H

ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

/100

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS 9758/01

Paper 1 **20 August 2024**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Question	Marks
1	/4
2	/5
3	/6
4	/8
5	/9
6	/9
7	/10
8	/11
9	/12
10	/12
11	/14
Total	100

This document consists of **24** printed pages and **2** blank pages.

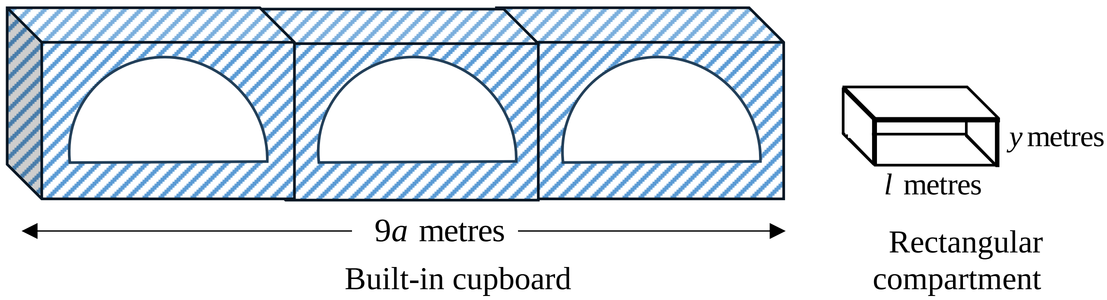
Anglo-Chinese Junior College

1 The complex numbers z and w satisfy the following equations.

$$\mathrm{i}w^2 + 2wz = 2\mathrm{i}$$
$$z + \mathrm{i}z = 2 + \mathrm{i}w$$

Find z and w, giving your answers in the form of $a+\mathrm{i}b$ where a and b are real numbers. [4]

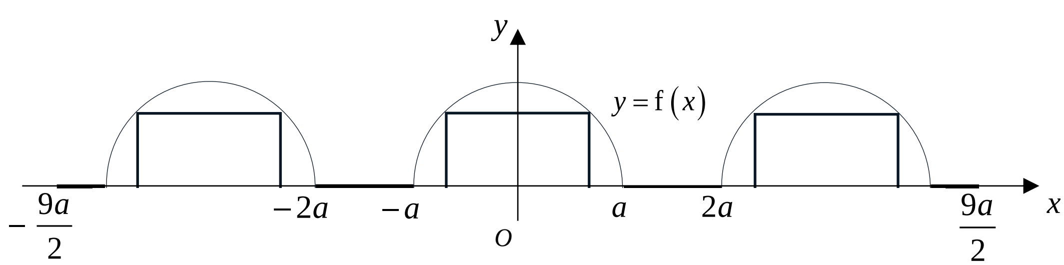
2



An interior designer designed a built-in cupboard for his client as shown above. The built-in cupboard of length $9a$ metres, $a > 0$, has three equal sections and each section has a semi-elliptical hole in the centre. The designer wants to fit a hollow rectangular compartment, for storage into each of the elliptical hole. Each rectangular compartment with negligible thickness, has a length of l metres, where $l < 2a$, a height of y metres, and a fixed depth. The cross-section for part of the built-in cupboard is shown in the diagram below and the elliptical holes are modelled by the equation

$$\mathrm{f}(x) = \begin{cases} \sqrt{1 - \dfrac{x^2}{a^2}} & \text{for } -a \le x \le a, \\ 0 & \text{for } a \le x \le 2a, \end{cases}$$

and $\mathrm{f}(x+3a) = \mathrm{f}(x)$ for $-\dfrac{9a}{2} \le x \le \dfrac{9a}{2}$, where a is a real constant.



(a) Write down, in terms of l and a, the value of $\mathrm{f}(x)$ when $x = 3a + \dfrac{1}{2}l$. [1]

(b) The interior designer wishes to maximise the rectangular compartment storage space. Show that the length of the compartment l, is $\sqrt{2}a$ metres, when the space is maximised. Find also the corresponding height of the compartment. (You do not need to show that the value is a maximum.) [4]

3 Elly started planking as an exercise and she continues the exercise every day to build her core muscles. If she meets her target duration, she increases the target duration of the exercise by an additional 4 seconds on the next day. On any day, she will stop her exercise once she meets her target duration for the day. However, Elly does not always meet her target. Each day when Elly misses her target, she decreases her target duration by 5% on the following day. On Day 1, Elly carries out 20 seconds of planking, and she hopes to reach her target of 2 minutes by the end of 30 days.

(a) Assume that Elly met her targets for the first 11 days but missed her target duration from Day 12 to Day 15. Determine whether Elly will be able to reach her target of 2 minutes by the end of 30 days, if she met all her targets from Day 16 onwards. [3]

Due to the difficulty level, Elly decides to restart the programme by increasing the target duration of the exercise by a% each day, regardless of whether she meets her target.

(b) Find in terms of a, the total target duration Elly has completed by the end of 30 days if she carries out 20 seconds of planking on Day 1. [2]
[You may assume that on any day, she will stop her exercise once she meets her target duration for that day.]

(c) If the total target duration she has completed by the end of 30 days is at least 30 minutes, find, to the nearest integer, the least value of a. [1]

4 **(i)** Using standard series from the List of Formulae (MF26), show that for x^4 and higher powers to be neglected,

$$\mathrm{f}(x)=\ln\left(\frac{1+2x}{1-2x}\right)\approx 4x+\frac{16}{3}x^3 .$$ [3]

(ii) Use your series from part **(i)** to estimate $\int_0^{0.04} \ln\left(\frac{1+2x}{1-2x}\right)\mathrm{d}x$, correct to 8 decimal places. [1]

(iii) Use your calculator to find $\int_0^{0.04} \ln\left(\frac{1+2x}{1-2x}\right)\mathrm{d}x$, correct to 8 decimal places. [1]

(iv) Comparing your answers to parts **(ii)** and **(iii)**, and with reference to the value of x, comment on the accuracy of your approximations. [2]

(v) Explain why a Maclaurin series for $\mathrm{g}(x)=\ln\left(\frac{x+2}{x-2}\right)$ cannot be found. [1]

5 A curve has equation $y=\dfrac{5e^x}{\sqrt{4e^x-3}}$. The line $y=5$ intersects the curve at points A and B.

(i) Find the exact x-coordinates of the points A and B. [3]

(ii) Using the substitution $u=e^x$, find the exact volume generated when the area bounded by the curve and the line $y=5$ is rotated about the x-axis through 360°. Give your answer in the form $\dfrac{25\pi}{8}(a\ln 3-b)$, where a and b are constants to be determined. [6]

6 **Do not use a calculator in answering this question.**

(a) **(i)** One of the roots of the equation $aw^4-16w^3+21w^2-aw+5=0$, where a is real, is $2-i$. Find the value of a and the other roots. [4]

(ii) Hence solve $5w^4-aw^3+21w^2-16w+a=0$. [2]

(b) The complex number z is given by

$$z=\frac{\left(\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)\right)^4}{-k\left(\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)\right)},$$

where k is a positive real constant.

Find $|z|$ and $\arg z$. [3]

7 **(i)** Show that $\dfrac{r^2-3r+1}{r!}=\dfrac{1}{(r-2)!}-\dfrac{2}{(r-1)!}+\dfrac{1}{r!}$.

Hence find $\displaystyle\sum_{r=3}^{n}\frac{r^2-3r+1}{r!}$ in terms of n. [3]

(ii) It is given that $\displaystyle\sum_{r=3}^{5}\frac{r^2-3r+1}{r!}=\sum_{r=2}^{a+1}(2r-3)$. Find the value of a. [3]

(iii) State the value of $\displaystyle\sum_{r=3}^{\infty}\frac{r^2-3r+1}{r!}$. Hence evaluate $\displaystyle\sum_{r=7}^{\infty}\frac{r^2-r-1}{(r+1)!}$. [4]

8 The curve C is defined by the parametric equations

$$x = \theta - \cos^2\theta \text{ and } y = \theta - \sin\theta \text{ where } 0 \le \theta \le \pi .$$

(a) Show algebraically that the gradient of C is never negative for all points on C. [2]

(b) Find the equation of tangent that is parallel to y – axis. [2]

(c) If θ is sufficiently small for θ^3 and higher powers to be neglected, show that

$$\frac{dy}{dx} \approx a + a\theta + b\theta^2,$$

where a and b are constants to be determined. [3]

The line D has cartesian equation $y = x + \frac{1}{4}$.

(d) Find the exact x-coordinates of the point(s) of intersection(s) of curve C and line D. [4]

9 The function f is given by

$$\text{f}(x) = (x-a) + \frac{1}{|x-a|}, \text{for } x \in \mathbb{R}, x \ne a,$$

where a is a positive constant.

(i) Using differentiation,

(a) find, in terms of a, the coordinates of the stationary point(s) of $y = \text{f}(x)$ for $x > a$.

[2]

(i) **(b)** show that $y = \text{f}(x)$ has no stationary points for $x < a$. [2]

(ii) Sketch the curve of $y = \text{f}(x)$, showing clearly the equations of asymptotes, the coordinates of the points where the curve crosses the axes and coordinates of any turning point(s). [3]

(iii) Describe a sequence of transformations which transforms the curve of $y = \text{f}(x)$ on to the curve of $y = 2x - 2a + \frac{1}{|2x|}$. [3]

The function g is given by

$$\text{g}(x) = (x-a) + \frac{1}{|x-a|}, \text{ for } x \in \mathbb{R},\ x < a,$$

where a is a positive constant.

(iv) By considering the graphs of $y = \text{g}(x)$ and $y = \text{g}^{-1}(x)$, solve the inequality $\text{g}^{-1}(x) > \text{g}(x)$, giving your answer in terms of a. [2]

10 Second-hand smoking in public spaces have resulted in negative effects such as coronary heart disease, lung cancer, and other diseases. Designated smoking rooms are often being built to contain the smoke. A room containing 30 m^3 of air is originally free of carbon monoxide. Let V m^3 be the volume of carbon monoxide in the room at time t minutes after the smoke starts entering the room. Let C be the concentration of carbon monoxide in the room at time t.

Initially, there is no carbon monoxide in the room. However, smoke containing 5% of carbon monoxide is blown into the room at the rate of 0.002 m^3/min. The rate at which the carbon monoxide leaves the room is $\dfrac{C}{500}$ m^3/min.

(i) Express $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ in terms of C. [1]

(ii) Hence, given that $C=\dfrac{V}{30}$, show that $\dfrac{\mathrm{d}C}{\mathrm{d}t}=\dfrac{1}{15000}(0.05-C)$. [2]

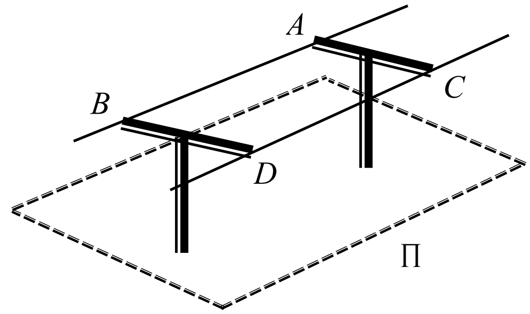
(iii) By solving the differential equation in part **(ii)**, show that the concentration of carbon monoxide in the room at time t is $C=0.05\left(1-\mathrm{e}^{-\frac{1}{15000}t}\right)$. [4]

(iv) Explain in context what will happen to the concentration of carbon monoxide in the long run? [1]

(v) Sketch the curve of C against t. [2]

(vi) Medical research has shown that when the volume of carbon monoxide in the room reaches 0.0036 m^3, a person exposed to it can lead to loss of consciousness. Find the time for the concentration of carbon monoxide to reach this level, giving your answer to nearest integer. [2]

11



At a ski resort, engineers are installing cables for a new cable car system to transport skiers to ski slopes. The system involves installing cables running between support towers. Cables are laid in straight lines and the widths of cables can be neglected. The cable *AB* is used to transport skiers up the slope and another parallel cable *CD* is used to transport skiers down the slope. Straight lines are used to represent the cables and a plane Π is used to model the ski slope.

The cable *AB* has vector equation $\mathbf{r} = \begin{pmatrix} 5 \\ -5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$ where $\lambda \in \mathbb{R}$ and $-5 < \lambda < 15$.

The parallel cable *CD* passes through the point $(3, -18, 10)$. The cartesian equation of the ski slope Π is $x - 2y + 3z = 5$.

(i) Find the distance between the cables *AB* and *CD*. [2]

(ii) The length of the cable from point *A* to point *B* is 100 units. Find the length of the projection of *AB* on the ski slope Π. [3]

(iii) Find the coordinates of a point *P* on the cable *AB* which is at a perpendicular distance of $2\sqrt{14}$ units from the ski slope Π. [3]

(iv) There is a viewing gallery on the mountain that overlooks the ski slope. The engineers wish to install a huge glass window plane at the viewing gallery. The window plane is perpendicular to the ski slope and is parallel to the cable *AB*. Given that $(10,\ 10,\ 20)$ is a point on the window plane, find the cartesian equation of the window plane. [3]

(v) Due to a power outage while testing the system, a cable car got stuck on the cable *AB* at the point *Q* with coordinates $(-7,\ 7,\ 25)$. The maintenance team wishes to reach the point *Q* as quickly as possible. Find the coordinates of the point on the ski slope closest to the point *Q* from where the team should launch their operation. [3]

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JC2 PRELIMINARY EXAMINATION

Higher 2

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CANDIDATE NAME

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INDEX NUMBER

MATHEMATICS 9758/02

Paper 2 **23 August 2024**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

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4	/8
5	/10
6	/8
7	/8
8	/10
9	/10
10	/12
11	/12

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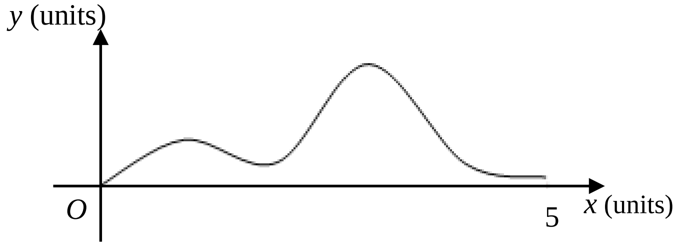
Section A: Pure Mathematics [40 marks]

1 The diagram shows part of the graph $y = x^{\cos 2x}$, for $0 \le x \le 5$, which represents the path of a roller coaster. The horizontal distance travelled by the roller coaster is denoted by x units and its vertical distance travelled is denoted by y units.



(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^{\cos 2x}\left[\dfrac{\cos 2x}{x} - (2\sin 2x)\ln x\right]$. [2]

(b) At the point on the graph where $x = \pi$, find the rate at which the roller coaster is moving vertically when it is moving horizontally at a rate of 8 units per hour. [2]

(c) Find the acute angle that the tangent to the graph where $x = \pi$ makes with the horizontal. [1]

2 Referred to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively and they lie on plane π.

(a) Show that $\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}$ is a vector perpendicular to the plane π. [2]

(b) Prove that the equation of plane π can be written as

$$\mathbf{r}\bullet(\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}) = \mathbf{a}\bullet(\mathbf{b}\times\mathbf{c}),$$

explaining clearly the reason for any result that you use in your proof. [2]

(c) Given that $\mathbf{a} = \mathbf{i}$, $\mathbf{b} = \mathbf{j}$ and $\mathbf{c} = \mathbf{k}$, show that the equation of the plane π can be written as $\mathbf{r}\bullet\begin{pmatrix}1\\1\\1\end{pmatrix} = 1$. Hence, find the cartesian equations of the planes which are at a distance of 5 units from plane π. [3]

3 **(a)** Show that $\int 2t\cos^2 t\,\mathrm{d}t = \frac{1}{4}\left(2t\sin 2t + 2t^2 + \cos 2t\right) + c$, where c is an arbitrary constant. [3]

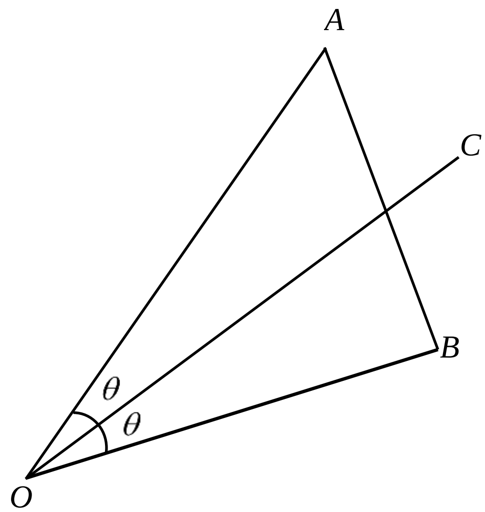
(b) A curve C has parametric equations

$$x = 2t\sin t,\ \ y = \cos t \ \text{ for } \frac{3\pi}{4} \le t \le \pi.$$

(i) Sketch the graph of C. Give in exact form the coordinates of the end points. [2]

(ii) Find the exact area enclosed by C, the y-axis, the x-axis and the line $x = \frac{3\pi}{2\sqrt{2}}$. [5]

4



The origin O and the points A, B and C lie in the same plane, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. It is given that $\angle AOC = \angle BOC = \theta$, where θ is an acute angle.

(a) Show that $\mathbf{c}\bullet\hat{\mathbf{a}} = \mathbf{c}\bullet\hat{\mathbf{b}}$ where $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are unit vectors in the directions of vectors **a** and **b** respectively. [1]

(b) If **c** can be written as $m\hat{\mathbf{a}} + n\hat{\mathbf{b}}$, where m and n are constants, use the result from **(a)** to show that $m = n$. [3]

(c) Write down the equation of the line passing through the points A and B. [1]

(d) Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $m = n$, show that the position vector of the point of intersection of the line passing through A and B and the line passing through O and C is $t\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right)$, where t is a constant to be determined. [3]

5 The function h is defined by $\mathrm{h}: x \mapsto [\ln(x+2)]^2 + 1,\ x \in \mathbb{R},\ x > -2$.

(a) The function h^{-1} exists if the domain of h is restricted to $-2 < x \le k$.
State the greatest possible value of k. [1]

The function f is defined by

$$\mathrm{f}(x) = \begin{cases} [\ln(x+2)]^2 + 1, & \text{for } x \in \mathbb{R}, -2 < x \le -1, \\ \dfrac{1}{x+2}, & \text{for } x \in \mathbb{R}, x > -1. \end{cases}$$

(b) Sketch the graph of $y = \mathrm{f}(x)$. [1]

(c) Given that f^{-1} exists, find f^{-1} in a form similar to f. [4]

(d) Show that f^2 exists and find its range. [2]

(e) If $\mathrm{f}^2(2) = \mathrm{f}(x)$, find x. [2]

Section B: Probability and Statistics [60 marks]

6 The events A and B are such that $\mathrm{P}(A) = a$ and $\mathrm{P}(B) = b$. A and B are independent events.

(a) Find an expression for $\mathrm{P}(A' \cap B')$ in terms of a and b, and hence prove that A' and B' are independent events. [2]

It is given that $\mathrm{P}(A' | B') = 0.85$ and $\mathrm{P}(B') = 0.8$.

(b) Find $\mathrm{P}(A \cap B')$. [2]

For a third event C, it is given that A and C are mutually exclusive and $\mathrm{P}(A' \cap C') = 0.52$.

(c) Find $\mathrm{P}(C)$. [1]

(d) Hence find the set of possible values of $\mathrm{P}(A' \cap B' \cap C')$. [3]

7 Taylor is planning some surprise treats for her fans during her upcoming concert. She is creating a setlist of 12 songs which consists of the following:

- 6 songs chosen from her entire discography which will include her number one hit song,
- 3 surprise duets with a special guest artist,
- 3 pre-recorded songs.

(a) In how many ways can Taylor arrange the setlist of 12 songs by considering the following:
- her number one hit song is the last song in the setlist,
- the 3 surprise duets are to be performed back-to-back,
- the 3 pre-recorded songs are all separated from each other by at least one song. [2]

Another segment of Taylor's concert involves a medley consisting of dancers from different countries. It is given that her dance entourage is made up of dancers from 6 different countries. There are 5 dancers from each country.

(b) Find the number of ways she can form a team of 10 dancers from 3 different countries with at least 2 dancers from each country. [4]

The final segment of Taylor's concert is a high-energy dance routine involving her most talented team of 10 dancers. The choreography requires the dancers to be positioned at 10 different spots on the stage. 5 of the spots form a circle with each spot illuminated in blue by the spotlights. The remaining 5 spots form another circle with each spot illuminated with a distinct colour by the spotlights.

(c) Given that the two circles do not overlap, find the number of possible arrangements for the 10 dancers at the 10 spots. [2]

8 A refrigerator manufacturer claims that the mean lifespan of refrigerators of a particular model is 12 years. A consumers association representative suspects that the mean lifespan of the refrigerators is actually less than 12 years.

The durations x, in years, of a random sample of 45 refrigerators are summarised as follows.

$$\sum(x-12)=-4.3 \qquad \sum(x-12)^2=17.08$$

(a) Calculate unbiased estimates of the population mean and variance of the lifespan of the refrigerators. [2]

(b) State hypotheses that can be used to test if the mean lifespan of the refrigerators is less than 12 years, defining any parameters you use. Test, at the 5% level of significance, whether the mean lifespan of the refrigerators is less than 12 years. [4]

The manufacturer switches to a different coolant and decides to test whether the mean lifespan of the refrigerators has changed from 12 years. He records the durations of a large random sample of n refrigerators and finds that their mean lifespan is 12.4 years and variance is 4.1 square years.

A test at 5% significance level, is carried out on the new random sample. The test shows that there is sufficient evidence that the mean lifespan of refrigerators has changed.

(c) Find the range of possible values of n. [4]

9 **In this question you should state the parameters of any distribution you use.**

The times, in minutes, taken for male runners to complete a marathon follow the distribution $N(196, 24^2)$.

(a) Calculate the expected number of male runners who take more than 180 minutes to complete a marathon in a randomly chosen batch of 80 male runners. [2]

(b) It is given that at most 10% of the fastest male runners will be eligible to join the competition. Find the qualifying time, to the nearest minute, to join the competition. [1]

The times, in minutes, taken for female runners to complete a marathon follow the distribution $N(210, 30^2)$.

(c) Find the probability that the total time taken by a randomly selected male runner and 3 randomly selected female runners is between 700 and 800 minutes. [3]

To help the group of marathon runners improve their timings for the actual competition, a sponsor provides all runners with a set of running apparel to help them reduce air drag. This reduces the timing of each male runner by 5% and reduces the timing of each female runner by 6%.

(d) Find the probability that, after being equipped with the new apparel, the total time taken by 2 randomly chosen female runners differs from twice the time taken by a randomly selected male runner by less than 17 minutes. [4]

10 A female social media influencer is analysing the performance of her past videos posted online. She wants to understand the relationship between the number of video views, v, and the number of followers gained, f, from her past video posts. The data from her earliest 9 videos is given in the table below.

v	1000	5000	8000	10 000	20 000	30 000	40 000	50 000	60 000
f	15	163	10	278	389	456	492	541	560

(a) Due to a technical issue, one of the 9 videos had no audio which affected the number of followers gained. Draw a scatter diagram of these 9 data points and circle the data point that likely represents the video with no audio. [2]

The female social media influencer decides to **exclude** the data point that represents the video with no audio from her analysis. For parts **(b)**, **(c)** and **(d)** of this question, you should **exclude** this data point.

(b) Use your scatter diagram to explain with reasons the conclusion that the influencer should reach regarding the relationship between f and v. [1]

(c) By referring to the scatter diagram and calculating the relevant product moment correlation coefficients, determine whether the relationship between f and v is modelled better by $f = a + bv$ or $f = a + b\ln v$, where a and b are constants.

Explain how you decide which model is better and state the equation of the least squares regression line in this case, giving your answer to 3 decimal places. [5]

(d) Use the appropriate least squares regression line in **(c)** to estimate the number of followers gained when the number of video views is 100 000. Comment on the reliability of this estimate. [2]

(e) Explain why in the 'method of least squares', the distances which are used in finding the least squares regression line are squared. [1]

A male social media influencer also decides to analyse the performance of his past videos posted online and found that the sum of the squares of the distances which are used in finding the least squares regression line is zero.

(f) What can you deduce about the data points of this male influencer? [1]

11 A candy shop is having a lucky draw to generate publicity. On average, 4% of candy bars produced contain a lucky draw ticket each.

(a) The shop owner orders r candy bars on a particular day. The number of candy bars that contain a lucky draw ticket each is the random variable D.

(i) State, in the context of the question, two assumptions needed to model D by a binomial distribution. [2]

You are now given that D can be modelled by distribution $\mathrm{B}(r, 0.04)$.

(ii) Find the value of r if the variance is 1.92. [1]

The shop owner orders k candy bars on another day.

(b) The probability that there are more than 3 lucky draw tickets among the k candy bars is at least 0.34. Determine the minimum value of k. [3]

The lucky draw box contains five numbered vouchers. Two of the vouchers are numbered 0, the three others are numbered 1, 2 and 4 respectively. A voucher is taken one at a time, at random and without replacement, until the second voucher labelled 0 is taken out. The random variable A is the sum of the numbers on the vouchers taken.

(c) State the possible values that A can take and determine the probability distribution of A. [4]

A customer plays this lucky draw once and receives \$$A$.

(d) Find the probability that the customer receives at least \$5, given that he has taken out at least 4 vouchers. [2]

Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Question wrongly (R)**	**Presentation (P)**

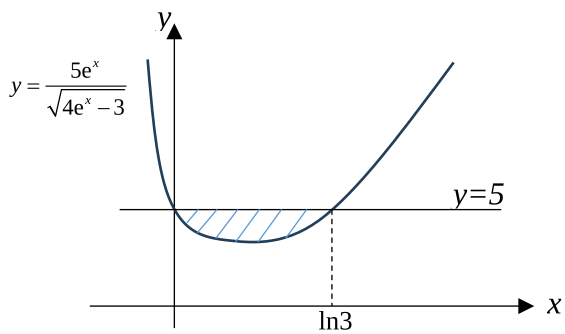
2024 JC2 H2 Mathematics Preliminary Examination Paper 1 Markers' Report

Qn	Solutions	Markers' Comments
1	$\mathrm{i}w^2+2wz=2\mathrm{i} \quad ---(1)$ $z+\mathrm{i}z=2+\mathrm{i}w \quad ---(2)$ From eq (2): $z(1+\mathrm{i})=2+\mathrm{i}w$ $\Rightarrow z=\frac{2+\mathrm{i}w}{1+\mathrm{i}}\cdot\frac{1-\mathrm{i}}{1-\mathrm{i}}=\frac{2-2\mathrm{i}+\mathrm{i}w-\mathrm{i}^2w}{1^2-\mathrm{i}^2}=\frac{1}{2}(2-2\mathrm{i}+\mathrm{i}w+w)$ Sub into eq (1): $\mathrm{i}w^2+2wz=2\mathrm{i}$ $\mathrm{i}w^2+w(2-2\mathrm{i}+\mathrm{i}w+w)=2\mathrm{i}$ $(1+2\mathrm{i})w^2+(2-2\mathrm{i})w-2\mathrm{i}=0$ $w=\frac{-(2-2\mathrm{i})\pm\sqrt{(2-2\mathrm{i})^2-4(1+2\mathrm{i})(-2\mathrm{i})}}{2(1+2\mathrm{i})}$ $=\frac{-2+2\mathrm{i}\pm\sqrt{-16}}{2(1+2\mathrm{i})}$ $=\frac{-2+2\mathrm{i}\pm4\mathrm{i}}{2(1+2\mathrm{i})}$ $=1+\mathrm{i}$ or $-\frac{3}{5}+\frac{1}{5}\mathrm{i}$ $z=1$ or $\frac{3}{5}-\frac{6}{5}\mathrm{i}$	**Common mistakes** 1. If $(4w-w^2)+3w^2i=-2+2i$ Equate real: $(4w-w^2)=-2$ Equate imag: $3w^2=2$ Taking wi, zi as imaginary and w, z is real 2. Trying to solve the question by just letting z = a + ib and w = u + iv equating real and imaginary and forming 4 equations. **Students wrongly rejected z = 1 claiming that it is not real.**
2(a)	When $x=3a+\frac{1}{2}l$, $\mathrm{f}(x)=\mathrm{f}(3a+\frac{1}{2}l)$ $=\mathrm{f}(\frac{1}{2}l)$ $=\sqrt{1-\frac{\left(\frac{1}{2}l\right)^2}{a^2}}$ $=\sqrt{1-\frac{l^2}{4a^2}}$	Students need to learn how to use the information of $\mathrm{f}(x+3a)=\mathrm{f}(x)$ to get $\mathrm{f}(3a+\frac{1}{2}l)=\mathrm{f}(\frac{1}{2}l)$ AND read the domain of piecewise function. Common mistake: Most students substituted $x=3a+\frac{1}{2}l$ into the function directly without realizing that it does not fall within the given domain.

2(b)	Let A be the cross-sectional area of the rectangular compartment. $A = l\sqrt{1-\frac{l^2}{4a^2}}$ ` $\frac{\mathrm{d}A}{\mathrm{d}l} = \sqrt{1-\frac{l^2}{4a^2}} + l\left(\frac{-\frac{2l}{4a^2}}{2\sqrt{1-\frac{l^2}{4a^2}}}\right)$ $= \sqrt{1-\frac{l^2}{4a^2}} - \frac{l^2}{4a^2\sqrt{1-\frac{l^2}{4a^2}}}$ To maximize A, $\frac{\mathrm{d}A}{\mathrm{d}l}=0$ $\sqrt{1-\frac{l^2}{4a^2}} - \frac{l^2}{4a^2\sqrt{1-\frac{l^2}{4a^2}}} = 0$ $\sqrt{1-\frac{l^2}{4a^2}} = \frac{l^2}{4a^2\sqrt{1-\frac{l^2}{4a^2}}}$ $1-\frac{l^2}{4a^2} = \frac{l^2}{4a^2}$ $2a^2 = l^2$ Since $l > 0$, $l = \sqrt{2}\,a$ (shown). Thus the height of the compartment is $\frac{1}{\sqrt{2}}$ cm.	Students need to learn to read the question carefully and be aware that there are two variables here, x and l, whereas a is a real constant. To maximise the rectangular compartment storage space, they need to find A in terms of l first. Unfortunately, many students failed to do so, they maximise the height instead, hence zero marks awarded.

	Alternatively $$A = 2x\sqrt{1-\frac{x^2}{a^2}}$$ $$\frac{dA}{dx} = 2\sqrt{1-\frac{x^2}{a^2}} + 2x\left(\frac{-\frac{2x}{a^2}}{2\sqrt{1-\frac{x^2}{a^2}}}\right)$$ $$= 2\sqrt{1-\frac{x^2}{a^2}} - \frac{2x^2}{a^2\sqrt{1-\frac{x^2}{a^2}}}$$ To maximize A, $\frac{dA}{dx} = 0$ $1-\frac{x^2}{a^2} = \frac{x^2}{a^2} \Rightarrow x^2 = \frac{a^2}{2} \Rightarrow x = \frac{a}{\sqrt{2}}$ since $x > 0$.	
3(a)	Let U_n be the Elly's targeted time of nth day $U_{12} = 20+11(4) = 64$ $U_{13} = 64(0.95)$ $U_{15} = 64(0.95)^{4-1}$ $U_{16} = 64(0.95)^{5-1}$ $U_{17} = 64(0.95)^4 + 4$ $U_{20} = 64(0.95)^4 + (5-1)(4)$ $\vdots$ $U_{30} = 64(0.95)^4 + 14(4) = 108.1284$ < 120 seconds Since the maximum time Elly can achieve is 108.12 seconds, thus, she is not able to.	Note: $U_{12} = 20+11(4) = 64$, not $U_{12} = 60(0.95)$. As the target is always based on the previous day's outcome. Likewise for U_{16}, it is should **not** be $U_{16} = 64(0.95)^3 + 4$.
3(b)	$$S_{30} = \frac{20\left[1-\left(1+\frac{a}{100}\right)^{30}\right]}{1-\left(1+\frac{a}{100}\right)} = \frac{2000}{a}\left[\left(1+\frac{a}{100}\right)^{30} - 1\right]$$	Wrong notations used: • $1.0a$, $0.0a$ etc. Wrong common ratio used: • a, $\frac{a}{100}$ etc There are 30 terms in the series, not 29.

3(c)	$\dfrac{2000}{a}\left[\left(1+\dfrac{a}{100}\right)^{30}-1\right]\geq 1800$ From GC, $a \geq 6.73$. Thus $a \approx 7$ (to the nearest integer)	Not possible to solve algebraically.
4(i)	Since $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots$ $\ln\left(\dfrac{1+2x}{1-2x}\right) = \ln(1+2x) - \ln(1-2x)$ $= 2x - \dfrac{4x^2}{2} + \dfrac{8x^3}{3} - \dfrac{16x^4}{4}$ $-\left(-2x - \dfrac{4x^2}{2} - \dfrac{8x^3}{3} - \dfrac{16x^4}{4}\right) + \ldots$ $\approx 4x + \dfrac{16}{3}x^3$ (shown)	Note the **meaning** of standard series in MF26.
4(ii)	$\int_0^{0.04} \ln\left(\dfrac{1+2x}{1-2x}\right) dx \approx \int_0^{0.04} 4x + \dfrac{16}{3}x^3 \, dx$ $= 0.00320341$ (to 8 d.p)	This is 8 decimal places and 6 significant figures. Do not give up to 10 decimal places.
4(iii)	$\int_0^{0.04} \ln\left(\dfrac{1+2x}{1-2x}\right) dx = 0.00320342$ (to 8 d.p)	As above.
4(iv)	When the value of x is close to zero, the approximation of both is as accurate as up to 8 decimal places.	Both points are necessary in order to get the full credit.
4(v)	For Maclaurin series, $g(0)$ is required. When $x = 0$, $\ln(-1)$ is undefined. Thus the Maclaurin series for $g(x) = \ln\left(\dfrac{x+2}{x-2}\right)$ cannot be found.	Stating $\ln(x-2) = \ln\left[(-2)\left(1-\dfrac{x}{2}\right)\right]$ $= \ln(-2) + \ln\left(1-\dfrac{x}{2}\right)$ is unacceptable as $\ln\left[(-2)\left(1-\dfrac{x}{2}\right)\right] \neq \ln(-2) + \ln\left(1-\dfrac{x}{2}\right)$ because logarithm properties are valid only for positive real numbers.

5(i)	$\frac{5e^x}{\sqrt{4e^x-3}}=5$ $5e^x=5\sqrt{4e^x-3}$ $e^{2x}=4e^x-3$ $e^{2x}-4e^x+3=0$ $\left(e^x-1\right)\left(e^x-3\right)=0$ $e^x=1$ or $e^x=3$ $\ln e^x=\ln 1$ or $\ln e^x=\ln 3$ $x=0$ or $x=\ln 3$	Need to improve knowledge of indices and solving equations. **Common mistakes** • $e^x\times e^x=e^{x^2}$ • $e^x(4-e^x)=3$ $e^x=3$ *or* $4-e^x=3$ $(e^x=1)$ Hence $x=\ln 3$ or $x=0$ • $e^{2x2}=4e^x-3$ $2x\ln e=4x\ln e-\ln 3$ 2) Not simplifying ln 1 = 0 i.e leaving answer as x = ln 3 or x = ln 1 3) Wasting time finding the y coordinate when x = 1 or ln 3 i.e when x = ln 3 $y=\frac{5e^{\ln 3}}{\sqrt{4e^{\ln 3}-3}}=5$ 4) Poor presentation like $(e^x)^2=4e^x-3$ Let $x=e^x$ $x^2=4x-3$
5(ii)	 Required Volume $=\pi\left(5^2\right)\left(\ln 3\right)-\pi\int_0^{\ln 3}\left(\frac{5e^x}{\sqrt{4e^x-3}}\right)^2 dx$ -------(1) Using $u=e^x$, $\frac{du}{dx}=e^x$,	**Common mistake** 1) Use area formula $25\pi\ln 3-\int_0^{\ln 3}\frac{5e^x}{\sqrt{4e^x-3}}\,dx$ 2) Use wrong formula like $2\pi\int_0^{\ln 3}[5-\frac{5e^x}{\sqrt{4e^x-3}}]^2\,dx$ 3) Error when changing variable • Vol = $\pi\int_1^3 5^2-(\frac{5u}{\sqrt{4u-3}})^2\ \frac{1}{u}du$ Which is incorrect and should be

	when $x = 0$, $u = 1$ when $x = \ln 3$, $u = 3$ $\int_0^{\ln 3}\left(\frac{5e^x}{\sqrt{4e^x-3}}\right)^2 dx = \int_{u=1}^{u=3}\left(\frac{5u}{\sqrt{4u-3}}\right)^2\left(\frac{1}{u}\right)du$ $= \int_1^3 \frac{25u}{4u-3}\, du$ $= 25\int_1^3 \frac{u}{4u-3}\, du$ $= 25\int_1^3 \frac{1}{4} + \frac{\frac{3}{4}}{4u-3}\, du$ $= 25\left[\frac{u}{4} + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\ln\lvert 4u-3\rvert\right]_1^3$ $= 25\left[\frac{3}{4} + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\ln 9 - \left(\frac{1}{4} + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\ln 1\right)\right]$ $= 25\left[\frac{3}{4} + \left(\frac{3}{16}\right)\ln 3^2 - \left(\frac{1}{4} + 0\right)\right]$ $= 25\left[\frac{1}{2} + \left(\frac{3}{8}\right)\ln 3\right]$ ----(2) Sub (2) into (1): Required Volume $= \pi\left(5^2\right)\left(\ln 3\right) - \pi\int_0^{\ln 3}\left(\frac{5e^x}{\sqrt{4e^x-3}}\right)^2 dx$ $= 25\pi\left(\ln 3\right) - 25\pi\left[\frac{1}{2} + \left(\frac{3}{8}\right)\ln 3\right]$ $= 25\pi\left(\ln 3\right) - 25\pi\left[\frac{1}{2} + \left(\frac{3}{8}\right)\ln 3\right]$ $= 25\pi\left[\ln 3 - \frac{1}{2} - \left(\frac{3}{8}\right)\ln 3\right]$ $= 25\pi\left[\left(\frac{5}{8}\right)\ln 3 - \frac{1}{2}\right]$ $= \frac{25\pi}{8}\left[5\ln 3 - 4\right]$ $\therefore a = 5,\ b = 4.$	$\pi\int_1^3 [5^2 - (\frac{5u}{\sqrt{4u-3}})^2]\ \frac{1}{u} du$ 4) Vol = $\pi\int_0^{\ln 3} (\frac{5e^x}{\sqrt{4e^x-3}})^2\ dx =$ $\pi\int_1^3 (\frac{5u}{\sqrt{4u-3}}.\frac{1}{u})^2\ du =$

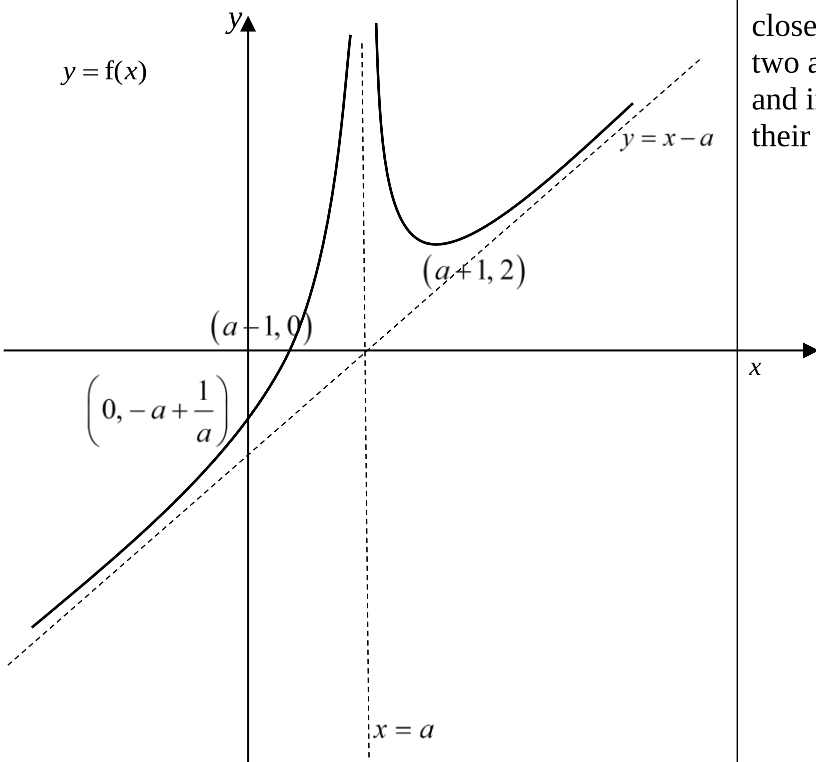
6(a)(i)	Since the coefficients of the polynomial equation are all real, $w=2+\mathrm{i}$ is also a root. $\left(w-(2-\mathrm{i})\right)\left(w-(2+\mathrm{i})\right)=(w-2)^2-\mathrm{i}^2=w^2-4w+5$ $aw^4-16w^3+21w^2-aw+5=0$ $\left(w^2-4w+5\right)\left(aw^2+bw+1\right)=0$ By comparing coefficient of w^2: $21=5a-4b+1$ $5a-4b=20$ By comparing coefficient of w : $-a=-4+5b$ $a+5b=4 \Rightarrow 5a+25b=20$ $\therefore b=0;\quad a=4$ $\left(w^2-4w+5\right)\left(4w^2+1\right)=0$ $w=2\pm\mathrm{i},\pm\frac{1}{2}\mathrm{i}$	Many students successfully stated $w=2+\mathrm{i}$ as a root. Well done! Common mistakes: (1) students did not realise the coeff. of $w^4=a$, hence factorized the polynomial wrongly (2) students used GC when question stated :"**Do not use a calculator in answering this question.**"
6(a)(ii)	$aw^4-16w^3+21w^2-aw+5=0$ $a\left(\frac{1}{w}\right)^4-16\left(\frac{1}{w}\right)^3+21\left(\frac{1}{w}\right)^2-a\left(\frac{1}{w}\right)+5=0$ $5w^4-aw^3+21w^2-16w+a=0$ Replace w by $\frac{1}{w}$, $\frac{1}{w}=2\pm\mathrm{i},\pm\frac{1}{2}\mathrm{i}$ $w=\frac{2}{5}\pm\frac{1}{5}\mathrm{i},\pm2\mathrm{i}$	Question stated "**Hence**", so students need to use replacement method. Some students simply used the GC to solve, hence zero marks awarded.
(b)	$z=\dfrac{\left(\cos\left(\frac{\pi}{3}\right)-\mathrm{i}\sin\left(\frac{\pi}{3}\right)\right)^4}{-k\left(\cos\left(\frac{\pi}{12}\right)+\mathrm{i}\sin\left(\frac{\pi}{12}\right)\right)}$ $=\dfrac{\left(\mathrm{e}^{-\mathrm{i}\frac{\pi}{3}}\right)^4}{k\mathrm{e}^{\mathrm{i}\pi}\left(\mathrm{e}^{\mathrm{i}\frac{\pi}{12}}\right)}$ $=\frac{1}{k}\mathrm{e}^{-\mathrm{i}\frac{29\pi}{12}}$ $\equiv\frac{1}{k}\mathrm{e}^{-\mathrm{i}\frac{5\pi}{12}}$	Most students can use the properties of modulus and argument well. Students must learn to apply $-1=\mathrm{e}^{\mathrm{i}\pi}$. Common mistakes: (1) $\lvert z\rvert=-\frac{1}{k}$ (2) make argument into principal range by $+\pi$ or $-\pi$, instead of $+2\pi$ or -2π.

	$\lvert z\rvert=\frac{1}{k}$ $\arg z=-\frac{5\pi}{12}$	
7(i)	$\frac{1}{(r-2)!}-\frac{2}{(r-1)!}+\frac{1}{r!}=\frac{r(r-1)-2r+1}{r!}$ $=\frac{r^2-3r+1}{r!}$ $\sum_{r=3}^{n}\frac{r^2-3r+1}{r!}=\sum_{r=3}^{n}\frac{1}{(r-2)!}-\frac{2}{(r-1)!}+\frac{1}{r!}$ $=\left[\begin{array}{l}\frac{1}{1!}-\frac{2}{2!}+\frac{1}{3!}\\+\frac{1}{2!}-\frac{2}{3!}+\frac{1}{4!}\\+\frac{1}{3!}-\frac{2}{4!}+\frac{1}{5!}\\\vdots\\+\frac{1}{(n-4)!}-\frac{2}{(n-3)!}+\frac{1}{(n-2)!}\\+\frac{1}{(n-3)!}-\frac{2}{(n-2)!}+\frac{1}{(n-1)!}\\+\frac{1}{(n-2)!}-\frac{2}{(n-1)!}+\frac{1}{n!}\end{array}\right]$ $=\frac{1}{1!}-\frac{2}{2!}+\frac{1}{2!}+\frac{1}{(n-1)!}-\frac{2}{(n-1)!}+\frac{1}{n!}$ $=\frac{1}{2}-\frac{1}{(n-1)!}+\frac{1}{n!}$	Prove from RHS is much easier. MOD generally well done. Students who are listed the first 3 and last 3 terms have higher success rate because cancellation pattern can be seen clearly.
7(ii)	$\sum_{r=3}^{5}\frac{r^2-3r+1}{r!}=\sum_{r=2}^{a+1}(2r-3)$ $\frac{7}{15}=\frac{a}{2}[1+2(a+1)-3]$ $=a^2$ $a=\sqrt{\frac{7}{15}}\ ,-\sqrt{\frac{7}{15}}$ (both rejected) Hence no solution since a is a positive integer value.	AP No. of terms $=a+1-2+1$ $=a$ $S_n=\frac{n}{2}[\text{first term}+\text{last term}]$ $=\frac{a}{2}[1+2(a+1)-3]$

		Common mistake: (1) Wrong sum of AP formula. (2) no of terms of AP calculated wrongly
7(iii)	As $n \to \infty, \frac{1}{(n-1)!} \to 0, \frac{1}{n!} \to 0,$ $\sum_{r=3}^{\infty} \frac{r^2-3r+1}{r!} = \frac{1}{2}$ $\sum_{r=7}^{\infty} \frac{r^2-r-1}{(r+1)!} = \sum_{r=8}^{\infty} \frac{r^2-3r+1}{r!}$ $= \sum_{r=3}^{\infty} \frac{r^2-3r+1}{r!} - \sum_{r=3}^{7} \frac{r^2-3r+1}{r!}$ $= \frac{1}{2} - \left(\frac{1}{2} - \frac{1}{6!} + \frac{1}{7!}\right)$ $= \frac{1}{840}$ or 0.00119	$\sum_{r=3}^{\infty} \frac{r^2-3r+1}{r!} = \frac{1}{2}$ Generally well done! Question stated "**Hence**", so students need to use replacement method of r by r-1. Otherwise, marks will be deducted.
8(a)	$x = \theta - \cos^2\theta$, $y = \theta - \sin\theta$ $\frac{dy}{dx} = \frac{1-\cos\theta}{1+2\cos\theta\sin\theta}$ For $0 \le \theta \le \pi$, $-1 \le \cos\theta \le 1$ and $-1 \le \sin 2\theta \le 1$. Thus $0 \le 1-\cos\theta \le 2$ and $0 \le 1+\sin 2\theta \le 2$ for $0 \le \theta \le \pi$. $\therefore \frac{dy}{dx} = \frac{1-\cos\theta}{1+\sin 2\theta} \ge 0$ for $0 \le \theta \le \pi$. Thus never negative.	Never negative implies "$\ge$", not "$>$". More detailed working needs to be presented for "shown" question.
8(b)	For tangents that are parallel to y – axis, $\frac{dy}{dx} \to \infty$. $1+\sin 2\theta = 0$ $\sin 2\theta = -1$ $2\theta = \frac{3\pi}{2}$ $\theta = \frac{3\pi}{4}$ since $0 \le \theta \le \pi$ When $\theta = \frac{3\pi}{4}$, equation of tangent is $x = \frac{3\pi}{4} - \frac{1}{2}$.	$\theta = \frac{3\pi}{4}$ since $0 \le \theta \le \pi$. Thus $\theta = -\frac{\pi}{4}$ is not acceptable. Equation of vertical lines are in the form of x = $\square$.

8(c)	$\frac{dy}{dx} = \frac{1-\cos\theta}{1+\sin 2\theta}$ $\approx \frac{1-\left(1-\frac{\theta^2}{2}\right)}{1+2\theta}$ $= \frac{\theta^2}{2}(1+2\theta)^{-1}$ $\approx \frac{\theta^2}{2}(1-2\theta)$ $= \frac{\theta^2}{2} - \theta^3$ $\approx \frac{\theta^2}{2}$ $a = 0,\ b = \frac{1}{2}$	Note that binomial expansion should be use.
8(d)	Solving $x = \theta - \cos^2\theta$, $y = \theta - \sin\theta$ and $y = x + \frac{1}{4}$: Thus $\theta - \sin\theta = \theta - \cos^2\theta + \frac{1}{4}$ $-\sin\theta = \sin^2\theta - 1 + \frac{1}{4}$ $\sin^2\theta + \sin\theta = \frac{3}{4}$ $\left(\sin\theta + \frac{1}{2}\right)^2 = 1$ $\sin\theta = -\frac{1}{2} \pm 1 = \frac{1}{2}$ or $-\frac{3}{2}$ (reject since $0 \le \theta \le \pi$) $\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ When $\theta = \frac{\pi}{6}$, $x = \frac{\pi}{6} - \cos^2\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \frac{3}{4}$ When $\theta = \frac{5\pi}{6}$, $x = \frac{5\pi}{6} - \cos^2\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} - \frac{3}{4}$	Do not attempt to convert to cartesian equation. For $\sin\theta = \frac{1}{2}$, $\theta = \frac{5\pi}{6}$ is missed out. Always solve a trigonometric equation by considering the 4 quadrants. Do not simply find the principal angle. Always check out for more solutions when solving for angles.

9(i)(a)	$\mathrm{f}(x)=(x-a)+\dfrac{1}{\lvert x-a\rvert}$, for $x\in\mathbb{R}, x\neq a$, if $x>a$ $y=\mathrm{f}(x)=(x-a)+\dfrac{1}{x-a}$ Let $\mathrm{f}'(x)=1-\dfrac{1}{(x-a)^2}=0$ $(x-a)^2=1$ $x=\pm1+a$ since $x>a$, $x=1+a$, (reject $-1+a$) When $x=1+a$, $y=2$ Turning point at $(1+a,\ 2)$	**<u>Common mistake</u>** Student differentiated the function f(x) directly. A handful of students gave two stationary points as their final answer.
9(i)(b)	$\mathrm{f}(x)=(x-a)+\dfrac{1}{\lvert x-a\rvert}$, for $x\in\mathbb{R}, x\neq a$, $\mathrm{f}(x)=(x-a)+\dfrac{1}{\lvert x-a\rvert}$, for $x\in\mathbb{R}, x\neq a$, if $x<a$ $y=\mathrm{f}(x)=(x-a)-\dfrac{1}{x-a}$ Let $\mathrm{f}'(x)=1+\dfrac{1}{(x-a)^2}=0$ $(x-a)^2=-1$ Since $(x-a)^2\geq0$, the equation has no real solutions. Hence there are no turning points.	Students are required to explain clearly that there are <u>no real solutions</u> for x and hence there are no stationary points.

9(ii)	 Asymptotes : $y = x - a$ and $x = a$ Intercepts: If $x < a$, $y = \mathrm{f}(x) = (x-a) - \frac{1}{x-a}$ when $x = 0$, $y = -a + \frac{1}{a}$ when $y = 0$, $(x-a) - \frac{1}{x-a} = 0$ $(x-a)^2 = 1$ $x = a - 1$, since $x < a$ If $x > a$, when $y = 0$, $(x-a) + \frac{1}{x-a} = 0$ $(x-a)^2 + 1 = 0$ (no solution)	Students need to be pay close attention to where the two asymptotes intersect and indicate accordingly in their sketch. Some students were unsure on how to find the axial intercepts and hence omitted them in their sketch.

9(iii)	$f(x)=(x-a)+\frac{1}{\lvert x-a\rvert} \rightarrow f(x+a)=x+\frac{1}{\lvert x\rvert}$ $\rightarrow f(2x+a)=2x+\frac{1}{\lvert 2x\rvert}$ $\rightarrow f(2x+a)-2a=2x-2a+\frac{1}{\lvert 2x\rvert}$ Sequence of transformations in the following order, 1. Translation by *a* units in the negative *x*-direction. 2. Scaling by factor $\frac{1}{2}$ parallel to the x-axis. 3. Translation by 2*a* units in the negative y-direction. **Alternative Method** 1. Scaling by factor $\frac{1}{2}$ parallel to the x-axis 2. Translation by $\frac{a}{2}$ units in the negative *x*-direction 3. Translation by 2a units in the negative y-direction.	Most students were able to determine the correct sequence in general but some were rather sloppy or vague in their descriptions. e.g. "Transform/Shift by *a* units…" "Scale by ½ units in the *x*-axis."
9(iv)	$g^{-1}(x)>g(x)$ To get points of intersection of $g^{-1}(x)$ and $g(x)$; solve $g(x)=x$ $x-a-\frac{1}{x-a}=x$ $-a=\frac{1}{x-a}$ $x-a=-\frac{1}{a}$ $x=a-\frac{1}{a}$	A lot of students tried to find $g^{-1}(x)$ first but were not successful in solving the inequality.

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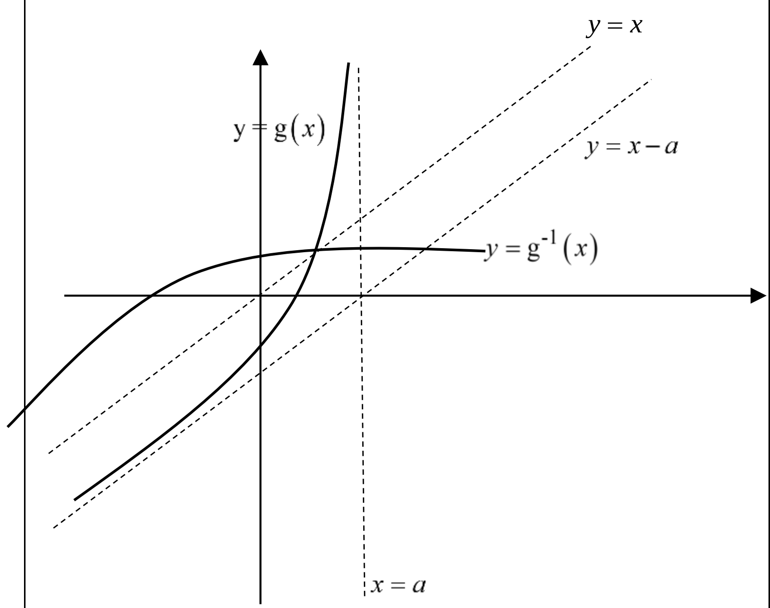
<tr><td></td><td>

By considering graph, $\mathrm{g}^{-1}(x) > \mathrm{g}(x)$ for $x < a - \frac{1}{a}$</td><td></td></tr>
<tr><td>10(i)</td><td>Rate in = $0.05(0.002)$
Rate out = $\frac{C}{500}$
$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.05(0.002) - \frac{C}{500}$
$= \frac{1}{10000} - \frac{C}{500}$</td><td>Students did not read the question carefully and mistaken the inflow rate as 0.002.</td></tr>
<tr><td>10(ii)</td><td>Since $C = \frac{V}{30}$
$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{1}{30}\frac{\mathrm{d}V}{\mathrm{d}t}$
$= \frac{1}{30}\left(\frac{1}{10000} - \frac{C}{500}\right)$
$= \frac{1}{15000}(0.05 - C)$</td><td>Most students were able to use implicit differentiation or chain rule to derive the differential equation.</td></tr>
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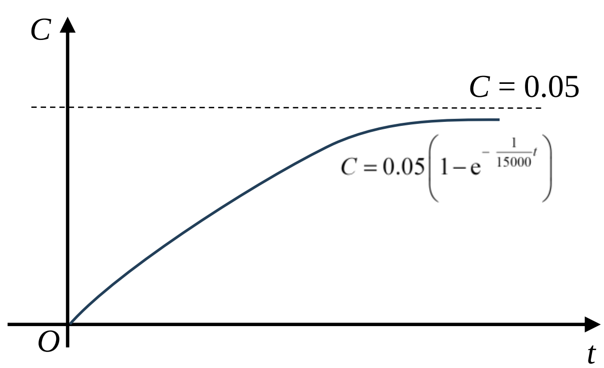
(iii)	$\frac{\mathrm{d}C}{\mathrm{d}t}=\frac{1}{15000}(0.05-C)$ $\int \frac{1}{0.05-C}\,\mathrm{d}\theta=\int \frac{1}{15000}\,\mathrm{d}t$ $-\ln\lvert 0.05-C\rvert=\frac{1}{15000}t+A$ $0.05-C=\pm \mathrm{e}^{-\frac{1}{15000}t-A}$ $=B\mathrm{e}^{-\frac{1}{15000}t}$, where $B=\pm \mathrm{e}^{-A}$ $C=0.05-B\mathrm{e}^{-\frac{1}{15000}t}$ When $t=0, C=0 \Rightarrow B=0.05$ $\therefore C=0.05-0.05\mathrm{e}^{-\frac{1}{15000}t}=0.05\left(1-\mathrm{e}^{-\frac{1}{15000}t}\right)$	**<u>Common mistake</u>** $\int \frac{1}{0.05-C}\,\mathrm{d}\theta$ $=\ln\lvert 0.05-C\rvert+D$ For ease of solving, students should solve for the arbitrary constant <u>only after</u> they have "removed" the modulus. (i.e. solve for B instead of A)
(iv)	When $t\to\infty, \mathrm{e}^{-\frac{1}{15000}t}\to 0, C\to 0.05$ The concentration of carbon monoxide **increases and approaches** to 5%.	Students need to explain <u>how</u> the concentration approaches to 5% (i.e. in an increasing manner) Saying that the concentration approaches/tends to 5% is not sufficient.
(v)		Note that the curve is not supposed to intersect the asymptote. The graph should only lie in the first quadrant based on the context of the question.
(vi)	When $V=0.0036, C=\frac{V}{30}=0.00012$ $\Rightarrow 0.00012=0.05\left(1-\mathrm{e}^{-\frac{1}{15000}t}\right)$ $\Rightarrow t=36.043=36$ minutes	Many students did not read the question carefully and mistaken the value of C as 0.00012.

11(i)	Let $\overrightarrow{OX}=\begin{pmatrix}5\\-5\\7\end{pmatrix}$ and $\overrightarrow{OY}=\begin{pmatrix}3\\-18\\10\end{pmatrix}$ $\overrightarrow{XY}=\overrightarrow{OY}-\overrightarrow{OX}=\begin{pmatrix}-2\\-13\\3\end{pmatrix}$ Distance between parallel lines AB and CD $=\left\|\dfrac{\overrightarrow{XY}\times\begin{pmatrix}-2\\2\\3\end{pmatrix}}{\sqrt{(-2)^2+(2)^2+(3)^2}}\right\|$ $=\dfrac{\left\|\begin{pmatrix}-2\\-13\\3\end{pmatrix}\times\begin{pmatrix}-2\\2\\3\end{pmatrix}\right\|}{\sqrt{17}}=\dfrac{\left\|\begin{pmatrix}-45\\0\\-30\end{pmatrix}\right\|}{\sqrt{17}}=\dfrac{\sqrt{(-45)^2+(-30)^2}}{\sqrt{17}}$ $=\dfrac{\sqrt{2925}}{\sqrt{17}}=15\sqrt{\dfrac{13}{17}}=13.1\text{ (to 3sf)}$	**Common mistake** Distance between parallel lines AB and CD = $\left\|\overrightarrow{XY}\times\begin{pmatrix}-2\\2\\3\end{pmatrix}\right\|$. Student should realise that they need to use a unit vector i.e divide by $\left\|\begin{pmatrix}-2\\2\\3\end{pmatrix}\right\|=\sqrt{2^2+2^2+3^2}=\sqrt{17}$
11(ii)	**Method 1** Let angle between line AB and normal of plane be θ $\cos\theta=\left\|\dfrac{\begin{pmatrix}-2\\2\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{(-2)^2+(2)^2+(3)^2}\sqrt{(1)^2+(-2)^2+(3)^2}}\right\|$ $=\left\|\dfrac{-2-4+9}{\sqrt{17}\sqrt{14}}\right\|=\dfrac{3}{\sqrt{17}\sqrt{14}}$ $\theta=78.787^0$ Length of projection of AB on plane =100 sin $78.787^0=98.1$units	**Common mistake for method 1** Length of projection = 100 cos 78.8 or 100 sin 11.21

	Alternative solution Let angle between line AB and the plane be θ $\sin\theta = \left\lvert \dfrac{\begin{pmatrix}-2\\2\\3\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{(-2)^2+(2)^2+(3)^2}\sqrt{(1)^2+(-2)^2+(3)^2}} \right\rvert$ $= \left\lvert \dfrac{-2-4+9}{\sqrt{17}\sqrt{14}} \right\rvert = \dfrac{3}{\sqrt{17}\sqrt{14}}$ $\theta = 11.213^0$ Length of projection of AB on plane $=100 \cos 11.213^0$ $= 98.1$ units	
	Method 2 $\left\lvert \begin{pmatrix}-2\\2\\3\end{pmatrix} \right\rvert = \sqrt{2^2+2^2+3^2} = \sqrt{17}$ $\overrightarrow{AB} = \pm\dfrac{100}{\sqrt{17}}\begin{pmatrix}-2\\2\\3\end{pmatrix}$	**Common mistake** $\overrightarrow{AB} = \begin{pmatrix}-2\\2\\3\end{pmatrix}$ $\left\lvert \begin{pmatrix}-2\\2\\3\end{pmatrix} \right\rvert = \sqrt{17}$ & $\lvert\overrightarrow{AB}\rvert = 100$ So $\overrightarrow{AB} \neq \begin{pmatrix}-2\\2\\3\end{pmatrix}$

	Length of projection of AB on the ski slope $\prod =$ $\left\lvert \dfrac{\overrightarrow{AB}\times\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{(1)^2+(-2)^2+(3)^2}} \right\rvert =$ $\dfrac{100}{\sqrt{17}}\left\lvert \dfrac{\begin{pmatrix}-2\\2\\3\end{pmatrix}\times\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{(1)^2+(-2)^2+(3)^2}} \right\rvert = \dfrac{100}{\sqrt{17}}\left\lvert \dfrac{\begin{pmatrix}12\\9\\2\end{pmatrix}}{\sqrt{14}} \right\rvert = 98.1$	
	Method 3 $\overrightarrow{AB} // \begin{pmatrix}-2\\2\\3\end{pmatrix} \Leftrightarrow \overrightarrow{AB} = \lambda\begin{pmatrix}-2\\2\\3\end{pmatrix}$ $\left\lvert\overrightarrow{AB}\right\rvert^2 = \lambda^2(2^2+2^2+3^2) = 100 \quad \therefore \lambda = \pm\dfrac{100}{\sqrt{17}}$. Hence $\overrightarrow{AB} = \pm\dfrac{100}{\sqrt{17}}\begin{pmatrix}-2\\2\\3\end{pmatrix}$ Length of projection of AB on the ski slope $\prod =$ $\left\lvert \dfrac{\overrightarrow{AB}\times\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{(1)^2+(-2)^2+(3)^2}} \right\rvert =$ $\dfrac{100}{\sqrt{17}}\left\lvert \dfrac{\begin{pmatrix}-2\\2\\3\end{pmatrix}\times\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{(1)^2+(-2)^2+(3)^2}} \right\rvert = \dfrac{100}{\sqrt{17}}\left\lvert \dfrac{\begin{pmatrix}12\\9\\2\end{pmatrix}}{\sqrt{14}} \right\rvert = 98.1$	**Should realise that** $\left\lvert\overrightarrow{AB}\right\rvert^2 \neq (5-2\lambda)^2+(-5+2\lambda)^2$ $+(7+3\lambda)^2 = 100$ Because $\overrightarrow{AB} \neq \begin{pmatrix}5-2\lambda\\-5+2\lambda\\7+3\lambda\end{pmatrix}$ Should realise that $\overrightarrow{OP} = \begin{pmatrix}5-2\lambda\\-5+2\lambda\\7+3\lambda\end{pmatrix}$ is the position vector of a point P on the line with respect to the origin and is not $\overrightarrow{AB}$

<table>
<tr>
<td>11(iii)</td>
<td>Since point P is on the line
Let $\overrightarrow{OP}=\begin{pmatrix}5-2\lambda\\-5+2\lambda\\7+3\lambda\end{pmatrix}$ and $\overrightarrow{OM}=\begin{pmatrix}5\\0\\0\end{pmatrix}$ is a point on the plane.
$\overrightarrow{MP}=\overrightarrow{OM}-\overrightarrow{OP}=\begin{pmatrix}-2\lambda\\-5+2\lambda\\7+3\lambda\end{pmatrix}$
Distance of P from plane
$=\left|\dfrac{\overrightarrow{MP}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{(1)^2+(-2)^2+(3)^2}}\right|=2\sqrt{14}$
$\left|\dfrac{\begin{pmatrix}-2\lambda\\-5+2\lambda\\7+3\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{14}}\right|=2\sqrt{14}$
$\left|-2\lambda-2(-5+2\lambda)+3(7+3\lambda)\right|=2(14)$
$3\lambda+31=\pm 28$
$\lambda=-1 \; or \; \lambda=-\dfrac{59}{3}=-19\dfrac{2}{3}(reject)$ since $-5<\lambda<15$
$\therefore$ Coordinates $of \;\; P \; (7,-7,4)$</td>
<td>Common mistake
1) When finding $\overrightarrow{OM}$, some students use any point on line or $\begin{pmatrix}5\\-5\\7\end{pmatrix}$

2) Did not consider the absolute sign when finding distance of P from plane. Thus
$\left(\dfrac{\begin{pmatrix}-2\lambda\\-5+2\lambda\\7+3\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}}{\sqrt{14}}\right)=2\sqrt{14}$
And
$-2\lambda-2(-5+2\lambda)+3(7+3\lambda)$
=2(14)
Thus ending with one answer for λ. Students need to remember to consider the $\left|\;\right|$ absolute sign.

Others:
1) Some wasted time finding point of intersection of line AB and plane $\prod$ and used it as point M on the plane</td>
</tr>
</table>

11(iv)	Vector perpendicular to plane $=\begin{pmatrix}1\\-2\\3\end{pmatrix}\times\begin{pmatrix}-2\\2\\3\end{pmatrix}=\begin{pmatrix}-12\\-9\\-2\end{pmatrix}=-\begin{pmatrix}12\\9\\2\end{pmatrix}$ Equation of window plane $\mathrm{r}\bullet\begin{pmatrix}12\\9\\2\end{pmatrix}=\begin{pmatrix}10\\10\\20\end{pmatrix}\bullet\begin{pmatrix}12\\9\\2\end{pmatrix}=250$ Cartesian Equation of plane is $12x+9y+2z=250$	**Common mistake** 1) Have problems finding the correct answer $\begin{pmatrix}1\\-2\\3\end{pmatrix}\times\begin{pmatrix}-2\\2\\3\end{pmatrix}$ 2) Left the answer in parametric form $r=\begin{pmatrix}10\\10\\20\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\3\end{pmatrix}+\mu\begin{pmatrix}-2\\2\\3\end{pmatrix}$ Or scalar product form
11(v)	Equation of line passing through Q and perpendicular to plane $\mathbf{r}=\begin{pmatrix}-7\\7\\25\end{pmatrix}+\mu\begin{pmatrix}1\\-2\\3\end{pmatrix}$ Point of intersection of this line and plane $\begin{pmatrix}-7+\mu\\7-2\mu\\25+3\mu\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}=5$ $(-7+\mu)-2(7-2\mu)+3(25+3\mu)=5$ $14\mu+54=5$ $\mu=-\frac{7}{2}=-3.5$ Foot of perpendicular is $\left(-\frac{21}{2},14,\frac{29}{2}\right)$ **Alternative Method** Let foot of perpendicular from Q to plane be F. M(5,0,0) is a point on the plane.	**Common mistake** 1) Foot of perpendicular is the point of intersection of $\mathbf{r}=\begin{pmatrix}-7\\7\\25\end{pmatrix}+\mu\begin{pmatrix}-2\\2\\3\end{pmatrix}$ and $\boldsymbol{r}\bullet\begin{pmatrix}1\\-2\\3\end{pmatrix}=5$ 2) Did not give answer in coordinate form 3) Alternative Method $\overrightarrow{\mathrm{QF}}=\left\lvert\overrightarrow{\mathrm{QM}}\bullet\hat{\mathbf{n}}\right\rvert\hat{\mathbf{n}}$ often seen

	Projected vector of $\overrightarrow{QM}$ onto the normal to plane is $\overrightarrow{QF}$ $\overrightarrow{QF} = \left(\overrightarrow{QM} \bullet \hat{\mathbf{n}}\right)\hat{\mathbf{n}}$ $= \left(\frac{\left(\begin{pmatrix} 12 \\ -7 \\ -25 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}\right)}{\sqrt{(-1)^2 + (-2)^2 + (3)^2}} \right) \frac{\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}}{\sqrt{(-1)^2 + (-2)^2 + (3)^2}}$ $= \frac{-7}{2}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ $\overrightarrow{OF} = \overrightarrow{QF} + \overrightarrow{OQ} = \frac{-7}{2}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} -7 \\ 7 \\ 25 \end{pmatrix} = \begin{pmatrix} -21/2 \\ 14 \\ 29/2 \end{pmatrix}$ Foot of perpendicular is $\left(-\frac{21}{2}, 14, \frac{29}{2}\right)$	

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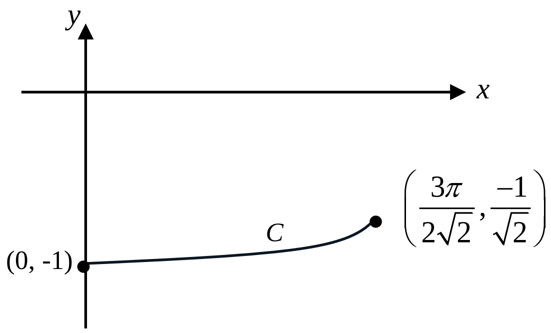
Qn	Solution	
1(a)	$y = x^{\cos 2x}$ $\ln y = (\cos 2x)\ln x$ $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos 2x}{x} + (-2\sin 2x)\ln x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = x^{\cos 2x}\left[\frac{\cos 2x}{x} - (2\sin 2x)\ln x\right]$ (shown)	Some used $y = e^{\cos 2x \ln x}$ to get to $\frac{dy}{dx} = e^{\cos 2x \ln x}\left[\cos 2x\left(\frac{1}{x}\right) + \ln x(-2\sin 2x)\right]$ Some students made the mistake of applying the formula for $y = a^{f(x)}$, not realising that x is a variable and not a constant.
1(b)	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = x^{\cos 2x}\left[\frac{\cos 2x}{x} - (2\sin 2x)\ln x\right] \times 8$ $= \pi^{\cos 2\pi}\left[\frac{\cos 2\pi}{\pi} - (2\sin 2\pi)\ln \pi\right] \times 8$ $= \pi\left[\frac{1}{\pi} - 0\right] \times 8$ $= 8$	Most students can apply this chain rule.
1(c)	Since $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\pi} = 1$, gradient of tangent is 1. Thus the angle that the tangent makes with the horizontal is $\frac{\pi}{4}$ or $45°$.	Some students concluded that gradient is equal to one, but did not know how to proceed.
2(a)	Vector perpendicular to π $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC}$ $= (\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})$ $= (\mathbf{b}\times\mathbf{c}) - (\mathbf{b}\times\mathbf{a}) - (\mathbf{a}\times\mathbf{c}) + (\mathbf{a}\times\mathbf{a})$ Since $-(\mathbf{b}\times\mathbf{a}) = \mathbf{a}\times\mathbf{b}, -(\mathbf{a}\times\mathbf{c}) = \mathbf{c}\times\mathbf{a}$ and $\mathbf{a}\times\mathbf{a} = 0$ $\therefore \mathbf{n} = \mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}$ (*proved*) **Alternative solution:** Vector perpendicular to π $\mathbf{n} = \overrightarrow{BA} \times \overrightarrow{BC}$ $= (\mathbf{a}-\mathbf{b})\times(\mathbf{c}-\mathbf{b})$ $= (\mathbf{a}\times\mathbf{c}) - (\mathbf{a}\times\mathbf{b}) - (\mathbf{b}\times\mathbf{c}) + (\mathbf{b}\times\mathbf{b})$ $= -(\mathbf{c}\times\mathbf{a}) - (\mathbf{a}\times\mathbf{b}) - (\mathbf{b}\times\mathbf{c})$ $= -(\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a})$ Since $\mathbf{a}\times\mathbf{c} = -(\mathbf{c}\times\mathbf{a})$ and $\mathbf{b}\times\mathbf{b} = 0$ $\therefore \mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}$ is a vector perpendicular to plane.(*proved*)	The points *A, B* and *C* lie on plane π. To find vector perpendicular to plane (ie normal of plane) we take the cross product of two vectors parallel to plane. Vectors parallel to plane can be $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (or vector joining any two points on plane) Note that **a**, **b** and **c** **need not be** vectors parallel to plane. i.e. if point A is on the plane, **vector a need not** be on the plane.

(b)	Equation of plane π : $\mathbf{r.n = a.n}$ $\mathbf{r}\bullet(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})$ $=\mathbf{a}\bullet(\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a})$ $=\mathbf{a}\bullet(\mathbf{a}\times\mathbf{b})+\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})+\mathbf{a}\bullet(\mathbf{c}\times\mathbf{a})$ $=\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})$ Since $\mathbf{a}\times\mathbf{b}$ is a vector perpendicular to $\mathbf{a}$, so $\mathbf{a}\bullet(\mathbf{a}\times\mathbf{b})=0$ $\mathbf{c}\times\mathbf{a}$ is a vector perpendicular to $\mathbf{a}$, so $\mathbf{a}\bullet(\mathbf{c}\times\mathbf{a})=0$	Since vector perpendicular to plane is already found in (a) we use the formula $\mathbf{r.n = a.n}$ to find equation of plane. Property of cross product: $\mathbf{a}\times\mathbf{b}$ is a **vector** which is perpendicular to both vectors **a** and **b**. Dot product of two perpendicular vectors is 0.
(c)	$\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}$ $=\mathbf{i}\times\mathbf{j}+\mathbf{j}\times\mathbf{k}+\mathbf{k}\times\mathbf{i}=\mathbf{k}+\mathbf{j}+\mathbf{i}=\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})=\mathbf{i}\bullet(\mathbf{j}\times\mathbf{k})=\mathbf{i}\bullet\mathbf{i}=\lvert\mathbf{i}\rvert^2=1$ Equation of plane π is $\mathbf{r}\bullet\begin{pmatrix}1\\1\\1\end{pmatrix}=1.$ Planes which are at a distance of 5 units from plane π are parallel to it. Let the equation of the planes be $\mathbf{r}.\begin{pmatrix}1\\1\\1\end{pmatrix}=d$ Distance between parallel planes $\frac{\lvert d-1\rvert}{\left\lvert\begin{pmatrix}1\\1\\1\end{pmatrix}\right\rvert}=\frac{\lvert d-1\rvert}{\sqrt{3}}=5$ $d-1=\pm5\sqrt{3}$ $d=1\pm5\sqrt{3}$ Catesian equations of planes are $x+y+z=1\pm5\sqrt{3}$ **Alternative Method** Planes which are at a distance of 5 units from plane π are parallel to it. Let the equation of the planes be $\mathbf{r}.\begin{pmatrix}1\\1\\1\end{pmatrix}=d$ Distance of parallel planes from O = $\frac{\lvert d\rvert}{\sqrt{3}}$	$\mathbf{i}\times\mathbf{j}=\begin{pmatrix}1\\0\\0\end{pmatrix}\times\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}=\mathbf{k}$ etc. The right-hand side of the equation should also be shown. Distance between two parallel planes $\mathbf{r.n}=d_1$ and $\mathbf{r.n}=d_2$ is $\frac{\lvert d_1-d_2\rvert}{\lvert\mathrm{n}\rvert}$.(from lecture notes) Note: To use this formula the equations of the two planes must first be expressed with the exact same normal vector $\mathbf{n}$. Distance of plane $\mathbf{r.n}=d$ from O is $\frac{\lvert d\rvert}{\lvert\mathbf{n}\rvert}$.

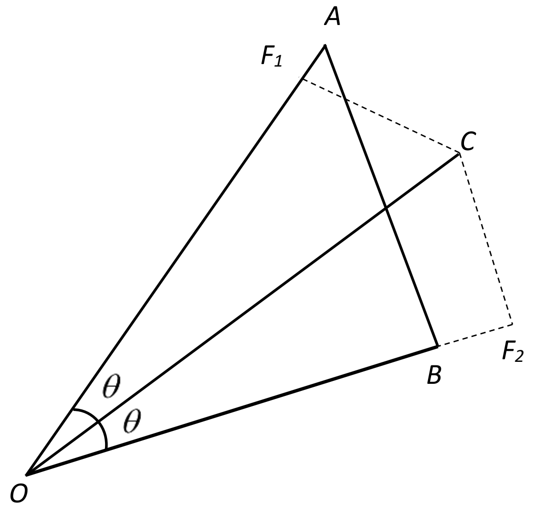
<table>
<tr><td></td><td>
Distance of π from O = $\frac{1}{\sqrt{3}}$
Distance between two planes
= $\frac{d}{\sqrt{3}} - \frac{1}{\sqrt{3}}$ =5 (if $d > 0$) or $= \frac{-d}{\sqrt{3}} + \frac{1}{\sqrt{3}}$ =5 (if $d < 0$)
$\therefore d = 1 + 5\sqrt{3}$ or $d = 1 - 5\sqrt{3}$
Catesian equations of planes are $x + y + z = 1 \pm 5\sqrt{3}$

Alternative Method
Find a point P on the plane which are at a distance of 5 units from plane π .
$\overrightarrow{OP} = k + 5\hat{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \pm \frac{5}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \pm\frac{5}{\sqrt{3}} \\ \pm\frac{5}{\sqrt{3}} \\ 1 \pm \frac{5}{\sqrt{3}} \end{pmatrix}$
Let the equation of the planes be
$\mathbf{r}.\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \pm\frac{5}{\sqrt{3}} \\ \pm\frac{5}{\sqrt{3}} \\ 1 \pm \frac{5}{\sqrt{3}} \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{r}.\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 + \frac{15}{\sqrt{3}} = 1 + 5\sqrt{3} \; or \; 1 - \frac{15}{\sqrt{3}} = 1 - 5\sqrt{3}$
Catesian equations of planes are $x + y + z = 1 \pm 5\sqrt{3}$
</td><td></td></tr>
<tr><td>3(a)</td><td>
Let $u = 2t$ Let $\frac{\mathrm{d}v}{\mathrm{d}t} = \cos^2 t$
$\frac{\mathrm{d}u}{\mathrm{d}x} = 2$ $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\cos 2t + 1}{2}$
$v = \frac{1}{2}\left(\frac{\sin 2t}{2} + t\right)$
</td><td>
Use LIATE to choose u.
i.e algebraic over trigo.

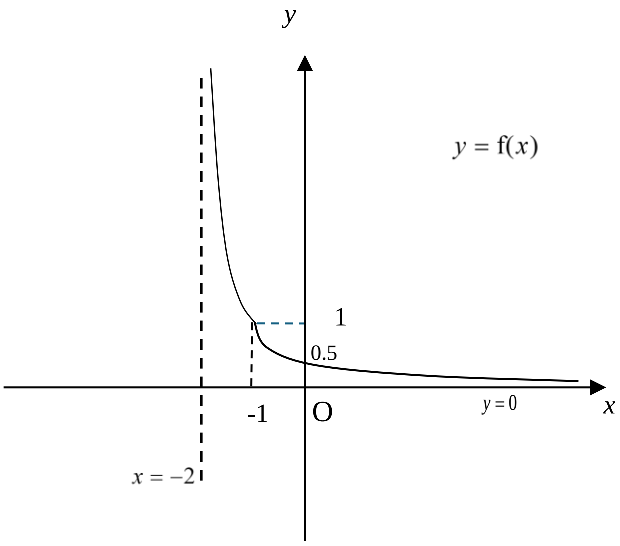
Recall the correct method to evaluate $\int \cos^2 t \; \mathrm{d}t$
i.e using double angle formula.
</td></tr>
</table>

	$\int 2t\cos^2 t\,\mathrm{d}t = (2t)\left(\frac{1}{2}\left(\frac{\sin 2t}{2}+t\right)\right)-\int \frac{1}{2}\left(\frac{\sin 2t}{2}+t\right)(2)\,\mathrm{d}t$ $= t\left(\frac{\sin 2t}{2}+t\right)-\int \frac{\sin 2t}{2}+t\,\mathrm{d}t$ $= \frac{t\sin 2t}{2}+t^2+\frac{\cos 2t}{4}-\frac{t^2}{2}+c$ $= \frac{t\sin 2t}{2}+\frac{t^2}{2}+\frac{\cos 2t}{4}+c$ $= \frac{1}{4}\left(2t\sin 2t+2t^2+\cos 2t\right)+c$ (shown)	Recall the correct integration by parts formula.
3(bi)	 when $t=\frac{3\pi}{4}$, $x=2\left(\frac{3\pi}{4}\right)\sin\left(\frac{3\pi}{4}\right)=\frac{3\pi}{2\sqrt{2}}$ when $t=\pi$, $y=\cos\left(\frac{3\pi}{4}\right)=\frac{-1}{\sqrt{2}}$	Note: It is necessary to copy the shape of the curve accurately from the GC. Curves which show y-coordinates of curve below $y=-1$ were penalised.
3(bii)	Required area $=-\int_0^{\frac{3\pi}{2\sqrt{2}}} y\,\mathrm{d}x$	Since the area required is below the x-axis it is necessary to include a negative sign when using the area under the curve formula. i.e $-\int_{x_1}^{x_2} y\,\mathrm{d}x$. Limits in this case are x-coordinates. Since y is in terms of t, it is necessary to express dx in terms of dt. The limits must also be converted accordingly, to become values of t when $x=0,\quad t=\pi$ (lower limit) $x=\frac{3\pi}{2\sqrt{2}}, t=\frac{3\pi}{4}$ (upper limit) as seen from graph.

	$=-\int_{\pi}^{\frac{3\pi}{4}}(\cos t)(2t\cos t+2\sin t)\,\mathrm{d}t$ $=-\int_{\pi}^{\frac{3\pi}{4}}2t\cos^2 t+2\sin t\cos t\,\mathrm{d}t$ $=\int_{\frac{3\pi}{4}}^{\pi}2t\cos^2 t+\sin 2t\mathrm{d}t$ $=\frac{1}{4}\left[2t\sin 2t+2t^2+\cos 2t\right]_{\frac{3\pi}{4}}^{\pi}+\left[-\frac{\cos 2t}{2}\right]_{\frac{3\pi}{4}}^{\pi}$ $=\frac{1}{4}\left[\left(0+2\pi^2+1\right)-\left(\frac{3\pi}{2}\sin\left(\frac{3\pi}{2}\right)+\frac{18\pi^2}{16}+\cos\left(\frac{3\pi}{2}\right)\right)\right]+\left[-\frac{1}{2}-0\right]$ $=\frac{1}{4}\left[2\pi^2+1+\frac{3\pi}{2}-\frac{9\pi^2}{8}-0\right]-\frac{1}{2}$ $=\frac{\pi^2}{2}-\frac{9\pi^2}{32}+\frac{3\pi}{8}-\frac{1}{4}$ $=\frac{7\pi^2}{32}+\frac{3\pi}{8}-\frac{1}{4}$ **Alternative method** (with respect to y -axis) $=\int_{-1}^{-\frac{1}{\sqrt{2}}}x\,\mathrm{d}y+\text{area of rectangle}$ $=\int_{\pi}^{\frac{3\pi}{4}}2t\sin t\,(-\sin t)\mathrm{d}t+\left(\frac{3\pi}{2\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $=\int_{\pi}^{\frac{3\pi}{4}}-2t\sin^2 t\,\mathrm{d}t+\frac{3\pi}{4}$ $=\int_{\frac{3\pi}{4}}^{\pi}2t\left(1-\cos^2 t\right)\mathrm{d}t+\frac{3\pi}{4}$ $=\int_{\frac{3\pi}{4}}^{\pi}2t-\int_{\frac{3\pi}{4}}^{\pi}2t\cos^2 t\,\mathrm{d}t+\frac{3\pi}{4}$ $=\left[t^2\right]_{\frac{3\pi}{4}}^{\pi}-\frac{1}{4}\left[2t\sin 2t+2t^2+\cos 2t\right]_{\frac{3\pi}{4}}^{\pi}+\frac{3\pi}{4}$ $=\frac{7\pi^2}{16}-\frac{1}{4}\left[2\pi^2+1+\frac{3\pi}{2}-\frac{9\pi^2}{32}\right]+\frac{3\pi}{4}$ Required area $=\frac{7\pi^2}{32}+\frac{3\pi}{8}-\frac{1}{4}$	When the limits are interchanged and the negative sign is removed. Remember to use the result from the part (a) above. Since the area required is on the right of the y-axis it is **not** necessary to include a negative sign when using the area under the curve formula. i.e $\int_{y_1}^{y_2}x\,\mathrm{d}y$. Limits in this case are y-coordinates. Since x is in terms of t, it is necessary to express dy in terms of dt. The limits must also be converted accordingly, to become values of t when $y=-1,\quad t=\pi$ (lower limit) $y=\frac{-1}{\sqrt{2}}, t=\frac{3\pi}{4}$ (upper limit) as seen from graph. When the limits are interchanged and the negative sign is removed. Remember to use the result from the part (a) above.
4(a)	Since the angle between **c** and **a** and angle between **c** and **b** is θ $\mathbf{c}\bullet\hat{\mathbf{a}}=\lvert\mathbf{c}\rvert\lvert\hat{\mathbf{a}}\rvert\cos\theta=\lvert\mathbf{c}\rvert\cos\theta=\lvert\mathbf{c}\rvert\lvert\hat{\mathbf{b}}\rvert\cos\theta=\mathbf{c}\bullet\hat{\mathbf{b}}$ $\therefore\mathbf{c}\bullet\hat{\mathbf{a}}=\mathbf{c}\bullet\hat{\mathbf{b}}$ OR	Basic formula for angle between two vectors is $\cos\theta=\frac{\mathbf{c}\bullet\mathbf{a}}{\lvert\mathbf{c}\rvert\lvert\mathbf{a}\rvert}$ *or* $\cos\theta=\frac{\mathbf{c}\bullet\hat{\mathbf{a}}}{\lvert\mathbf{c}\rvert\lvert\hat{\mathbf{a}}\rvert}$

	$\mathbf{c}\bullet\hat{\mathbf{a}} = \lvert\mathbf{c}\rvert\lvert\hat{\mathbf{a}}\rvert\cos\theta = \lvert\mathbf{c}\rvert\cos\theta$ since $\lvert\hat{\mathbf{a}}\rvert = 1$ $\mathbf{c}\bullet\hat{\mathbf{b}} = \lvert\mathbf{c}\rvert\lvert\hat{\mathbf{b}}\rvert\cos\theta = \lvert\mathbf{c}\rvert\cos\theta$ since $\lvert\hat{\mathbf{b}}\rvert = 1$ $\therefore \mathbf{c}\bullet\hat{\mathbf{a}} = \mathbf{c}\bullet\hat{\mathbf{b}}$ **Alternative method** $\mathbf{c}\bullet\hat{\mathbf{a}} =$ length of projection of **c** on **a** $\mathbf{c}\bullet\hat{\mathbf{b}} =$ length of projection of **c** on **b** Since the triangles OCF_1 and OCF_2 are congruent, $OF_1 = OF_2$ 	Also $\lvert\hat{\mathbf{a}}\rvert = 1$ $\hat{\mathbf{a}} = \frac{\mathbf{a}}{\lvert\mathbf{a}\rvert}$ or $\mathbf{a} = \lvert\mathbf{a}\rvert\hat{\mathbf{a}}$ are results that may be used.
4(b)	$\mathbf{c} = m\hat{\mathbf{a}} + n\hat{\mathbf{b}}$ $(m\hat{\mathbf{a}} + n\hat{\mathbf{b}})\bullet\hat{\mathbf{a}} = (m\hat{\mathbf{a}} + n\hat{\mathbf{b}})\bullet\hat{\mathbf{b}}$ $m\hat{\mathbf{a}}\bullet\hat{\mathbf{a}} + n\hat{\mathbf{b}}\bullet\hat{\mathbf{a}} = m\hat{\mathbf{a}}\bullet\hat{\mathbf{b}} + n\hat{\mathbf{b}}\bullet\hat{\mathbf{b}}$ $m\lvert\hat{\mathbf{a}}\rvert^2 + n\hat{\mathbf{b}}\bullet\hat{\mathbf{a}} = m\hat{\mathbf{a}}\bullet\hat{\mathbf{b}} + n\lvert\hat{\mathbf{b}}\rvert^2$ $m - n = m\hat{\mathbf{a}}\bullet\hat{\mathbf{b}} - n\hat{\mathbf{b}}\bullet\hat{\mathbf{a}}$ since $\lvert\hat{\mathbf{a}}\rvert = \lvert\hat{\mathbf{b}}\rvert = 1$ $m - n = (m-n)\hat{\mathbf{a}}\bullet\hat{\mathbf{b}}$ $(m-n)(1 - \hat{\mathbf{a}}\bullet\hat{\mathbf{b}}) = 0$ $\Rightarrow m - n = 0$ *or* $1 - \hat{\mathbf{a}}\bullet\hat{\mathbf{b}} = 0$ $\therefore m = n$ *or* $\hat{\mathbf{a}}\bullet\hat{\mathbf{b}} = 1$ (*shown*) $\lvert\hat{\mathbf{a}}\rvert\lvert\hat{\mathbf{b}}\rvert\cos 2\theta = 1$ $\cos 2\theta = 1$ $\theta = 0^0 (reject)$	It is necessary to follow the instruction given in the question, mainly to use the result from (a). Note that $\hat{\mathbf{a}}\bullet\hat{\mathbf{a}} = \lvert\hat{\mathbf{a}}\rvert^2 = 1$ It is wrong to compare coefficients to deduce that $m = n$. It is necessary to show that $(1 - \hat{\mathbf{a}}\bullet\hat{\mathbf{b}})$ is not 0.
4(c)	Equation of line through *A* and *B* $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$	Equation of line is $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ **NOT** $l = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$

4(d)	Equation of line passing through O and C $\mathbf{r} = m\hat{\mathbf{a}} + m\hat{\mathbf{b}} = m\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right)$ Equate equations of lines to find point of intersection, $\mathbf{a} + \lambda\left(\mathbf{b} - \mathbf{a}\right) = m\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right)$ $(1-\lambda)\mathbf{a} + \lambda\mathbf{b} = m\left(\frac{\mathbf{a}}{\lvert\mathbf{a}\rvert} + \frac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right)$ $= m\left(\frac{\mathbf{a}}{3} + \frac{\mathbf{b}}{2}\right)$ Since $\mathbf{a}$ and $\mathbf{b}$ are not parallel , $1 - \lambda = \frac{m}{3}, \quad \lambda = \frac{m}{2}$ $\Rightarrow m = \frac{6}{5}$ $\therefore \mathbf{c} = m\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right) = \frac{6}{5}\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right), \quad \therefore t = \frac{6}{5}$ **Alternative solution for last part** Equate equations of lines to find point of intersection, $\mathbf{a} + \lambda\left(\mathbf{b} - \mathbf{a}\right) = m\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right)$ Since $\mathbf{a} = 3\hat{\mathbf{a}}$ and $\mathbf{b} = 2\hat{\mathbf{b}}$ $(1-\lambda)\left(3\hat{\mathbf{a}}\right) + \lambda\left(2\hat{\mathbf{b}}\right) = m\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right)$ Since $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are not parallel, $3 - 3\lambda = m, \quad 2\lambda = m$ $\Rightarrow m = \frac{6}{5}$ $\therefore \mathbf{c} = m\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right) = \frac{6}{5}\left(\hat{\mathbf{a}} + \hat{\mathbf{b}}\right), \quad \therefore t = \frac{6}{5}$	Method to find point of intersection of two lines. -Equate the vector equation of the two lines. - Equate the 'coefficients' of the non parallel vectors $\mathbf{a}$ and $\mathbf{b}$ and solve for λ and m. Use $\hat{\mathbf{a}} = \frac{\mathbf{a}}{\lvert\mathbf{a}\rvert}$ to write $\hat{\mathbf{a}}$ in terms of $\mathbf{a}$.
5(a)	Greatest possible value of $k = -1$.	For h^{-1} to exist, h needs to be a one-one function. The greatest value of k can be found easily by identifying the minimum point of the graph of $y = \mathrm{f}(x)$ using a GC. There is no need to find the minimum point by differentiation.

<table>
<tr><td>5(b)</td><td>
</td><td>Students need to remember to include and label the equations of the asymptotes in their sketch.

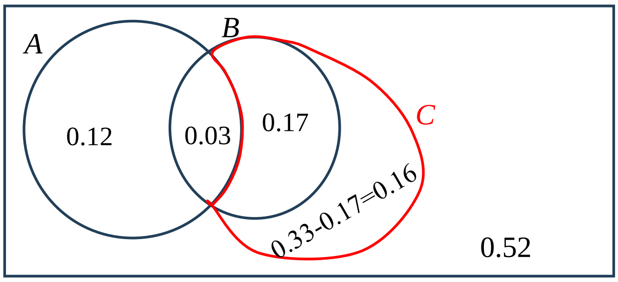
A significant number of students did not realise the presence of a vertical asymptote.</td></tr>
<tr><td>5(c)</td><td>$\mathrm{f}(x)=\begin{cases}(\ln(x+2))^2+1, & \text{for } x\in\mathbb{R}, -2<x\le -1,\\ \dfrac{1}{x+2}, & \text{for } x\in\mathbb{R},\ x>-1.\end{cases}$

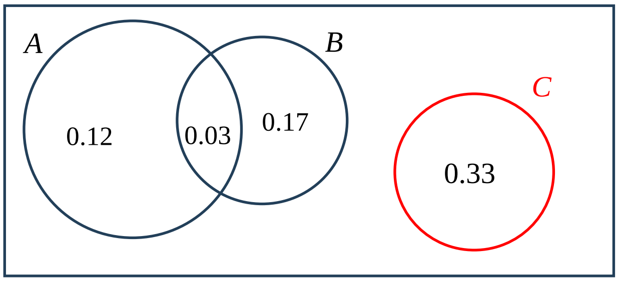
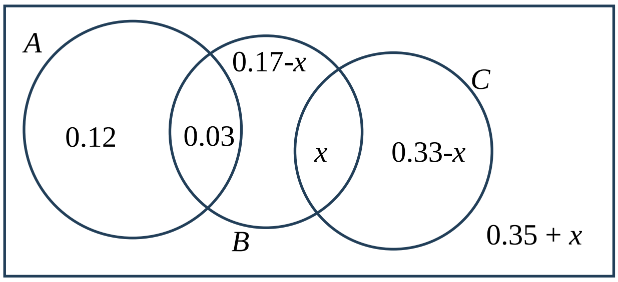
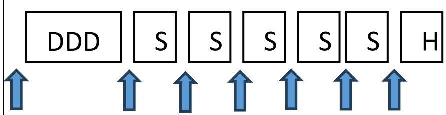
For $-2<x\le -1$
$y=(\ln(x+2))^2+1$
$\pm\sqrt{y-1}=\ln(x+2)$
since $-2<x\le -1 \Rightarrow 0<x+2\le 1 \Rightarrow \ln(x+2)\le 0$
$\therefore -\sqrt{y-1}=\ln(x+2)$
$x+2=e^{-\sqrt{y-1}}$
$x=-2+e^{-\sqrt{y-1}}$

For $x>-1$
$y=\dfrac{1}{x+2}$
$x=-2+\dfrac{1}{y}$
$D_{f^{-1}}=R_f$
$\therefore \mathrm{f}^{-1}(x)=\begin{cases}-2+e^{-\sqrt{x-1}}, & \text{for } x\ge 1,\\ -2+\dfrac{1}{x}, & \text{for } 0<x<1\end{cases}$</td><td>Many students did not know the reason why $\sqrt{y-1}=\ln(x+2)$ needs to be rejected.

A small number of students did not express f^{-1} in terms of x. A significant number of students have difficulty finding the domains of f^{-1}. There were also some <u>inappropriate</u> use of notations such as $1\le x\le\infty$.</td></tr>
</table>

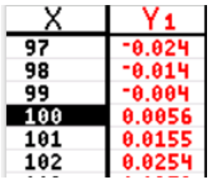
5(d)	For f^2 to exist $R_f \subseteq D_f$ $R_f = (0,\infty)$ $D_f = (-2,\infty)$.Hence,$R_f \subseteq D_f$. $\therefore f^2$ exists $R_{f^2} = \left(0,\frac{1}{2}\right)$	Students needs to be show clearly why $R_f \subseteq D_f$. A significant number of students have difficulty finding R_{f^2} correctly. Some incorrectly wrote $R_{f^2} = \left(\frac{1}{2},0\right)$
5(e)	$f^2(2) = f(x)$ $f^{-1}f^2(2) = f^{-1}f(x)$ $f(2) = x$ $\frac{1}{2+2} = x$ $x = \frac{1}{4}$ **Alternative Method** $f^2(2) = f(x)$ $ff(2) = f\left(\frac{1}{2+2}\right) = f\left(\frac{1}{4}\right) = \frac{1}{\frac{1}{4}+2} = \frac{4}{9} = f(x)$ $f(x) = \frac{1}{x+2} = \frac{4}{9}$ $x = \frac{1}{4}$	This part was well attempted by many students.
6(a)	$P(A' \cap B') = 1 - P(A \cup B)$ $= 1 - \left[P(A) + P(B) - P(A \cap B)\right]$ $= 1 - P(A) - P(B) + P(A)P(B)$ $= 1 - a - b + ab$ $= (1-a) - b(1-a)$ $= (1-a)(1-b)$ $= P(A')P(B')$ Since $P(A' \cap B') = P(A')P(B')$, hence A' and B' are independent events.	This part was well attempted by many students. However some students were not clear in the presentation of their steps to prove that $P(A' \cap B') == P(A')P(B')$. It is not sufficient to state that since *A* and *B* are independent events and hence *A'* and *B'* are independent events.
6(b)	Since *A* and *B* are independent events and *A'* and *B'* are independent events, $P(A' \mid B') = P(A') = 0.85$ $P(A) = 1 - P(A') = 0.15$ **Method 1** $P(A \cap B') = P(A) \times P(B')$ $= 0.15 \times 0.8$ $= 0.12$	This part was generally well attempted by many students with a variety of different methods observed. However some students did not know that $P(A' \mid B') = P(A')$ which resulted in a tedious process to find $P(A)$.

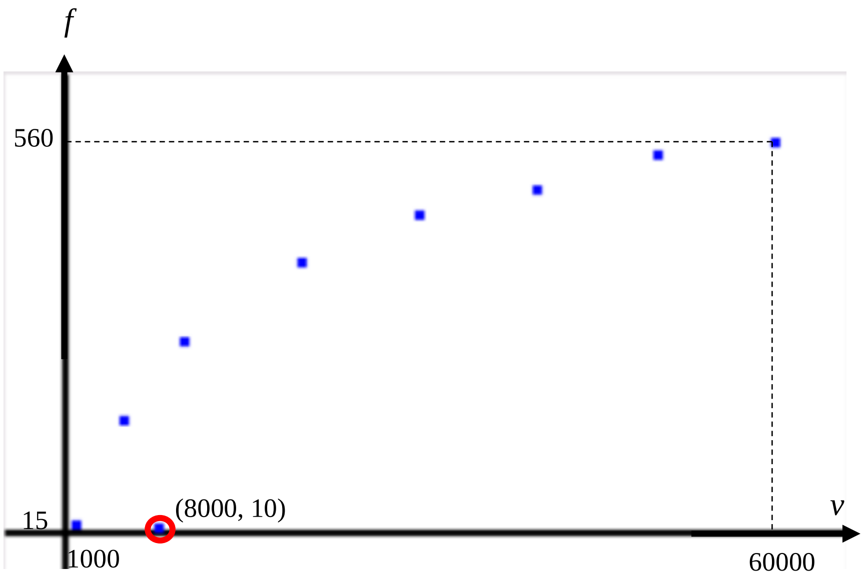
	Method 2 Since A' and B' are independent events, $\mathrm{P}(A'\mid B')=\mathrm{P}(A')=0.85$. $\mathrm{P}(A\cap B')=\mathrm{P}(A)-\mathrm{P}(A\cap B)$ $=\mathrm{P}(A)-\mathrm{P}(A)\times\mathrm{P}(B)$ $=(1-0.85)-(1-0.85)(1-0.8)$ $=0.15-(0.15)(0.2)$ $=0.12$ **Method 3** $\mathrm{P}(A\cap B')=1-\mathrm{P}(A'\cap B')-\mathrm{P}(B)$ $=1-[\mathrm{P}(A')\times\mathrm{P}(B')]-\mathrm{P}(B)$ $=1-[0.85\times0.8]-0.2$ $=1-0.68-0.2$ $=0.12$	
6(c)	$\mathrm{P}(A'\cap C')=1-\mathrm{P}(A\cup C)$ $=1-[\mathrm{P}(A)+\mathrm{P}(C)]$ $=1-0.15-\mathrm{P}(C)$ $=0.85-\mathrm{P}(C)$ Since $\mathrm{P}(A'\cap C')=0.52$, $0.52=0.85-\mathrm{P}(C)$ $\mathrm{P}(C)=0.33$	Some students did not understand what mutually exclusive events means and hence did not know that since $\mathrm{P}(A\cap C)=0$ hence $\mathrm{P}(A\cup C)=\mathrm{P}(A)+\mathrm{P}(C)$.
6(d)	**Method 1** When $\mathrm{P}(A'\cap B'\cap C')$ is maximum,  Maximum $\mathrm{P}(A'\cap B'\cap C')=1-0.15-0.17-0.16=0.52$	Students who tried solving using method 1 generally found more success than students who solve using method 2. For both methods, students generally have more difficulty finding the maximum $\mathrm{P}(A'\cap B'\cap C')$ than finding the minimum $\mathrm{P}(A'\cap B'\cap C')$. When finding the minimum $\mathrm{P}(A'\cap B'\cap C')$, a significant number of students failed to observe that $\mathrm{P}(C)>\mathrm{P}(B)$ and ended up drawing an incorrect Venn diagram which shows that event C being a subset of event B.

	When $\mathrm{P}(A' \cap B' \cap C')$ is minimum,  Minimum $\mathrm{P}(A' \cap B' \cap C') = 1 - 0.15 - 0.17 - 0.33 = 0.35$ Hence, $0.35 \le \mathrm{P}(A' \cap B' \cap C') \le 0.52$. **Method 2** Let $\mathrm{P}(B \cap C) = x$  Consider: $0 \le 0.17 - x \le 1$ $-1 \le x - 0.17 \le 0$ $-0.83 \le x \le 0.17$ $0 \le x \le 0.17$ and Consider: $0 \le 0.33 - x \le 1$ $-1 \le x - 0.33 \le 0$ $-0.67 \le x \le 0.33$ $0 \le x \le 0.33$ and Consider: $0 \le 0.35 + x \le 1$ $-0.35 \le x \le 0.65$ $0 \le x \le 0.65$ Combing results, $0 \le x \le 0.17$ Hence, $0.35 \le x + 0.35 \le 0.17 + 0.35$ $0.35 \le \mathrm{P}(A' \cap B' \cap C') \le 0.52$	There are students who did not understand what are mutually exclusive events and hence did not know that since $\mathrm{P}(A \cap C) = 0$. This resulted in an incorrect Venn diagram being drawn whereby event *A* and *C* intersects. For students who tried solving using method 2, some did not consider all cases and hence did not get the correct answer.
7(a)	**Method 1**  Number of ways $= 6! \times 3! \times {}^{7}C_{3} \times 3! = 907200$ Note: 6! ways to arrange 6 blocks consisting of one block of 3 back-to-back surprise duets and 5 other songs. 3! ways of to arrange within a block of of 3 back-to-back surprise duets.	Most students tried solving using method 1 with many students achieving success. Some students who tried the complement method which is not recommended for this part of the question. Few found success with many students either missing out on the complement Case B or had

	${}^{7}C_{3} \times 3!$: Out of the 7 available positions, choose 3 positions to slot in the recorded songs such that are they are all separated from each other by at least one song, with the hit song being the last song. Then within the 3 chosen positions, there are 3! ways to arrange the 3 pre-recorded songs among the 3 chosen positions. **Method 2** (Complement Method) Complement Case A: All three pre-recorded songs are back-to-back Number of ways $= 7! \times 3! \times 3! = 181440$ Complement Case B: Two of the three pre-recorded songs are back-to-back. Number of ways $= 6! \times 3! \times {}^{7}C_{2} \times 2! \times {}^{3}C_{2} \times 2! = 1088640$ Number of ways without restrictions $= 9! \times 3! = 2177280$ Required number of ways $= 2177280 - 181440 - 1088640 = 907200$ Note: 6! ways to arrange 6 blocks consisting of one block of 3 back-to-back surprise duets and 5 other songs. 3! ways of to arrange within a block of 3 back-to-back surprise duets. ${}^{7}C_{2} \times 2!$: Out of the 7 available positions, choose 2 positions to slot in a block of 2 back-to-back pre-recorded songs and a separate block of one pre-recorded song, with the hit song being the last song. Then within the 2 chosen positions, there are 2! ways to arrange the block of 2 back-to-back pre-recorded songs and a separate block of one pre-recorded song. ${}^{3}C_{2} \times 2!$: Out of the 3 pre-recorded songs, choose 2 pre-recorded songs to be back-to-back and then within the 2 chosen songs, they are 2! ways to permutate among themselves.	difficulty finding the number of ways for complement Case B.
7(b)	**Method 1** | | 1st Country | 2nd Country | 3rd Country | | Case 1 | 2 dancers | 4 dancers | 4 dancers | | Case 2 | 2 dancers | 3 dancers | 5 dancers | | Case 3 | 3 dancers | 3 dancers | 4 dancers | Case 1: ${}^{5}C_{2} \times {}^{5}C_{4} \times {}^{5}C_{4} \times \frac{3!}{2!} = 10 \times 5 \times 5 \times 3 = 750$ Case 2: ${}^{5}C_{2} \times {}^{5}C_{3} \times {}^{5}C_{5} \times 3! = 10 \times 10 \times 1 \times 6 = 600$ Case 3: ${}^{5}C_{3} \times {}^{5}C_{3} \times {}^{5}C_{4} \times \frac{3!}{2!} = 10 \times 10 \times 5 \times 3 = 1500$ Total no. of ways $= {}^{6}C_{3} \times [750 + 600 + 1500] = 20 \times 2850 = 57000$.	Generally well attempted by students who did method 1 with many students being able to consider the three main cases. The most common mistakes made by students involve omitting $\frac{3!}{2!}$ or 3! in their workings. Another common mistake involves students writing ${}^{6}C_{1}\,{}^{5}C_{1}\,{}^{4}C_{1}$ instead of ${}^{6}C_{3}$. These students did not realise that ${}^{6}C_{1}\,{}^{5}C_{1}\,{}^{4}C_{1}$ will result in <u>repeated combinations</u> of 3 countries chosen from the 6 countries.

	Method 2 (Complement) Complement / 1st Country / 2nd Country / 3rd Country Case 1: 5 dancers / 4 dancers / 1 dancers Case 2: 5 dancers / 5 dancers / 0 dancers Case 1: ${}^5C_5 \times {}^5C_4 \times {}^5C_1 \times 3! = 1\times 5\times 5\times 3! = 150$ Case 2: ${}^5C_5 \times {}^5C_5 \times {}^5C_0 \times \frac{3!}{2!} = 1\times 1\times 1\times 3 = 3$ Total no. of ways $= {}^6C_3 \times \left[{}^{15}C_{10} - 150 - 3\right] = 20\times 2850 = 57000.$	Students who did the complement method were generally successful and those who did not get the correct answer made similar mistakes mentioned in method 1.
7(c)	Total no. of ways $= \left[{}^{10}C_5 \times 5!\right]\times\left[{}^5C_5 \times (5-1)!\right] = [252\times 120]\times 24 = 725760.$	This part is not as well attempted as the earlier two parts. Common mistakes include: $\left[{}^{10}C_5 \times 5!\right] + \left[{}^5C_5 \times (5-1)!\right]$ ${}^{10}C_5 \times 5! \times 5!$ ${}^{10}C_5 \times 4!$
8(a)	Unbiased estimate of the population variance $s^2 = \frac{1}{45-1}\left(17.08 - \frac{(-4.3)^2}{45}\right) = 0.378843 = 0.379 \text{ (3 sf)}$ $\bar{x} = \frac{-4.3}{45} + 12 = 11.904 = 11.9 \text{ (3 sf)}$	Most students can recall and apply the correct formulas.
8(b)	To test $H_0 : \mu = 12$ Against $H_1 : \mu < 12$ at 5% sig level where μ represents the population mean lifespan of a refrigerator Under H_0, $\bar{X} \sim N\left(12, \frac{0.378845}{45}\right)$ approx. by Central Limit Theorem since n = 45 is large Value of test statistic, $z = \frac{11.90444 - 12}{\sqrt{\frac{0.378843}{45}}} = -1.04 \text{ (3 sf)}$ $p -$ value $= 0.14883 > 0.05$ $\therefore$ Do not reject H_0. There is insufficient evidence at 5% sig level, to conclude that the mean lifespan of the refrigerator is less than 12 years.	Some students do not know how to properly define μ. Some students made the mistake of putting the sample mean value 11.9 in the distribution. Should be $\bar{X} \sim N\left(12, \frac{0.378845}{45}\right)$. Some students got up to the correct p-value but concluded wrongly. Some students left out the 5% sig. level at the conclusion part.
8(c)	Unbiased estimate for population variance = $\frac{n}{n-1}(4.1)$ To test $H_0 : \mu = 12$ Against $H_1 : \mu \neq 12$ at 5% level of significance	Some students did not realise that this is a sample variance and need to find the unbiased estimate for the population variance.

	Under H_0, $\bar{Y} \sim N\left(12, \frac{4.1}{n-1}\right)$ approx. by Central Limit Theorem, since n is large Value of test statistic, $z = \frac{12.4-12}{\sqrt{\frac{4.1}{n-1}}}$ In order to reject H_0 $\frac{12.4-12}{\sqrt{\frac{4.1}{n-1}}} > 1.95996$ or $\frac{12.4-12}{\sqrt{\frac{4.1}{n-1}}} < -1.95996$ (reject) $\sqrt{\frac{4.1}{n-1}} < \frac{0.4}{1.95996}$ $n > 99.4 (3sf)$ or $n \geq 100$ <u>GC method</u>: $\frac{12.4-12}{\sqrt{\frac{4.1}{n-1}}} - 1.95996 > 0$ Let $y = \frac{12.4-12}{\sqrt{\frac{4.1}{n-1}}} - 1.95996$  When n = 99, $y = -0.004$ When n = 100, $y = 0.0056 > 0$ Therefore $n \geq 100$, where $n \in \mathbb{Z}$.	Some students made the mistake of putting the sample mean value 12.4 in the distribution. Should be $\bar{Y} \sim N\left(12, \frac{4.1}{n-1}\right)$. Some forgot to quote Central limit Theorem. Many students were able to get the z-values ± 1.95996. To reject H_0, the test-stats should be at the tail-ends i.e. $\frac{12.4-12}{\sqrt{\frac{4.1}{n-1}}} > 1.9599$ or $\frac{12.4-12}{\sqrt{\frac{4.1}{n-1}}} < -1.9599$
9(a)	Let M denote the amount of time taken by a male runner to complete a run for the training programme $P(M > 180) = 0.74751$ $= 0.748$ (3 sf) Expected number = 0.74751×80 $= 59.8$ (3 sf)	Some students could not understand the question and did find $P(M > 180)$. Many did not realise that E(X) is a statistical value and should be rounded off to 3sf and not a whole number.
9(b)	$P(M \leq a) \leq 0.1$ $a \leq 165.24$ $= 165$ (nearest minute)	Most students can do (b).
9(c)	Let F denote the amount of time taken by a female runner to complete a run for the training programme $M_1 + F_1 + F_2 + F_3 \sim N\left(196 + 210 \times 3, 24^2 + 30^2 + 30^2 + 30^2\right)$ $M_1 + F_1 + F_2 + F_3 \sim N(826, 3276)$	Most students can do (c). A mistake is at the variance of the $F_1 + F_2 + F_3$ which should be 3×30^2 and not $3^2 \times 30^2$

	$P(700 < M_1 + F_1 + F_2 + F_3 < 800) = 0.31097$ $= 0.311$ (3 sf)	
9(d)	Let $T = 0.94(F_1 + F_2) - 1.9M$ $E(T) = 0.94(210 + 210) - 1.9(196) = 22.4$ $\text{Var}(T) = 0.94^2(30^2 + 30^2) + 1.9^2(24^2) = 3669.8$ $P(\|T\| < 17)$ $= P(-17 < T < 17)$ $= 0.207$ (3 sf)	Some student mis-read the question and left out the modulus part. Some students misread and did 0.05 of M or 0.06 of F instead of 0.95 of M and 0.94 of F. Some students factored in the discount for the Mean and forgot to do that for the Variance.
10(a)		Most students can do (a), with a few missing out the instructions to circle the outlier. While the scatter plot did not state the axes, students should identify the independent/dependent variable based on the context.
10(b)	The influencer should observe from the scatter diagram which a curvilinear (non-linear) relationship between *f* and *v*. As *v* increases, *f* increases at a decreasing rate (*f* increases by decreasing amounts).	Most students can do (b), where they should state the trend of *f* and *v*. A portion of the students calculated the *r* value even though question stated to use the scatter diagram.

10(c)	For $f = a + bv$: $r = 0.92312 = 0.923$(3 s.f.) For $f = a + b\ln v$: $r = 0.99271 = 0.993$(3 s.f.) Since the scatter diagram shows a curvilinear (non-linear) relationship between *f* and *v*, hence $f = a + bv$ will not be a suitable model. Furthermore, the product moment correlation coefficient of the model $f = a + b\ln v$ is closer to one, hence, $f = a + b\ln v$ is a better model. $f = -976.6912461 + 138.5796656\ln v$ $f = -976.691 + 138.580\ln v$ (3 d.p)	A portion of students did not follow the instructions stated to omit the circled data in the passage, thus obtaining wrong *r* values. Students should also take note of the correct phrasing, to state that the *r* value is closer to 1 or –1 when comparing values instead of higher/stronger/weaker.
10(d)	When $v = 100000$, $f = -976.691 + 138.580\ln(100000)$ =619 (3s.f) This estimate is not reliable as $v = 100000$ does not lie within the *v* data range $(1000 \le v \le 60000)$. Hence, there is extrapolation when calculating *f*.	Most students can do (d), students are reminded to state the range of values.
10(e)	This is because the distances (residuals) which are used could either be positive or negative and summing them up might cause the values to cancel out. Hence the values need to be squared.	Some students indicated that distances cannot be negative, while not taking note that differences in distance can be negative. Students need to clear state what will happen instead of saying there will be a different impact.
10(f)	All the data points of this male influencer lie on the least squares regression line.	Students linked the value to either the *r* value or gradient. Students should also answer the question directly by describing the data points and not the variables.
11(ai)	The probability of a candy bar containing a lucky draw ticket is **constant** for all candy bars. The event that a candy bar containing a lucky draw ticket is independent of another candy bar.	Students should take note of the phrasing that they used and not be confused by the definition between random (equal probability for any trial) and probability of any trial being constant.

<table>
<tr>
<td>11(aii)</td>
<td>$r(0.04)(1-0.04)=1.92$
$r=50$</td>
<td>Most students can do (aii). Students who did not attempt should know that the formula is in the booklet for reference.</td>
</tr>
<tr>
<td>11(b)</td>
<td>Let X be the number of tickets obtained, out of n candy bars.
$X \sim B(k, 0.04)$
$\mathrm{P}(X>3)\geq 0.34$
$1-\mathrm{P}(X\leq 3)\geq 0.34$
$\mathrm{P}(X\leq 3)\leq 0.66$
<table>
<tr><td>k</td><td>$\mathrm{P}(X\leq 3)$</td><td></td></tr>
<tr><td>73</td><td>0.6655</td><td>>0.66</td></tr>
<tr><td>74</td><td>0.6564</td><td>≤ 0.66</td></tr>
<tr><td>75</td><td>0.6473</td><td></td></tr>
</table>
$\therefore k=74$</td>
<td>While most students can identify the distribution to be binomial, there were a number of careless mistakes when changing $\mathrm{P}(X>3)$ to the correct inequality. A portion of the students approached the question using normal distribution instead.
Students should also take note and present the table in their answer instead.</td>
</tr>
<tr>
<td>11(c)</td>
<td>Let the amount of money received be $A.
$\mathrm{P}(A=0)=\mathrm{P}(0,0)=\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$
$\mathrm{P}(A=1)=\mathrm{P}(01,\ 0)=2!\times\frac{2}{5}\times\frac{1}{4}\times\frac{1}{3}=\frac{1}{15}$
$\mathrm{P}(A=2)=\mathrm{P}(02,\ 0)=\frac{1}{15}$
$\mathrm{P}(A=3)=\mathrm{P}(012,\ 0)=3!\times\frac{2}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}=\frac{1}{10}$
$\mathrm{P}(A=4)=\mathrm{P}(04,\ 0)=\frac{1}{15}$
$\mathrm{P}(A=5)=\mathrm{P}(014,\ 0)=\frac{1}{10}$
$\mathrm{P}(A=6)=\mathrm{P}(024,\ 0)=\frac{1}{10}$
$\mathrm{P}(A=7)=\mathrm{P}(0124,\ 0)=4!\times\frac{2}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times 1=\frac{2}{5}$
<table>
<tr><td>A</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>$\mathrm{P}(A=a)$</td><td>$\frac{1}{10}$</td><td>$\frac{1}{15}$</td><td>$\frac{1}{15}$</td><td>$\frac{1}{10}$</td><td>$\frac{1}{15}$</td><td>$\frac{1}{10}$</td><td>$\frac{1}{10}$</td><td>$\frac{2}{5}$</td></tr>
</table></td>
<td>This part is not well attempted as students either left it blank or did not include the permutation of the numbers. Most students could identify all the 8 cases.

A good reminder that students can check their working by ensuring that the total probability adds up to 1.</td>
</tr>
<tr>
<td>11(d)</td>
<td>$\mathrm{P}(\mathrm{A}\geq 5 \mid \text{at least 4 vouchers})=\frac{\mathrm{P}(A=5,6\text{ or }7)}{\mathrm{P}(A=3,5,6\text{ or }7)}$
$=\frac{\frac{1}{10}+\frac{1}{10}+\frac{2}{5}}{\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{2}{5}}$
$=\frac{6}{7}$ or 0.857 (3 sf)</td>
<td>This part is not well attempted. For those who attempted, they might have either neglected the conditional probability, or confused themselves by considering the taking of 4 vouchers as P(A = 4).</td>
</tr>
</table>