + \left(-\frac{1}{x}\right)^3$$

$$= 8 + (3)(4)(-\tfrac{1}{x}) + (3)(2)\left(\frac{1}{x^2}\right) + \left(-\frac{1}{x^3}\right)$$

$$= 8 - \frac{12}{x} + \frac{6}{x^2} - \frac{1}{x^3}$$

4. The general formula for the $(r+1)^{\text{th}}$ term of the expansion of $(x+y)^n$ is given by the following formula:

$$\boldsymbol{T_{r+1} = \binom{n}{r} x^{n-r} y^r}$$

5. This formula helps us find specific terms (for example, the term in x^5) in a binomial expansion without having to do the entire expansion.

6. In particular, we can also find the term **independent of x,** i.e. the constant term, in the expansion. Equate the power of x in the simplified expression to 0 and solve for r.

Example

In the expansion of $\left(x^2 + \frac{2}{x}\right)^6$, find

(a) the coefficient of x^3,

(b) the constant term.

The $(r+1)^{\text{th}}$ term, $T_{r+1} = \binom{6}{r} (x^2)^{6-r} \left(\frac{2}{x}\right)^r$ or $= {}^6C_r (x^2)^{6-r} \left(\frac{2}{x}\right)^r$

$= \binom{6}{r} x^{12-2r} \cdot 2^r \cdot x^{-r}$ or $= {}^6C_r x^{12-2r} \cdot 2^r \cdot x^{-r}$

$= \binom{6}{r} x^{12-3r} \cdot 2^r$ or $= {}^6C_r x^{12-3r} \cdot 2^r$

(a) To find the coefficient of x^3, let $12 - 3r = 3$

$3r = 9$

$r = 3$

Therefore, the coefficient of $x^3 = \binom{6}{3} 2^3$ or $= {}^6C_3 2^3$

$= 160$

(b) To find the constant term, let $12 - 3r = 0$

$3r = 12$

$r = 4$

Therefore, the constant term $= \binom{6}{4} 2^4$ or $= {}^6C_4 2^4$

$= 240$

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Binomial Theorem – Practice Questions 1

1. Find the **first four terms**, in ascending powers of x, each of the following expansions.

   (a) $(1 + x^2)^{10}$

   (b) $\left(1 + \frac{x}{2}\right)^7$

7. (a) In the expansion of $\left(2 - \frac{x}{3}\right)^n$, show that the ratio of the coefficient of the 2nd term to that of the 4th term can be simplified to the expansion $\frac{216}{(n-1)(n-2)}$.

(b) Find the value of n if the ratio in (a) is 108 : 55.

(c) Hence, find the term x^5.

8. (a) Given that the constant term in the binomial expansion of $\left(x + \frac{k}{x}\right)^6$ is -160, find the value of the constant k.

(b) Using the value of k found in part (a), show that there is no constant term in the expansion of $\left(x + \frac{k}{x}\right)^6 (2x^2 + 3)$.

9. The coefficient of $\frac{1}{x^3}$ is 512 in the expansion of $\left(\frac{2}{x} + px^2\right)^9$, where $p < 0$.

(a) By working out the general term of $\left(\frac{2}{x} + px^2\right)^9$, find the value of p.

(b) Using the results in (a),

(i) show that the coefficient of the first term in the expansion of $\left(\frac{2}{x} + px^2\right)^9$ is also 512.

(ii) find the $\frac{1}{x^6}$ term in the expansion of $\left(\frac{2}{x} + px^2\right)^9$.

(c) Explain why the term $\frac{1}{x^4}$ does not exist in the expansion of $\left(\frac{2}{x} + px^2\right)^9 \left(\frac{1}{8x} + \frac{1}{12}x^2\right)$.

10. (a) By considering the general term in the binomial expansion of $\left(x^3 - \frac{2}{x}\right)^7$, explain why there are only odd powers of x in this expansion.

(b) Find the term independent of x in the expansion of $\left(x^3 - \frac{2}{x}\right)^7 \left(\frac{5}{x} - 2x^2\right)$.