Candidate Name : _____   CT Group : _____

                                                            Index no :   _____

PIONEER JUNIOR COLLEGE
JC 2 Preliminary Examination

MATHEMATICS
Higher 2 Paper 2   **(9740 / 2)**

Wednesday   21 Sept 2011

Additional material: Answer paper, List of Formulae MF15

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your full name, index number and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
Attach this question paper with your answers, and arrange your answers in numerical order.

For Examiner's Use					
Qn	**Marks**	**Qn**	**Marks**	**Qn**	**Marks**
1		7		13	
2		8		14	
3		9			
4		10			
5		11			
6		12			
Sub-total		Sub-total		Total	

_____
This question paper consists of 6 printed pages and 2 blank pages.

[Turn over]

[Turn Over]

Section A : Pure Mathematics [40 Marks]

1   The first four terms of a sequence are given by $T_1 = 1$, $T_2 = 2$, $T_3 = 4$, $T_4 = 8$. Given that T_r is a cubic polynomial in r, find T_r in terms of r.   [4]

2   A sequence of positive real numbers x_1, x_2, x_3, ... satisfies the recurrence relation $x_{n+1} = \sqrt{3 + x_n}$ for $n \geq 1$.
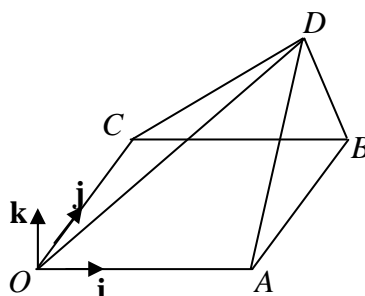
   (i)   If this sequence converges to α, determine the exact value of α.   [2]

   (ii)   By using a graphical method, prove that $x_{n+1} > x_n$ if $0 < x < \alpha$.   [2]

3   Without using a graphic calculator and showing your working clearly, solve the inequality $\dfrac{2x-1}{x+4} \leq 1$.   [3]

   Hence, solve the inequality $\dfrac{2 - e^x}{1 + 4e^x} \leq 1$, giving your answer in exact form.   [3]

4

   The diagram shows a pyramid with a square base $OABC$ of side 4 cm. The vertex D is 6 cm vertically above M, the mid-point of AB. Taking O as the origin and unit vectors **i**, **j** and **k** as indicated, find

   (i)   the position vector of L, the mid-point of CD,   [2]

   (ii)   the area of triangle OAL giving your answer in exact form,   [2]

   (iii)   the angle between the lines ML and OD.   [3]

5 **(i)** Find $\dfrac{d}{dx}\left(x\sqrt{4-x^2}+4\sin^{-1}\left(\dfrac{x}{2}\right)\right)$, giving your answer in its simplest form. [3]

(ii) Hence, or otherwise, show that $\dfrac{1}{2}\displaystyle\int_0^k \sqrt{4-x^2}\,dx = a\sqrt{4-k^2}+\sin^{-1}\left(\dfrac{k}{2}\right)$ where a is to be found in terms of k, $0 < k < 2$. [2]

(iii) Sketch the curve C with equation $4y^2 + x^2 = 4$ and show by shading, a region whose area is given by the integral in **(ii)**. [2]

(iv) Hence, find the area of the region lying inside C and between the lines $x = -1$ and $x = 1$, giving your answer in exact form. [3]

6 **(i)** Illustrate, on an Argand diagram, the locus of a point Q representing the complex number z, where $\left|\dfrac{z-3}{z+1}\right| \geq 1$. [2]

(ii) Illustrate, using the same Argand diagram, the locus of a point P representing the complex number z, where $-\dfrac{\pi}{6} \leq \arg\left(\dfrac{\sqrt{3}+i}{z}\right) \leq \dfrac{\pi}{6}$. [3]

(iii) Indicate, clearly, in your diagram the locus of z such that $-\dfrac{\pi}{6} \leq \arg\left(\dfrac{\sqrt{3}+i}{z}\right) \leq \dfrac{\pi}{6}$ and $\left|\dfrac{z-3}{z+1}\right| \geq 1$. [1]

(iv) Hence find the range of $\arg\left(z - 2\sqrt{3}i\right)$, giving your answer in exact form. [3]

Section B : Statistics [60 Marks]

7 A university faculty consists of 90 members of whom 9 are professors, 18 are senior lecturers and the rest are lecturers.

A random sample of 10 members is required. Explain how this may be obtained as

(i) a systematic random sample, [2]

(ii) a stratified random sample. [2]

State one advantage of the sampling method used in **(ii)**. [1]

8 A laboratory technician conducted an experiment and obtained the following observations of the variables x and y:

x	5	6	7	8	9	10	11	12	13	14
y	8	9	9	10	12	16	18	20	21	24

(i) Draw a scatter diagram to illustrate the data and find the product moment correlation coefficient for the sample. [3]

(ii) Comment on whether a linear model is appropriate. [1]

(iii) Fit a model of the form $\ln y = a + bx$ to the data, stating clearly the values of a and b. [1]

(iv) It is given that x represents a certain chemical (in mg) added to a solution, and y represents the reaction time (in minutes) taken. Use an appropriate regression line to give the best estimate of the amount of chemical added when the reaction time is 15 minutes. Explain your choice of regression line for this estimate. [2]

9 In a group of 100 students, 25 own an IPod, 40 own an IPhone, and 35 own either an IPod or an IPhone, but not both.

Find the probability that a student chosen at random

(i) owns both an IPod and an IPhone, [2]

(ii) does not own an IPod or an IPhone, [2]

(iii) owns an IPod, given that he owns an IPhone. [2]

10 Joshua tries to recall the 6-digit pin number for his ATM card.

(Assuming that the number 000 000 is even and valid.)

How many possible numbers can there be if he remembers that:

(i) the number starts and ends with the digit 5? [1]

(ii) the number is odd and the digits do not repeat? [2]

(iii) the digits do not repeat and there are exactly 3 odd digits? [2]

11 It is known that 1% of the pens produced by a company are defective. The company sells the pens in boxes of 10. A box can be rejected by customers if it contains more than 1 defective pen.

    (i) What is the probability that a randomly chosen box of pens is rejected? [2]

    (ii) A junior college bought 1000 boxes of pens. Using a suitable approximation, find the probability that at least 990 boxes are accepted. [3]

    (iii) A polytechnic bought 2000 boxes of pens. Estimate the probability that at most 12 boxes are rejected. [3]

12 Traffic police measured the speeds of vehicles travelling along a particular stretch of expressway to monitor the accident rate along the expressway.

For a sample of 24 vehicles, the speeds, x km/h, were summarised by

$$\Sigma(x - 30) = 1266 \qquad \text{and} \qquad \Sigma(x - 30)^2 = 78060.5$$

    (i) Calculate unbiased estimates for the population mean and variance. [2]

    (ii) Test, at the 3% level of significance, the hypothesis that the mean speed is 80 km/h against the alternative that it is greater than this. [4]

The supervisor of the traffic police found that there was an error in recording the results - the last digit 0 was missing in the record. So the correct sample size was 240, with $\Sigma(x - 30) = 12660$ and $\Sigma(x - 30)^2 = 780605$. Is there any change in the conclusion in **(ii)** ? [4]

13 Tai Ming travels to work by one of two possible routes. If he travels by route A, he takes bus '10' to reach the workplace directly. If he travels by route B, he takes bus '20' and then he walks the rest of the way. The time taken by bus and the time taken to walk are independent of each other.

The travelling times are all normally distributed with parameters (in minutes) as shown in the tables below.

ROUTE A	Mean	Standard Deviation
Time taken by bus '10'	14	2.5

ROUTE B	Mean	Standard Deviation
Time taken by bus '20'	6	1.7
Time taken to walk	6	2.1

(i) Find the probability that Tai Ming takes more than 12 minutes to travel by route *A*. [2]

(ii) Find the probability that the difference in time taken between the two routes is at least 3 minutes. [4]

Tai Ming travels by route A on three days and route B on two days. Find the probability that, for the five days, the total time taken for Tai Ming to travel to work is less than 70 minutes. [2]

14 The number of fish caught per hour by a particular angler has a Poisson distribution with mean 1.5. Find the probability that he catches exactly 3 fish in a particular hour. [2]

The angler's son catches 1 fish per hour on average. Assuming that the son's catches per hour also has Poisson distribution and that both of them fish at the same time, but sufficiently far apart for their results to be considered independent, find the probabilities that

(i) in a given hour, they catch a total of at least 2 fish. [2]

(ii) in a 2-hour period, the father catches more fish than his son given that they catch a total of 4 fish. [3]

Find the length of time, to the nearest minute, for which the probability that they catch at least one fish is more than 0.9. [2]

50 one-hour periods are chosen at random, and the number of fish caught by the angler in each of the periods is recorded. Find the probability that the mean number of fish caught by the angler per hour is more than 2. [2]