RVHS H2 Mathematics Remedial Programme

Topic: Vectors IV

Basic Mastery Questions

1. **RVHS CT 9758/2021/Q7**

The plane π passes through the points with coordinates $(-1, 0, 1)$, $(2, 1, -1)$ and $(1, -3, 2)$. The line l passes through the point $P(16, 20, 16)$ and is parallel to the vector $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(i) Find the cartesian equation of π. [3]

RVHS JC Skills Builder: Click here or scan this to view video example on how to find vector equation of a plane!

Also, click here or scan this to view video example on how to convert vector equation of a plane to Cartesian form!

(ii) Find the coordinates of the point of intersection of l and π. [2]

(iii) Find the acute angle between l and π. [2]

RVHS JC Skills Builder: Click here or scan this to view video example on how to find vector equation of a plane!

(iv) Determine a vector equation of the line of reflection of l in π. [4]

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Answers: (i) $5x + 7y + 11z = 6$ (ii) $(3, -6, 3)$ (iii) $\theta = 61.3°$ (iv) $\mathbf{r} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ 2 \\ 31 \end{pmatrix}, \mu \in \mathbf{R}$

2. **ACJC Promo 9758/2020/Q9**

The plane π passes through the point with position vector $\mathbf{i}-3\mathbf{k}$ and contains the line with equation $\mathbf{r}=\mathbf{i}+\mathbf{j}+\lambda(2\mathbf{i}-\mathbf{j}+\mathbf{k})$.

(i) Show that the cartesian equation of the plane π is $2x+3y-z=5$. [2]

(ii) Find the shortest distance from $P(-5, 6, -5)$ to the plane π. [2]

The line L that passes through P and is parallel to $3\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ intersects the plane π at the point Q.

(iii) Find the coordinates of Q. [2]

(iv) Hence or otherwise, find the length of projection of $\overrightarrow{PQ}$ on the plane π. [2]

Answers: (ii) $\frac{8}{\sqrt{14}}$ (iii) Q(7, -2, 3) (iv) $4\sqrt{\frac{117}{7}}=16.4$ (3 s.f.)

Standard Questions

1. **NJC CT 9758/2021/Q12**

Diagram I: Louvre Museum

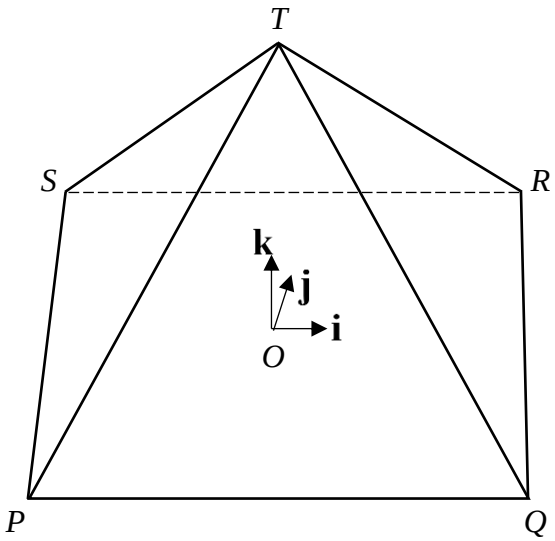

Diagram II

Diagram I shows the structural design of the Louvre Museum which is constructed entirely with glass segments and metal poles. As shown in Diagram II, the structure can be modelled by a right pyramid consisting of a square base *PQRS* of sides 30 metres on horizontal ground and the apex *T*. *O* is the centre of the square base and *T* is vertically above *O* with a perpendicular height of 20 metres. Points (x, y, z) are defined relative to the point $O(0, 0, 0)$, with units in metres. The unit vectors **i**, **j** and **k** are parallel to *PQ*, *QR* and *OT* respectively.

(i) Find a cartesian equation of face *QRT*. [3]

A torch at point $L(14, -14, 0)$ emits a ray of light in the direction $\begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$.

(ii) Find the acute angle between the ray of light and face *QRT*. [2]

The face *QRT* is a smooth surface. After the ray of light from the torch hits face *QRT*, it is reflected. The point *A* where the ray of light from the torch hits face *QRT* is called the point of incidence.

(iii) Show that the coordinates of *A* is $(14.25, -13.75, 1)$. [3]

(iv) Hence find a vector equation of the line which represents the reflected ray in face *QRT*. Assume that the ray of light from the torch, the reflected ray and the normal at the point of incidence lie on the same plane, and that the ray of light from the torch and the reflected ray make the same angle with the normal at the point of incidence. [4]

Answers: (i) $4x + 3z = 60$ (ii) $\theta = 49.0°$ (iv) $\mathbf{r} = \begin{pmatrix} 14.25 \\ -13.75 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1.03 \\ -0.25 \\ -0.04 \end{pmatrix}, t \in \mathbb{R}$