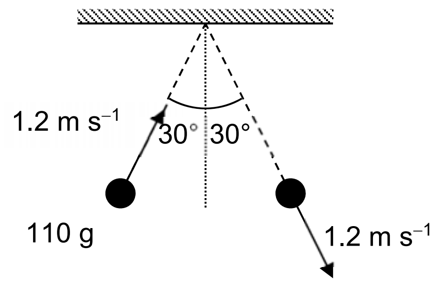
Example 1a: Change in Momentum

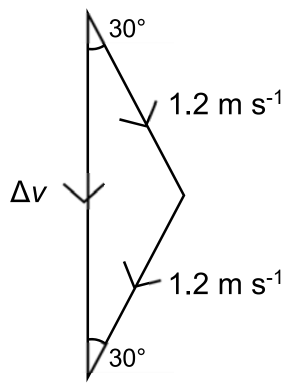
A 110 g billiard ball rebounds off a wall, with velocities as shown in the (top view) diagram. The ball moves on a horizontal plane. Evaluate the change in momentum of the ball.



Answer

$\Delta p = p_f - p_i = m(v_f - v_i)$

Draw the vector triangle.



$\Delta v = \sqrt{1.2^2 + 1.2^2 - 2(1.2)(1.2)\cos 120°}$

$\Delta p = m\Delta v = 0.110\sqrt{1.2^2 + 1.2^2 - 2(1.2)(1.2)\cos 120°} = 0.23 \text{ kg m s}^{-1}$ (downward)

Alternative Method

Horizontally, $\Delta p_H = m(v_{x,f} - v_{x,i})$

$= 0.110(1.2\sin 30° - 1.2\sin 30°)$

$= 0$

Vertically, $\Delta p_V = m(v_{y,f} - v_{y,i})$

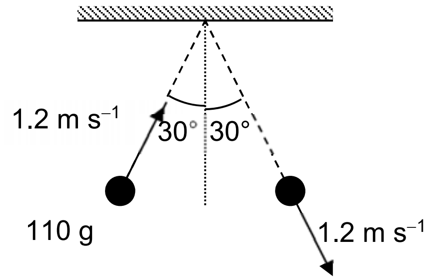
$= 0.110[1.2\cos 30° - (-1.2\cos 30°)]$

$= 0.23 \text{ kg m s}^{-1}$

$\Delta p = \Delta p_V = 0.23 \text{ kg m s}^{-1}$ (downward)

Example 1b: Rate of Change in Momentum

The time interval of contact between the billiard ball and the wall is 0.100 s. Find the average net force acting on the wall during the collision.



Answer:

In example 1a, we found $\Delta p = 0.23$ kg m s^{-1}

By Newton's 2nd Law,

$$\langle F_{\text{wall on ball}} \rangle = \frac{\Delta p}{\Delta t} = \frac{0.23}{0.100} = 2.3 \text{ N (downward)}$$

By Newton's 3rd Law,

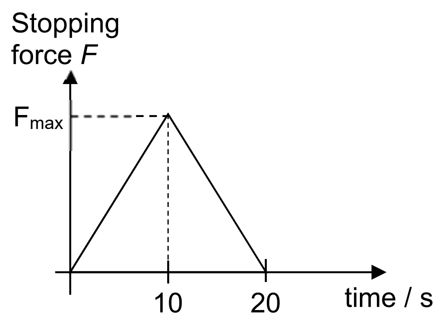
$$\langle F_{\text{wall on ball}} \rangle = -\langle F_{\text{ball on wall}} \rangle$$

So the average net force acting on the wall by the ball during the collision is 2.3 N directed upward (perpendicularly into the wall).

Example 2 (N89/II/8 modified)

In order to stop a car of mass 1500 kg travelling at 30 m s^{-1}, the driver applies his brakes so that F, the total stopping force, increases steadily to a maximum and then decreases to zero as shown in the figure. Calculate

(a) the momentum of the car when it is travelling at 30 m s^{-1},
(b) the impulse due to the braking force,
(c) the magnitude of the average stopping force, <F>,
(d) the value of F_{max}.



Answer:

(a) Momentum of car,

$p = mv = (1500)(30) = 45 \times 10^3$ kg m s^{-1}

(b) Impulse due to the braking force = change in linear momentum of car, Δp_{car}

$= p_f - p_i = 1500(0 - 30) = -45 \times 10^3$ kg m s^{-1}

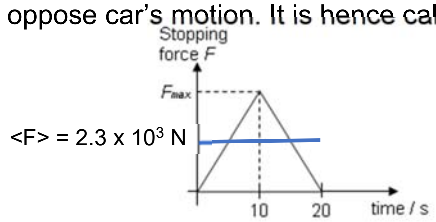
(c) $\langle F \rangle = \dfrac{\Delta p_{car}}{\Delta t} = \dfrac{-45x10^3}{20} = -2.3 \times 10^3$ N

Note: negative sign => force acts to oppose car's motion. It is hence called "stopping force".

(d) $|\Delta p_{car}|$ = area under F-t graph

$45 \times 10^3 = (0.5)(20)F_{\text{max}}$

$F_{\text{max}} = 4500$ N



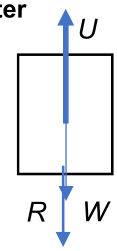
Example 3

A helicopter of mass M and weight W rises with vertical acceleration, a, due to the upward thrust U generated by its rotor. The crew and passengers of total mass m and total weight w, exerts a combined force R on the floor of the helicopter.

Draw an appropriate free-body diagram, and write down an equation for the motion of
(a) the helicopter, (b) the crew and passengers, (c) helicopter, crew and passengers

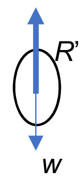
Answer:

(a) helicopter



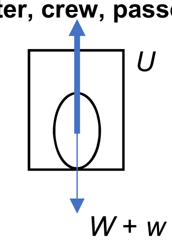
Resultant upwards force on helicopter = $U - W - R$
Hence, by Newton's Second Law,
$U - W - R = Ma$

(b) crew and passengers



Resultant upwards force on crew and passengers
= $R' - w$
Hence, by Newton's Second Law,
$R' - w = ma$

(c) helicopter, crew, passengers

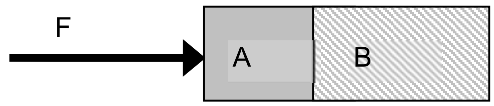


(1) $U - W - R = Ma$
(2) $R' - w = ma$

(1) + (2):
$U - (W+w) = (M+m)a$

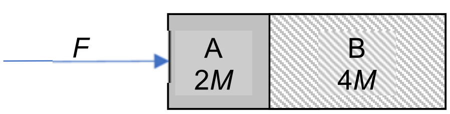
Example 4

Two blocks A and B, of masses 2M and 4M, respectively, are pushed along a smooth horizontal surface by a force of F as shown in the diagram.
What is the magnitude of the force exerted by block A on block B during the acceleration?



Answer:

Consider A + B:



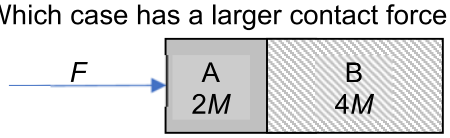
By Newton's second law:
$F = (2M + 4M)a$
$a = \frac{F}{6M}$

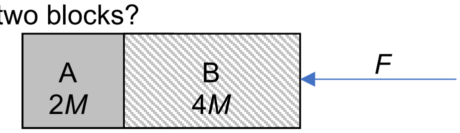
Consider B:

Note: the weight and normal contact force, despite equal in value, should be included in FBD for completeness.

By Newton's second law:
$F_{A\text{ on }B} = 4Ma = 4M\left(\frac{F}{6M}\right) = \frac{2}{3}F$

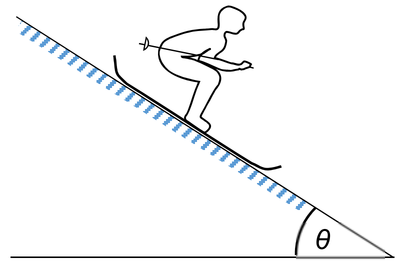
(Extension)
Which case has a larger contact force between the two blocks?





Example 5

A 75 kg skier is accelerating at 2.6 m s^{-2} down a slope at an angle θ = 30°. Find the friction *f* and the normal contact force *N* acting on the skier.

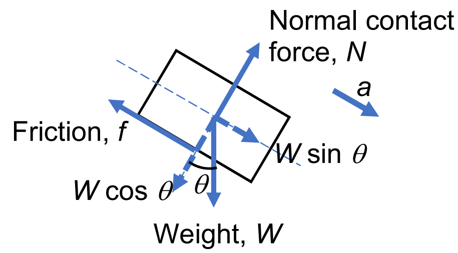


Step 1 Choose/Identify the system, in this case the skier, and draw a simple box to represent it.

Step 2 Draw the free-body diagram.

Step 3 For two-dimensional problems, resolve forces into two convenient, mutually perpendicular directions to simplify analysis.

- We choose the components *along* and *perpendicular to* the slope.
- The frictional force is parallel to the slope.
- The normal contact force is perpendicular to the slope.
- The weight has components along and perpendicular to the slope.



Step 4 Apply Newton's second law to the 2 perpendicular directions:

Parallel to the slope, taking downwards along the slope as positive

↘ $+x$

Direction // to slope: $(F_{net} = ma)$

$$W \sin\theta - f = Ma$$

$$(75)(9.81)\sin 30° - f = (75)(2.6)$$

$$f = 173 \text{ N}$$

Perpendicular to the slope, taking upwards away from the slope as positive

↗ $+y$

Direction ⊥ to slope: $(F_{net} = 0)$

$$N = Mg\cos\theta$$

$$= (75)(9.81)\cos 30°$$

$$= 637 \text{ N}$$

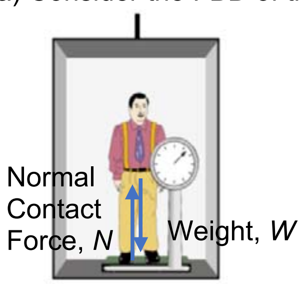
Don't know how to resolve the weight into the 2 components? Watch this:

Example 6 (Man in the lift)

An 80 kg man weighs himself by standing on a weighing scale inside a lift. What does the scale read if the lift
(a) is at rest,
(b) moves with an upward acceleration of 1.8 m s^{-2},
(c) the lift is moving upwards with a constant velocity of 2.2 m s^{-1},
(d) the lift slows from its velocity in (c) to rest at rate of 1.9 m s^{-2}.

Answer:
Note that the weighing scale actually does not measure the man's weight. It measures the **normal contact force pushing down on it**. As we do not have any other information about the weighing scale, we are unable to choose the weighing scale as the system and draw its FBD. Instead we will determine the normal contact force on the scale by using its third-law partner. Since the normal contact force **on the scale** is due to the **man**, its third-law partner is the **normal contact force on the man by the scale**. We will solve the question by choosing the man as the system and drawing the FBD of the man.

(a) Consider the FBD of the man:



The man is at rest.
Net force = 0
$N = mg$

By Newton's Third Law,
The scale reading reads mg.

Weighing scale reads the true weight of the man.

(c) This outcome is true for (c) as well since moving at constant speed also implies that the net force is zero.

(b) The lift moves with an upward acceleration a
Net force upwards

$$(F_{net} = ma)$$
$$N - mg = ma$$
$$N = m(g + a)$$
$$= (80)(9.81 + 1.8)$$
$$= 928.8 \text{ N}$$

Mass reading $= 928.8 \div 9.81 = 94.7$ kg

(d) The lift slows to rest at rate of a

$$(F_{net} = ma)$$
$$mg - N = ma$$
$$N = m(g - a)$$
$$= (80)(9.81 - 1.9)$$
$$= 632.8 \text{ N}$$

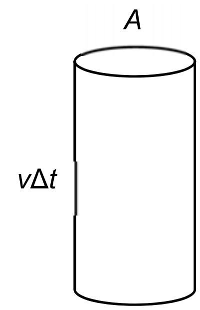
Mass reading $= 632.8 \div 9.81 = 64.5$ kg

This YouTube video sheds more light on apparent weight and true weight:
https://www.youtube.com/watch?v=AbNJv1VNWu8

Example 7:
A hovering jetpack with nozzles of total cross-sectional area A expels water at constant velocity v relative to the jetpack. Show that the force exerted by the expelled water on the jetpack is $F = \rho A v^2$, where ρ is the density of the water jet.

Answer:

Since the jetpack is hovering, the speed of water relative to the jetpack is the same as the speed of water observed by a ground observer. Hence in a short time interval Δt, the column of water ejected from the jetpack is as shown.



Consider the cylindrical column of water leaving the nozzle in duration of time Δt.

Volume of water leaving in time Δt $= (v\Delta t)A$
Mass of water leaving in time Δt $= \rho(v\Delta t)A$
Momentum of water "produced" in time Δt
$= \rho(v\Delta t)A \times v$ ………………………………(1)
$= \rho A v^2 \Delta t$

Average rate of change of momentum of water, $F = \dfrac{\Delta p}{\Delta t} = \dfrac{\rho A v^2 \Delta t}{\Delta t} = \rho A v^2$ (downward)

This is the force experienced by the water. By Newton's Third Law, the jetpack experiences a force of $\rho A v^2$ (upward).

Note: There is another convenient form of expression for the force which follows from (1),

Momentum of water "produced" in time $\Delta t = \rho(v\Delta t)A \times v = v\Delta m$

Average rate of change of momentum of water, $F = \frac{\Delta p}{\Delta t} = v\frac{\Delta m}{\Delta t}$
where $\frac{\Delta m}{\Delta t}$ is the rate at which water is being ejected from the jet pack.

Example 8 (Freedman)

An open-topped freight car with mass 24 000 kg is coasting along without friction along a level track. It is raining very hard and the rain is falling vertically downward. Originally the car is empty and moving with a speed of 4.00 m/s. What is the speed of the car after it has collected 3000kg of rainwater?

Answer:

Consider the rain and car as a system. Since horizontal momentum is conserved, the total horizontal momentum when the car is empty is the same as when the car is filled with rainwater. Hence

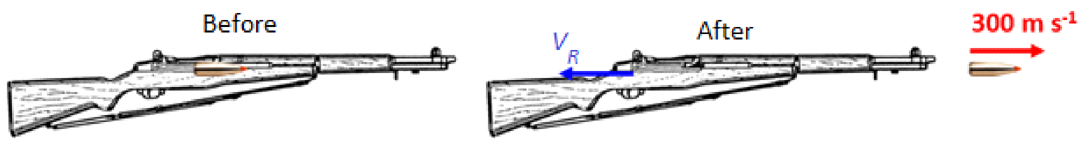
$$(\mathbf{24000})(\mathbf{4}) = (\mathbf{24000} + \mathbf{3000})\boldsymbol{v}$$

$$\boldsymbol{v} = \mathbf{3.56} \text{ m/s}$$

Example 9

A marksman holds a rifle of mass M = 3.00 kg loosely in his hands, so as to let it recoil freely when fired. He fires a bullet of mass m = 5.00 g horizontally with a velocity of v_B = 300 m s^{-1}.
What is the recoil velocity v_R of the rifle?

Answer:



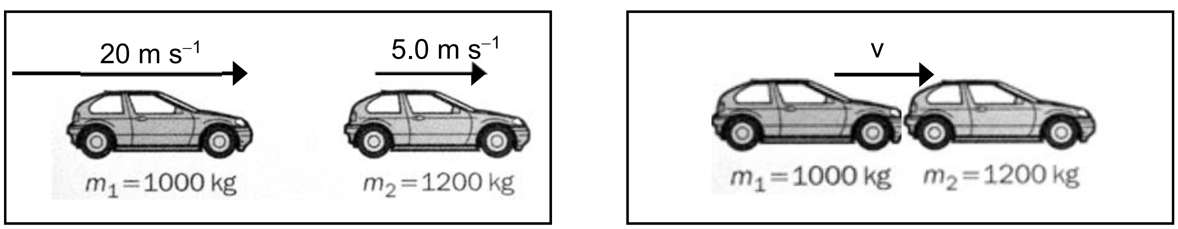
Note that since both the rifle and bullet are **stationary** before firing, the total initial momentum of the system is **zero**. Since momentum is a vector quantity, in order for the final total momentum to be zero, the **rifle must move in the direction opposite to the bullet** (recoil).

By the principle of conservation of linear momentum,

$$\Sigma p_i = \Sigma p_f$$
$$0 = m_B v_B + m_R(-v_R)$$
$$0 = (0.00500)(300) - (3.00)(v_R)$$
$$v_R = 0.500 \text{ m s}^{-1} \text{ (leftward)}$$

Example 10 (Perfectly Inelastic collision)

A car of mass 1000 kg moving at 20.0 m s^{-1} collides with a car of mass 1200 kg, moving at 5.0 m s^{-1} in the same direction.
(a) If the collision is perfectly inelastic, determine their final velocity after the collision.
(b) Calculate also the ratio of (total final kinetic energy) to (total initial kinetic energy).



before **after**

Answer:
(a)
By PCOM (taking rightward as positive)

$$\sum p_i = \sum p_f$$
$$(1000)(20) + (1200)(5.0) = (1000 + 1200)v$$
$$v = 11.8 \text{ m s}^{-1}$$

(b)

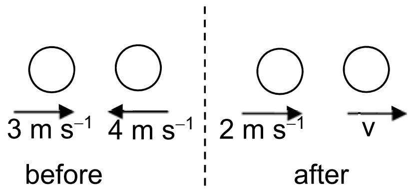
$$\text{Total initial KE} = \frac{1}{2}(1000)(20)^2 + \frac{1}{2}(1200)(5.0)^2 = 2.15 \times 10^5 \text{ J}$$

$$\text{Total final KE} = \frac{1}{2}(1000 + 1200)(11.8)^2 = 1.53 \times 10^5 \text{ J}$$

$$\text{Ratio} = \frac{1.53 \times 10^5}{2.15 \times 10^5} = 0.71$$

Example 11

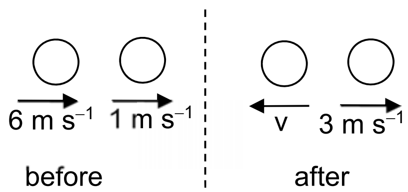
Assuming the following are **head-on elastic collisions**, solve for the unknown velocity *v* in each case. Note that the masses of the objects are not (necessarily) equal.



Elastic collision: $RSOA = RSOS$

$$3-(-4) = v-2$$

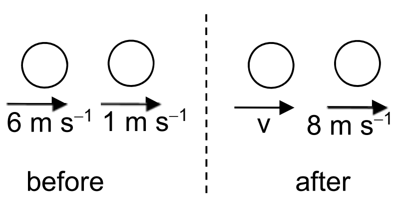
$$v = 9 \text{ m s}^{-1}$$



Elastic collision: $RSOA = RSOS$

$$6-1 = 3-(-v)$$

$$v = 2 \text{ m s}^{-1} \text{ (leftward)}$$



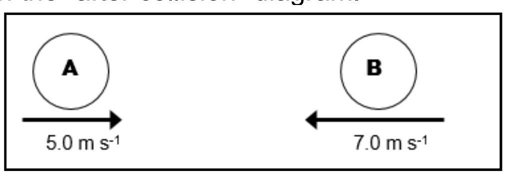
Elastic collision: $RSOA = RSOS$

$$6-1 = 8-v$$

$$v = 3 \text{ m s}^{-1}$$

Example 12 (Elastic collision)
Two spheres, A and B, of mass 4.0 kg and 6.0 kg, respectively, collide head-on. Their initial velocities are 5.0 m s^{-1} and 7.0 m s^{-1} in opposite directions, as shown in the diagram.
Assuming the collision is **elastic**, determine their final velocities.

Answer:
The directions of motion of the spheres after the collision are not given in the question. Hence, we will need to draw the "after collision" diagram to indicate the directions used in the analysis. It is ok if the directions chosen are incorrect as the final answer will tell us if the sphere are really moving in the predicted direction in the "after collision" diagram.



Before

After

Applying the principle of conservation of linear momentum:

$$(\rightarrow +ve) \quad : \quad m_A u_A + m_B u_B = m_A w + m_B v$$
$$4(5) + 6(-7) = 4w + 6v$$
$$2w + 3v = -11 \quad \text{........} \quad (1)$$

Since the collision is elastic, relative speed of approach equals realtive speed of separation:

$$5 + 7 = v - w$$
$$v - w = 12 \quad \text{........} \quad (2)$$

Solving simultaneously:

$$w = -9.4 \text{ m s}^{-1}, \; v = 2.6 \text{ m s}^{-1}$$

The negative sign for w indicates that the actual direction of motion is **opposite** to the "predicted" direction in the diagram, i.e. sphere A moves to the left after the collision.