2021 DHS-EJC H3 Math Prelim Solutions

Qn	Solution
1(i)	For $n=1$: $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{f}(x)\mathrm{g}(x)\right)=\mathrm{f}'(x)\mathrm{g}(x)+\mathrm{f}(x)\mathrm{g}'(x)$ For $n=2$: $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\mathrm{f}(x)\mathrm{g}(x)\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{f}(x)\mathrm{g}(x)\right)\right)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{f}'(x)\mathrm{g}(x)+\mathrm{f}(x)\mathrm{g}'(x)\right)$ $=\mathrm{f}''(x)\mathrm{g}(x)+\mathrm{f}'(x)\mathrm{g}'(x)$ $+\mathrm{f}'(x)\mathrm{g}'(x)+\mathrm{f}(x)\mathrm{g}''(x)$ $=\mathrm{f}''(x)\mathrm{g}(x)+2\mathrm{f}'(x)\mathrm{g}'(x)+\mathrm{f}(x)\mathrm{g}''(x)$
(ii) (a)	Let $\mathrm{f}(x)=\sin^{-1}x$ and $\mathrm{g}(x)=y$, then $\mathrm{f}'(x)=\frac{1}{\sqrt{1-x^2}}$ and $\mathrm{g}'(x)=\frac{\mathrm{d}y}{\mathrm{d}x}$. $\left(\sin^{-1}x\right)\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{y}{\sqrt{1-x^2}}=\mathrm{e}^x$ $\frac{\mathrm{d}}{\mathrm{d}x}\left(y\sin^{-1}x\right)=\mathrm{e}^x \Rightarrow y\sin^{-1}x=\mathrm{e}^x+c$ Given $y=1$ when $x=1$, $(1)\left(\sin^{-1}(1)\right)=\mathrm{e}^1+c \Rightarrow c=\sin^{-1}(1)-\mathrm{e}=\frac{\pi}{2}-\mathrm{e}$ $y=\frac{\mathrm{e}^x+\frac{\pi}{2}-\mathrm{e}}{\sin^{-1}x}$
(ii) (b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-(2\tan x)\frac{\mathrm{d}y}{\mathrm{d}x}-y=x$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\left(\frac{\sin x}{\cos x}\right)\frac{\mathrm{d}y}{\mathrm{d}x}-y=x$ $(\cos x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2(-\sin x)\frac{\mathrm{d}y}{\mathrm{d}x}+(-\cos x)y=x\cos x$ $\frac{\mathrm{d}^2}{\mathrm{d}x^2}(y\cos x)=x\cos x$ $\frac{\mathrm{d}}{\mathrm{d}x}(y\cos x)=\int x\cos x\ \mathrm{d}x=x\sin x-\int \sin x\ \mathrm{d}x$ $=x\sin x+\cos x+c$ $y\cos x=\int x\sin x+\cos x+c\ \mathrm{d}x$ $=\left(-x\cos x+\int\cos x\ \mathrm{d}x\right)+\sin x+cx$ $=\ -x\cos x+2\sin x+cx+d$ $y=\frac{-x\cos x+2\sin x+cx+d}{\cos x}=-x+2\tan x+\frac{cx+d}{\cos x}$ where c and d are arbitrary constants.

Qn	Solution
2(i)	$T_{k+1} = T_k + T_1 + k = T_k + k + 1$ $T_{k+1} - T_k = k + 1$ for $k \geq 1$ $T_n - T_1 = \sum_{k=1}^{n-1}(T_{k+1} - T_k)$ $T_n - 1 = \sum_{k=1}^{n-1}(k+1)$ $T_n = 1 + \sum_{k=1}^{n-1}(k+1) = 1 + 2 + ... + n = \frac{n(n+1)}{2}$ (AP) (S.R.) Mathematical Induction Proof is possible as well
(ii)	$T_{ab} = T_a T_b + T_{a-1} T_{b-1}$ $T_a T_b + T_{a-1} T_{b-1}$ $= \frac{a(a+1)}{2}\frac{b(b+1)}{2} + \frac{a(a-1)}{2}\frac{b(b-1)}{2}$ $= \frac{ab(a+1)(b+1) + ab(a-1)(b-1)}{4}$ $= \frac{ab}{4}(ab + a + b + 1 + ab - a - b + 1)$ $= \frac{ab}{4}(2ab + 2) = \frac{ab(ab+1)}{2} = T_{ab}$ (verified)
(iii)	3 possible cases for n: If n is a multiple of 3, $T_n = \frac{3k(3k+1)}{2}$ is a multiple of 3 since k and $3k+1$ are of opposite parities. If $n = 3k+1$, $T_n = \frac{(3k+1)(3k+2)}{2} = \frac{9k(k+1)}{2} + 1 \equiv 1 \pmod 9$ since k and $k+1$ are of opposite parities. If $n = 3k+2$, $T_n = \frac{(3k+2)(3k+3)}{2} = \frac{3(k+1)(3k+2)}{2}$ is a multiple of 3 since $k+1$ and $3k+2$ are of opposite parities. $\therefore$ $T_n = 0$ or $1 \pmod 3$ for all positive integers n. (Alternative) $T_1 = 1$, $T_2 = T_1 + T_2 + 1(1) = 3$, $T_3 = T_1 + T_2 + 1(2) = 1 + 3 + 2 = 6$. 3 possible cases for n: If n is a multiple of 3, $T_n = T_{3k}$ $= T_3 T_k + T_2 T_{k-1}$ (using (ii)) $= 6T_k + 3T_{k-1} = 3(2T_k + T_{k-1}) \equiv 0$ (mod 3) (continued next page)

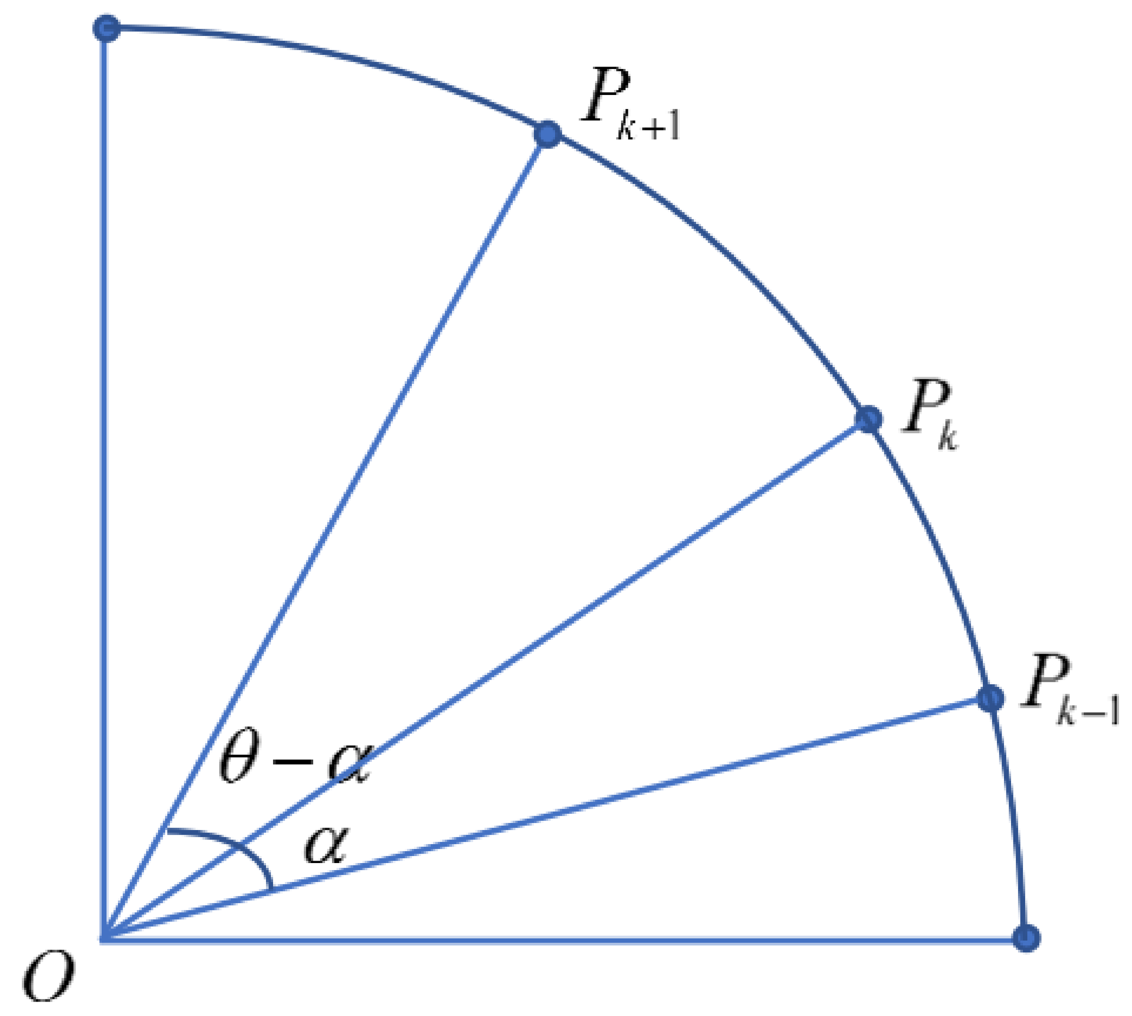
Qn	Solution
2(iii)	(continued) If $n = 3k+1$, $T_{3k+1} = T_{3k} + T_1 + 3k$ $\left(\text{using given recurrence relation}\right)$ $\equiv 0+1+0 \pmod 3$ $\left(\text{using results derived for } T_{3k} \text{ and } T_1 = 1\right)$ $\equiv 1 \pmod 3$ If $n = 3k+2$, $T_{3k+2} = T_{3k} + T_2 + (3k)(2)$ $\left(\text{using given recurrence relation}\right)$ $\equiv 0+3+0 \pmod 3$ $\left(\text{using results derived for } T_{3k} \text{ and } T_2 = 3\right)$ $\equiv 0 \pmod 3$
(iv)	$8T_n + 1 = 8\left(\frac{n(n+1)}{2}\right) + 1 = 4n^2 + 4n + 1 = (2n+1)^2$ which is a perfect square.
(v)	$(4T_n)^2 + (8T_n + 1) = 16T_n^{\ 2} + 8T_n + 1 = (4T_n + 1)^2$ Hence, $\left(\sqrt{8T_n+1},\ 4T_n,\ 4T_n+1\right)$ is the Pythagorean Triple required.
3(i)	When $p > 1$, $\int_1^\infty \frac{1}{n^p}\,\mathrm{d}n = \left[\frac{n^{1-p}}{1-p}\right]_1^\infty = \left[0 - \frac{1^{1-p}}{1-p}\right] = \frac{1}{p-1}$ is finite. Hence $\sum_{n=1}^\infty \frac{1}{n^p}$ converges for $p > 1$. When $p = 1$, $\int_1^\infty \frac{1}{n}\,\mathrm{d}n = [\ln n]_1^\infty$ is not finite. Hence $\sum_{n=1}^\infty \frac{1}{n^p}$ diverges for $p = 1$. When $0 \le p < 1$, $\int_1^\infty \frac{1}{n^p}\,\mathrm{d}n = \left[\frac{n^{1-p}}{1-p}\right]_1^\infty$ is not finite, since $1-p > 0$ and thus $\frac{n^{1-p}}{1-p} \to \infty$ as $n \to \infty$. Hence $\sum_{n=1}^\infty \frac{1}{n^p}$ diverges for $0 \le p < 1$. When $p < 0$, the function f is not monotonically increasing and hence the integral test cannot be applied. However, since $-p > 0$, $\sum_{n=1}^\infty \frac{1}{n^p} = \sum_{n=1}^\infty n^{-p} \ge \sum_{n=1}^\infty 1^{-p}$ and hence $\sum_{n=1}^\infty \frac{1}{n^p}$ diverges for $p < 0$. Combining all results, we have $\sum_{n=1}^\infty \frac{1}{n^p}$ converges for $p > 1$ and diverges otherwise.

Qn	Solution
3(ii)	$0 < \dfrac{n}{4n^4+1} \le \dfrac{n}{4n^4} = \dfrac{1}{4n^3}$ for $n \ge 1$. Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^3} = \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^3}$ converges (with $p = 3$), $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4n^4+1}$ converges by comparison test.
(iii)	$\displaystyle\int_1^{\infty} \frac{n}{4n^4+1}\,\mathrm{d}n = \frac{1}{4}\int_1^{\infty} \frac{4n}{\left(2n^2\right)^2+1}\,\mathrm{d}n$ $\displaystyle= \left[\frac{1}{4}\tan^{-1}(2n)\right]_1^{\infty} = \frac{\pi}{8} - \frac{1}{4}\tan^{-1}(2)$ which is finite. Hence $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4n^4+1}$ converges by integral test. (Usage of GC for approximate answer is accepted)
(iv)	$\left(2n^2+2n+1\right)\left(2n^2-2n+1\right)$ $=\left(2n^2+1+2n\right)\left(2n^2+1-2n\right)$ $=\left(2n^2+1\right)^2-(2n)^2$ $=4n^4+4n^2+1-4n^2$ $=4n^4+1$ (verified) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4n^4+1} = \lim_{k\to\infty}\sum_{n=1}^{k}\frac{n}{4n^4+1}$ $\displaystyle= \lim_{k\to\infty}\sum_{n=1}^{k}\frac{n}{\left(2n^2+2n+1\right)\left(2n^2-2n+1\right)}$ $\displaystyle= \lim_{k\to\infty}\frac{1}{4}\sum_{n=1}^{k}\left(\frac{1}{n^2+(n-1)^2} - \frac{1}{n^2+(n+1)^2}\right)$ $\displaystyle= \lim_{k\to\infty}\frac{1}{4}\left(1-\frac{1}{k^2+(k+1)^2}\right)$ (using MOD) $=\dfrac{1}{4}$
4(i)	Let $x=0,\ y=0$, $\mathrm{f}(0+0)=\mathrm{f}(0)+\mathrm{f}(0)$ $\mathrm{f}(0)=2\mathrm{f}(0)$ $\therefore \mathrm{f}(0)=0$
(ii)	Consider for any positive integer n, $\mathrm{f}(n)=\mathrm{f}(1+1+\cdots+1)$ $=\mathrm{f}(1)+\mathrm{f}(1)+\cdots+\mathrm{f}(1)=n\mathrm{f}(1)=n$

Qn	Solution
4(iii)	Let the positive rational number be $\frac{p}{q}$ with both p and q positive integers. Since p and q are real, $p=\mathrm{f}(p)=\mathrm{f}\left(q\left(\frac{p}{q}\right)\right)$ $=\mathrm{f}\left(\underbrace{\frac{p}{q}+\ldots+\frac{p}{q}}_{q\text{ of these terms}}\right)=\underbrace{\mathrm{f}\left(\frac{p}{q}\right)+\ldots+\mathrm{f}\left(\frac{p}{q}\right)}_{q\text{ of these terms}}=q\mathrm{f}\left(\frac{p}{q}\right)$ Dividing throughout by q, we obtain $\mathrm{f}\left(\frac{p}{q}\right)=\frac{p}{q}$ (shown)
(iv)	$\lim_{h\to0}\mathrm{f}(h)=\left(\lim_{h\to0}h\right)\left(\lim_{h\to0}\frac{\mathrm{f}(h)}{h}\right)=(0)(1)=0$ $\therefore\lim_{h\to0}\mathrm{f}(h)=0$ Let $h=x-k$, $\lim_{x\to k}\mathrm{f}(x)=\lim_{h\to0}\mathrm{f}(h+k)$ $=\lim_{h\to0}\left[\mathrm{f}(k)+\mathrm{f}(h)\right]=\mathrm{f}(k)+\lim_{h\to0}\mathrm{f}(h)=\mathrm{f}(k)$ Since $\lim_{x\to k}\mathrm{f}(x)=\mathrm{f}(k)$, f is a continuous function.
5(i) (a)	There are three types of flowers, R, S, T. Twelve flowers are to be selected. If there are no other restrictions, we can consider this to be a problem involving 3 distinct boxes (type of flowers), 12 identical objects (flower). Therefore, this can be done in $\binom{12+3-1}{3-1}=\binom{14}{2}=91$ ways.
(b)	Remove two of each type of flower. This reduces to the problem involving 3 distinct boxes (type of flower), 6 identical objects (flowers). Therefore, this can be done in $\binom{6+3-1}{2}=\binom{8}{2}=28$ ways.
(ii) (a)	A <u>sequence</u> of twelve flowers is to be formed, with the 3 types of flowers _ _ _ _ _ _ _ _ _ _ _ _ If there are no other restrictions, the number of ways this can be done is $3^{12}=531441$.
(b)	By Inclusion-Exclusion Principle, Required number of ways $=3^{12}-\binom{3}{2}\cdot2^{12}+\binom{3}{1}\cdot1^{12}=519156$

Qn	Solution
5(iii)	This problem can be modelled as distributing 10 identical objects into 3 identical boxes with at least 1 object in each box. No of ways = $P(10,3)=8$.
(iv)	Once the rose and tulip are put into 2 different vases, all the vases are now distinct. By putting one sunflower into the remaining vase, all 3 vases are now filled (and distinct), so the problem becomes that of distributing 9 identical objects into 3 distinct boxes. Therefore, this can be done in $\binom{9+3-1}{2}=\binom{11}{2}=55$ ways.
6(a)	(Method 1) Suppose $2p_2p_3\cdot p_k+1$ is a perfect square, then $2p_2p_3\cdot p_k+1=m^2$ for some integer *m*. Since $2p_2p_3\cdot p_k+1$ is odd, therefore m^2 is odd, which implies *m* is odd. Rearranging $2p_2p_3\cdot p_k+1=m^2$ we have $2p_2p_3\cdot p_k=m^2-1=(m+1)(m-1)$ Since *m* is odd, both $m+1$ and $m-1$ are even and hence the RHS is a multiple of 4. However, the LHS has only one factor of 2 as all other primes are odd. Thus the LHS is not a multiple of 4 and a contradiction ensues. (Method 2) Let $k\in\mathbb{N}$, then $M_k=2p_2p_3\cdot p_k+1$ can be written in the form $$M_k=2(2r+1)+1=4r+3,$$ for some positive integer r, since $p_2,\ldots,p_k$ are all odd primes, and thus the product $p_2\cdots p_k$ remain odd. On the other hand, if M_k is a perfect square, i.e. $M_k=q^2$, for some positive integer *q*, then each perfect square is only of the form 4*r* or $4r+1$, and not of the form $4r+3$. To prove this, we consider two cases: Case 1: *q* is even, i.e., $q=2n$, for some $n\in\mathbb{Z}^+$, then clearly $M_k=(2n)^2=4n^2$. Case 2: *q* is odd, i.e., $q=2n+1$, for some $n\in\mathbb{Z}^+$, then $M_k=4(n^2+n)+1$.

Qn	Solution
6(b)	By the binomial expansion theorem, we have $(x+y)^p = x^p + \binom{p}{1}x^{p-1}y + \binom{p}{2}x^{p-2}y^2 + \ldots + \binom{p}{p-1}xy^{p-1} + y^p$ We can show that if k is an integer, $1 \le k < p$, then $\binom{p}{k} = p \cdot \dfrac{(p-1)(p-2)\ldots(p-k+1)}{k!} \equiv 0 \pmod{p}$, and this is because that p is prime ensures gcd(*k!*,*p*)=1, and so $\dfrac{(p-1)(p-2)\ldots(p-k+1)}{k!} \in \mathbb{Z}^+$ as $\binom{p}{k} \in \mathbb{Z}^+$. $\binom{p}{1}x^{p-1}y + \binom{p}{2}x^{p-2}y^2 + \ldots + \binom{p}{p-1}xy^{p-1} \equiv 0 \pmod{p}$ $(x+y)^p = x^p + \binom{p}{1}x^{p-1}y + \binom{p}{2}x^{p-2}y^2 + \ldots + \binom{p}{p-1}xy^{p-1} + y^p$ $\equiv \left(x^p + y^p\right) \pmod{p}$
(c)	**Statement A**: Consider the first $p+1$ terms of the sequence modulo *p*. By pigeonhole principle, at least two of these terms must be congruent to each other modulo *p*. In other words, there are two positive integers *a*, *b* satisfying $0 < a < b \le p+1$ such that $\underbrace{77\ldots7}_{a \text{ digits}} \equiv \underbrace{77\ldots7}_{b \text{ digits}} \pmod{p}$ Taking the difference, we obtain $\underbrace{77\ldots7}_{(b-a)}\underbrace{00\ldots0}_{a} \equiv 0 \pmod{p}$ However, since $\underbrace{77\ldots7}_{(b-a)}\underbrace{00\ldots0}_{a} = \underbrace{77\ldots7}_{(b-a)} \times 10^a = \underbrace{77\ldots7}_{(b-a)} \times 2^a \times 5^a$, and since *p* is relatively prime to both 2 and 5, we must have $\underbrace{77\ldots7}_{(b-a) \text{ digits}} \equiv 0 \pmod{p}$. i.e. the $(b-a)^{\text{th}}$ term is divisible by *p*. **Statement B**: Suppose the c^{th} term is divisible by *p*. Consider the c^{th}, $2c^{\text{th}}$, $3c^{\text{th}}$, … terms of the sequence. Since these are of the form $\underbrace{77\ldots7}_{c \text{ digits}}\underbrace{77\ldots7}_{c \text{ digits}}\cdots\cdots\underbrace{77\ldots7}_{c \text{ digits}}$, they are all multiples of $\underbrace{77\ldots7}_{c \text{ digits}}$ and hence divisible by *p*. Thus, there are infinitely many terms in the sequence which are divisible by *p*.

Qn	Solution
7(a) (i)	 Let θ be angle $P_{k-1}OP_{k+1}$, which is fixed. Let α be the variable angle $0<\alpha<\theta$, corresponding to the position of P_k. Then $d\left(P_{k-1},P_k\right)=2\sin\frac{\alpha}{2}$ and $d\left(P_k,P_{k+1}\right)=2\sin\frac{\theta-\alpha}{2}$, so $d\left(P_{k-1},P_k\right)+d\left(P_k,P_{k+1}\right)=2\left(\sin\frac{\alpha}{2}+\sin\frac{\theta-\alpha}{2}\right)$ $=2\times2\sin\frac{\theta}{4}\cos\left(\frac{\alpha}{2}-\frac{\theta}{4}\right)$ $=4\sin\frac{\theta}{4}\cos\left(\frac{\alpha}{2}-\frac{\theta}{4}\right)$ Since $\sin\frac{\theta}{4}$ is a constant, this is maximized when $\cos\left(\frac{\alpha}{2}-\frac{\theta}{4}\right)=1$ i.e. when $\alpha=\frac{\theta}{2}$. Hence the required distance is maximum when P_k lies on the midpoint of the arc.
(ii)	Considering each set of 3 consecutive points P_{k-1}, P_k, P_{k+1} for $k=1,2,\ldots,n-1$, S_n is maximized when $d\left(P_0,P_1\right)=d\left(P_1,P_2\right)=\ldots=d\left(P_{n-1},P_n\right)$ When this is true, then $S_n=n\times d\left(P_0,P_1\right)=n\left[2\sin\frac{\pi}{4n}\right]=2n\sin\frac{\pi}{4n}$
(iii)	When $n\to\infty$, the length $S_n\to\frac{\pi}{2}$ the length of the quadrant. So $\lim_{n\to\infty}2n\sin\frac{\pi}{4n}=\frac{\pi}{2}$ $\lim_{n\to\infty}\frac{4n}{\pi}\sin\frac{\pi}{4n}=1$ $\lim_{n\to\infty}\frac{\sin\frac{\pi}{4n}}{\frac{\pi}{4n}}=1$ So letting $x=\frac{\pi}{4n}$ we get $\lim_{x\to0}\frac{\sin x}{x}=1$.

Qn	Solution
7(b)	Consider $\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$ $=\lim_{h\to 0}\frac{-2\sin\frac{(x+h)+x}{2}\sin\frac{(x+h)-x}{2}}{h}$ $=\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$ $=\lim_{h\to 0}\left[-\sin\left(x+\frac{h}{2}\right)\right]\times\left[\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right]$ $=\left[-\sin(x+0)\right](1)=-\sin x$
8(a) (i)	Since $a\mid(2b+1)$, $2b+1=ka$ for some integer *k*. $1\le\gcd(a,b)\le\gcd(ka,b)=\gcd(2b+1,b)=\gcd(1,b)=1$ Therefore $\gcd(a,b)=1$ i.e. *a* and *b* are coprime. Alternatively, since $b\mid(2a+1)$, $2a+1=kb$ for some integer *k*. $1\le\gcd(a,b)\le\gcd(a,kb)=\gcd(a,2a+1)=\gcd(a,1)=1$ Therefore $\gcd(a,b)=1$ i.e. *a* and *b* are coprime.
(ii)	Since $a\mid 2b+1$ and $a\mid 2a$, we have $a\mid(2a+2b+1)$. Likewise, $b\mid 2a+2b+1$. Since *a* and *b* are coprime, $ab\mid(2a+2b+1)$.
(iii)	Since $2a+1$ is a multiple of *b*, and $2a+1$ is odd, *b* must be odd. Likewise *a* must be odd by a similar argument
(iv)	$a^2<ab\le 2a+2b+1\le 2a+2(2a+1)+1=6a+3$
(v)	Using GC, the only possible integer solutions for $a^2<6a+3$ is $a=1,3$ or 5 If $a=1$, $2a+1=3$ so $b\mid 3$. This results in $b=1$ or $b=3$. Since $a<b$, $b\ne 1$. A quick verification verifies that $(a,b)=(1,3)$ is a valid solution to the problem. If $a=3$, $2a+1=7$ so $b\mid 7$. This results in $b=1$ or $b=7$. Since $a<b$, $b\ne 1$. A quick verification verifies that $(a,b)=(3,7)$ is a valid solution to the problem. If $a=5$, $2a+1=11$ so $b\mid 11$. This results in $b=1$ or $b=11$. Since $a<b$, $b\ne 1$. Also, since $5\nmid 2(11)+1$, $(a,b)=(5,11)$ is not a solution to the problem. Hence the possible solutions to (a,b) are $(1,3)$, $(3,7)$.

Qn	Solution
8(b) (i)	$\dfrac{n}{p_1 p_2 \cdots p_{k-1} p_k}$
(ii)	$\phi(m) = m - \left(\dfrac{m}{a} + \dfrac{m}{b} + \dfrac{m}{c}\right) + \left(\dfrac{m}{ab} + \dfrac{m}{ac} + \dfrac{m}{bc}\right) - \dfrac{m}{abc}$ $= m\left(\dfrac{abc - (ab + ac + bc) + (a + b + c) - 1}{abc}\right)$ $= m\left(\dfrac{a-1}{a}\right)\left(\dfrac{b-1}{b}\right)\left(\dfrac{c-1}{c}\right)$ (shown)
(iii)	$\phi(n) = n\left(\dfrac{p_1 - 1}{p_1}\right)\left(\dfrac{p_2 - 1}{p_2}\right)\cdots\left(\dfrac{p_k - 1}{p_k}\right)$
(iv)	$\phi(2^3 \times 5 \times 7^4 \times 11^2 \times 47^2)$ $= \left(2^3 \times 5 \times 7^4 \times 11^2 \times 47^2\right)\left(\dfrac{1}{2}\right)\left(\dfrac{4}{5}\right)\left(\dfrac{6}{7}\right)\left(\dfrac{10}{11}\right)\left(\dfrac{46}{47}\right)$ $= 7830936960$