**AM Paper 2 2018**

**(1i)** $x^2+2=k\left(x-\frac{1}{2}\right)$

$x^2+2=kx-\frac{k}{2}$

$x^2-kx+2+\frac{k}{2}=0$

$b^2-4ac>0$

$k^2-4(1)\left(2+\frac{k}{2}\right)>0$

$k^2-8-2k>0$

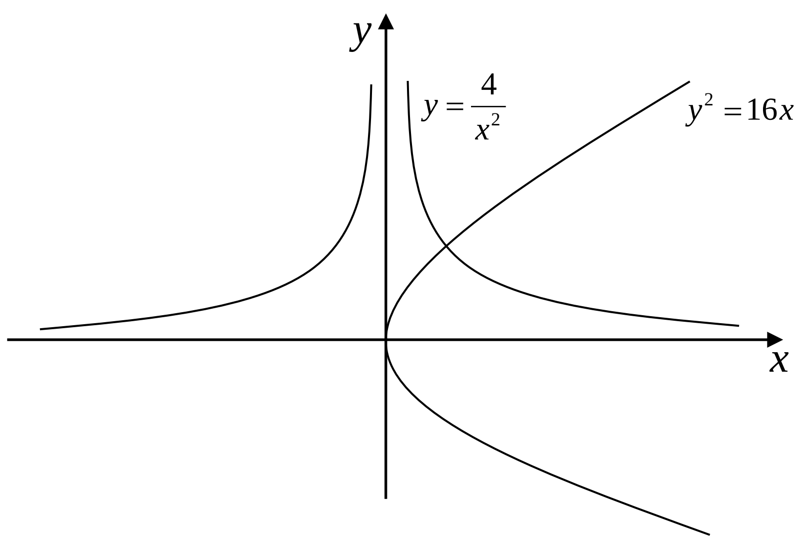
$k^2-2k-8>0$

$(k-4)(k+2)>0$

$k<-2 \;\; or \;\; k>4$

**(ii)** Since $k$ > 4 here, the roots are real and distinct.

**(2)**




$y^2=16x$ ---------------(1)

$y=\frac{4}{x^2}$ ----------------(2)

$\left(\frac{4}{x^2}\right)^2=16x$

$$\frac{16}{x^4} = 16x$$
$$x^5 = 1$$
$$x = 1$$
$$y = \frac{4}{1^2} = 4$$

The point of intersection is (1, 4) **[A1]**

**(3i)** When $t$ = 0, $M$ = 30

$$30 = k - 6\ln(0+1)$$

$$k = 30$$

**(3ii)** $15 = 30 - 6\ln(t+1)$

$$6\ln(t+1) = 30 - 15$$

$$t + 1 = e^{\frac{15}{6}}$$
11.18

**4**

$x^2 + y^2 = 37300$ - - - - (1)

$3x + 3y + (x - y) = 860$

$4x + 2y = 860$

$y = 430 - 2x$ - - - - (2)

(2) in (1) : $x^2 + (430 - 2x)^2 = 37300$

$$5x^2 - 1720x + 147600 = 0$$

$$x = \frac{1720 \pm \sqrt{(-1720)^2 - 4(4)(147600)}}{2(4)}$$

$$x = 180 \;\; or \;\; 164$$

$$y = 70 \;\; or \;\; 102$$

The tables are of lengths 180 cm and 70 cm, respectively or 164 cm and 102 cm.

**(5a)** $\log_3 x + 2 - 3\log_x 3 = 0$

$$\log_3 x + 2 - 3\left(\frac{1}{\log_3 x}\right) = 0$$

Let $y = \log_3 x$

$$y + 2 - \frac{3}{y} = 0$$

$$y^2 + 2y - 3 = 0$$

$$(y+3)(y-1) = 0$$

$$y = -3 \ \ or \ \ y = 1$$

$$\log_3 x = -3 \ \ or \ \ \log_3 x = 1$$

$$x = \frac{1}{27} \ \ or \ \ x = 3$$

**(5b)** $4^{2x} - 4^{x+\frac{1}{2}} = 4^{x+2} - 17$

$$4^x - 4^x\left(4^{\frac{1}{2}}\right) = 4^x\left(4^2\right) - 17$$

$$4^{2x} - 2\left(4^x\right) = 16\left(4^x\right) - 17$$

Let $y = 4^x$

$$y^2 - 18y + 17 = 0$$

$$(y-1)(y-17) = 0$$

$4^x = 1$ or $4^x = 17$

$x = 0$ $\quad$ $x = \frac{\lg 17}{\lg 4}$

$= 2.04$

**6** **(a)** If $\left(\sqrt{a} - \sqrt{3}\right)^2 = 5 - 4\sqrt{b}$, find the value of $a$ and of $b$. [4]

$$a - 2\sqrt{3a} + 3 = 5 - 4\sqrt{b}$$

$$a + 3 - 2\sqrt{3a} = 5 - 4\sqrt{b}$$

$$a + 3 = 5$$

$$a = 2$$

$$2\sqrt{3a} = 4\sqrt{b}$$

$$\sqrt{12a} = \sqrt{16b}$$

$$12a = 16b$$

$$b = \frac{3}{2}$$

**(6b)** A cuboid of volume $\left(18 + 11\sqrt{2}\right) cm^3$ stands on a square base of side $\left(1 + \sqrt{2}\right) cm$. Find the height of the cuboid. [4]

$$\text{Height of cuboid} = \frac{18+11\sqrt{2}}{\left(1+\sqrt{2}\right)^2}$$
$$= \frac{18+11\sqrt{2}}{3+2\sqrt{2}}$$
$$= \frac{18+11\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$
$$= \left(10-3\sqrt{2}\right)cm$$

**(7ai)** $\cos A = \frac{1}{\sqrt{5}}$

Using Pythagoras Theorem, opp = $2$

$$\sin A = \frac{2}{\sqrt{5}}$$
$$= \frac{2\sqrt{5}}{5}$$

**(aii)** $$\frac{2\cos\frac{\pi}{6}}{\tan\left(90^o - A\right)} = \frac{2\left(\frac{1}{2}\right)}{\frac{1}{\tan A}}$$
$$= 2$$

**(7bi)** $$\frac{\cos x}{1-\sin x} - \frac{1}{\cos x} = \tan x$$
$$LHS = \frac{\cos x}{1-\sin x} - \frac{1}{\cos x}$$
$$= \frac{\cos^2 x - (1-\sin x)}{\cos x(1-\sin x)}$$
$$= \frac{1-\sin^2 x - 1 + \sin x}{\cos x(1-\sin x)}$$
$$= \frac{\sin x - \sin^2 x}{\cos x(1-\sin x)}$$
$$= \frac{\sin x(1-\sin x)}{\cos x(1-\sin x)}$$
$$= \tan x$$

**(7bii)** $\dfrac{\cos x}{1-\sin x} = \dfrac{1}{\cos x} - 5$

$\dfrac{\cos x}{1-\sin x} - \dfrac{1}{\cos x} = -5$

$\tan x = -5$

Reference $\angle x = 78.7^o$

$x = 101.3^o \; and \; 281.3^o$

**(8i)** Equation of *AD* is $y = \left(\dfrac{12-6}{15}\right)x + 6$

$$y = \frac{2}{5}x + 6 \text{ ------------------------(1)}$$

$$y = -\frac{8}{5}x + 16 \text{ ----------------------(2)}$$

Solving simultaneously,

$$\frac{2}{5}x + 6 = -\frac{8}{5}x + 16$$

$$x = 5, y = 8$$

$$B(5,8)$$

**(8ii)** when $y = 0$, $E(10,0)$.

When $x = 10$, $y = \dfrac{2}{5}(10) + 6 = 10$

$C(10,10)$

**(8iii)** Area of $\Delta ABE$ = $\dfrac{1}{2}\begin{vmatrix} 0 & 10 & 5 & 0 \\ 6 & 0 & 8 & 6 \end{vmatrix}$

= 25 units$^2$

$BE = \sqrt{6^2 + 10^2} = \sqrt{136}$ units

Shortest distance = $\dfrac{2 \times 25}{\sqrt{136}}$

= 4.29 units

**(8iv)** Gradient of ⊥ bisector = $-\frac{5}{4}$

Mid-point of *OD* = $\left(\frac{0+15}{2}, \frac{0+12}{2}\right)$

$= \left(7\frac{1}{2}, 6\right)$

Equation of ⊥ bisector is $y-6=-\frac{5}{4}\left(x-7\frac{1}{2}\right)$

$$y=-\frac{5}{4}x+\frac{123}{8}$$

When $y=0,\ x=12.3$. Hence, it does not pass through *E*.

--------------------------------------------------------The End --------------------------------------------------------