Chapter 2: Probability

1.	CJC JC2 Prelim 8865/2019/Q8
	(a) Two mutually exclusive events A and B are such that $\mathrm{P}(A)$ and $\mathrm{P}(B)$ are both non-zero. Determine if A and B are independent events. Justify your answer. [1]
	(b) It is given that events C and D are such that $\mathrm{P}(C \mid D)=0.4$, $\mathrm{P}(C \cap D)=0.15$ and $\mathrm{P}(C \cup D)=0.82$. Find the value of $\mathrm{P}(C)$. [4]
	Answer: (b) 0.595

1.	CJC JC2 Prelim 8865/2019/Q8 (Solutions)
(a)	A and B are mutually exclusive events, $\mathrm{P}(A \cap B)=0$. However, $\mathrm{P}(A)$ and $\mathrm{P}(B)$ are both non-zero so $\mathrm{P}(A)\mathrm{P}(B) \neq 0$. Since $\mathrm{P}(A \cap B) \neq \mathrm{P}(A)\mathrm{P}(B)$, A and B are not independent.
(b)	$\mathrm{P}(C \mid D)=0.4$ $\frac{\mathrm{P}(C \cap D)}{\mathrm{P}(D)}=0.4$ $\frac{0.15}{\mathrm{P}(D)}=0.4$ $\mathrm{P}(D)=0.375$ $\mathrm{P}(C \cup D)=0.82$ $\mathrm{P}(C)+\mathrm{P}(D)-\mathrm{P}(C \cap D)=0.82$ $\mathrm{P}(C)+0.375-0.15=0.82$ $\mathrm{P}(C)=0.595$

2.	RVHS JC2 Prelim 8865/2019/Q7
	Events A and B are such that $\mathrm{P}(A)<\mathrm{P}(A \mid B)$. (i) Determine, with a reason, whether A and B are independent. [1] (ii) Determine, with a reason, whether A and B are mutually exclusive. [1] (iii) Given $\mathrm{P}(A)=\frac{2}{5}$, $\mathrm{P}(A \mid B)=\frac{2}{3}$ and $\mathrm{P}(B)=\frac{3}{8}$, and using a Venn Diagram or otherwise, find $\mathrm{P}(A \cup B')$. [4]
	Answer: (iii) $\frac{7}{8}$

2.	RVHS JC2 Prelim 8865/2019/Q7 (Solutions)
(i)	If A and B are independent,

$$\mathrm{P}(A|B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{\mathrm{P}(A)\mathrm{P}(B)}{\mathrm{P}(B)}=\mathrm{P}(A)$$

Since $\mathrm{P}(A)<\mathrm{P}(A|B)$, *A*, *B* are not independent.

(ii) If *A* and *B* are mutually exclusive,

$$\mathrm{P}(A|B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{0}{\mathrm{P}(B)}=0$$

Thus $\mathrm{P}(A)<\mathrm{P}(A|B)=0$ which is not possible, therefore *A*, *B* are not mutually exclusive.

Alternative Method:

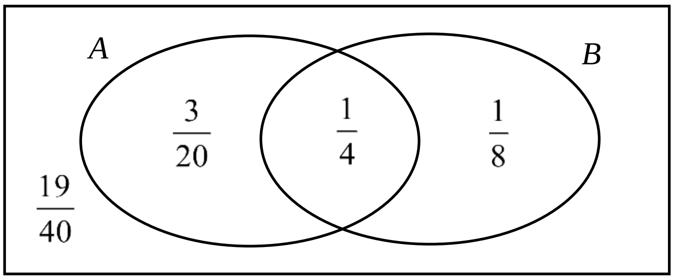
$$\mathrm{P}(A)<\mathrm{P}(A|B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$$

$$0\le \mathrm{P}(A)\mathrm{P}(B)<\mathrm{P}(A\cap B)$$

$$\Rightarrow 0<\mathrm{P}(A\cap B)$$

Thus not mutually exclusive.

(iii)



$$\mathrm{P}(A|B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$$

$$\mathrm{P}(A\cap B)=\frac{2}{3}\times\frac{3}{8}=\frac{1}{4}$$

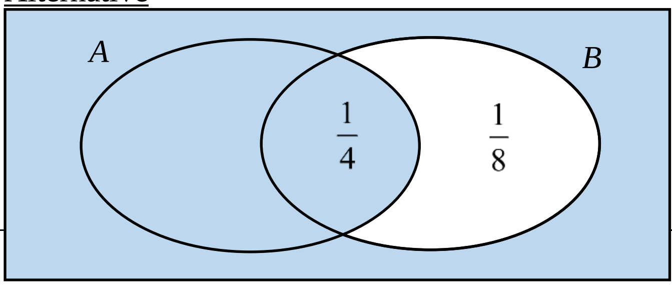
$$\mathrm{P}(A)-\mathrm{P}(A\cap B)=\mathrm{P}(A\cap B')=\frac{2}{5}-\frac{1}{4}=\frac{3}{20}\ (*)$$

$$\mathrm{P}(B)-\mathrm{P}(A\cap B)=\mathrm{P}(A'\cap B)=\frac{3}{8}-\frac{1}{4}=\frac{1}{8}\ (**)$$

$$\mathrm{P}(A'\cap B')=1-\mathrm{P}(A\cup B)=1-\left(\frac{3}{20}+\frac{1}{4}+\frac{1}{8}\right)=\frac{19}{40}\ (***)$$

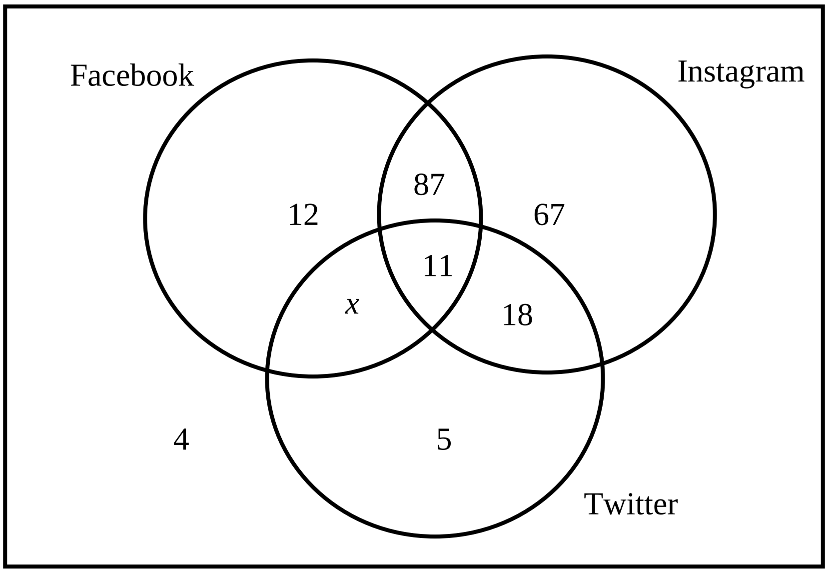
$$\mathrm{P}(A\cup B')=\frac{3}{20}+\frac{1}{4}+\frac{19}{40}=\frac{7}{8}$$

<u>Alternative</u>



	$\mathrm{P}(A\mid B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ $\mathrm{P}(A\cap B)=\dfrac{2}{3}\times\dfrac{3}{8}=\dfrac{1}{4}$ (*) $\mathrm{P}(B)-\mathrm{P}(A\cap B)=\mathrm{P}(A'\cap B)=\dfrac{3}{8}-\dfrac{1}{4}=\dfrac{1}{8}$ (**) $\mathrm{P}(A\cup B')=1-\mathrm{P}(A'\cap B)=1-\dfrac{1}{8}=\dfrac{7}{8}$

3. ASRJC JC2 Prelim 8865/2019/Q7



A group of students are surveyed on whether they use any social media platforms. The numbers of students using different combinations of these platforms are shown in the above Venn diagram. The number of students who use Facebook and Twitter only is x. One of the students is chosen at random.

- F is the event that the student uses Facebook.
- G is the event that the student uses Instagram.
- T is the event that the student uses Twitter.

(i) Write down the expression for $\mathrm{P}(F)$ in terms of x. [1]

(ii) Given that $\mathrm{P}(F\cap T)=\dfrac{32}{225}$, show that $x=21$. [1]

(iii) Determine whether F and T are independent. [1]

(iv) Find $\mathrm{P}(F'\cup T)$ [1]

(v) Explain, in the context of this question, what is meant by $\mathrm{P}((G\cup T)\mid F)$, and find its value. [3]

Two students from the group are chosen at random, without replacement.

(vi)	Find the probability that both of these students each use exactly two of the three social media platforms. [2]
	Answer: (i) $\frac{110+x}{204+x}$ (iv) $\frac{14}{25}$ (v) $\frac{119}{131}$ (vi) $\frac{5}{16}$
3.	ASRJC JC2 Prelim 8865/2019/Q7 (Solutions)
(i)	Total number of students $=12+87+67+11+18+5+4+x$ $=204+x$ $\mathrm{P}(F)=\frac{12+87+11+x}{204+x}=\frac{110+x}{204+x}$ $\mathrm{P}(F\cap T)=\frac{11+x}{204+x}$ $\mathrm{P}(F\cap T)=\frac{11+x}{204+x}=\frac{32}{225}$ $225(11+x)=32(204+x)$ $225x-32x=32(204)-225(11)$ $x=\frac{4053}{193}=21$
(iii)	$\mathrm{P}(F)=\frac{12+87+11+21}{204+21}=\frac{131}{225}$ $\mathrm{P}(T)=\frac{11+18+5+21}{204+21}=\frac{11}{45}$ $\mathrm{P}(F)\times\mathrm{P}(T)=\frac{131}{225}\times\frac{11}{45}=\frac{1441}{10125}\neq\frac{32}{225}$ Since $P(F\cap T)\neq\mathrm{P}(F)\times\mathrm{P}(T)$, F and T are not independent.
(iv)	$\mathrm{P}(F'\cup T)=\frac{5+18+67+11+21+4}{204+21}=\frac{126}{225}=\frac{14}{25}$
(v)	$\mathrm{P}((G\cup T)\mid F)$ refers to the probability that a student who uses Facebook, also uses at least one of either Instagram or Twitter [Or The probability of a student using either Instagram or Twitter or both, when he already uses Facebook.]

	$\text{P}\left((G \cup T) \mid F\right)$ $= \dfrac{\text{P}\left((G \cup T) \cap F\right)}{\text{P}(F)} = \dfrac{\frac{21+11+87}{225}}{\frac{131}{225}}$ $= \dfrac{119}{131}$
(vi)	Required Probability $= \dfrac{21+87+18}{225} \times \dfrac{21+87+18-1}{225-1}$ $= \dfrac{126}{225} \times \dfrac{125}{224}$ $= \dfrac{5}{16}$

4. HCI JC2 Prelim 8865/2019/Q12

During a flu season, 120 patients who display flu symptoms consulted a doctor. Of the 120 patients, 75 are females of whom 15 have actually contracted the flu. The number of male patients who have contracted the flu is x. One of the patients is chosen at random.

- A is the event that the patient is a male.
- B is the event that the patient has contracted flu.

(i) Write down expressions for P(B) and $\text{P}(A \cap B')$ in terms of x. [2]

(ii) Explain, in the context of this question, what is meant by $\text{P}(A|B)$. [1]

Given further that $\text{P}(A|B) = \dfrac{4}{7}$, show that $x = 20$. [2]

(iii) Are events A and B independent? Justify your answer. [1]

(iv) Two patients are chosen at random, without replacement. Find the probability that exactly one of them has flu. [2]

(v) A diagnostic test for the flu is used. The test has a probability of 0.93 of giving a positive result when the patient has the flu, and a probability of 0.06 of giving a positive result when the patient does not have the flu. The test is administered to a patient chosen at random from the 120 patients.

(a) Draw a tree diagram to represent this information. [2]

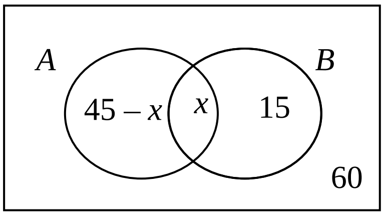
(b) Find the probability that the patient is tested positive. [2]

(c) Find the probability that the patient has contracted the flu, given that he or she is tested positive. [2]

Answer: (i) $\frac{45-x}{120}$ (iv) $\frac{5}{12}$ (vb) $\frac{251}{800}$ (vc) $\frac{217}{251}$

4. HCI JC2 Prelim 8865/2019/Q12 (Solutions)

(i)



$\mathrm{P}(B)=\frac{15+x}{120}$

$\mathrm{P}(A\cap B')=\frac{45-x}{120}$

(ii) It is the probability of selecting a male patient, given that the patient has contracted flu.

$\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{4}{7}$

$\frac{x}{15+x}=\frac{4}{7}$

$7x=60+4x$

$3x=60$

$x=20$ (Shown)

(iii) For A and B to be independent,

$\mathrm{P}(A|B)=\mathrm{P}(A)$

But $\mathrm{P}(A)=\frac{45}{120}=\frac{3}{8}\neq\frac{4}{7}$.

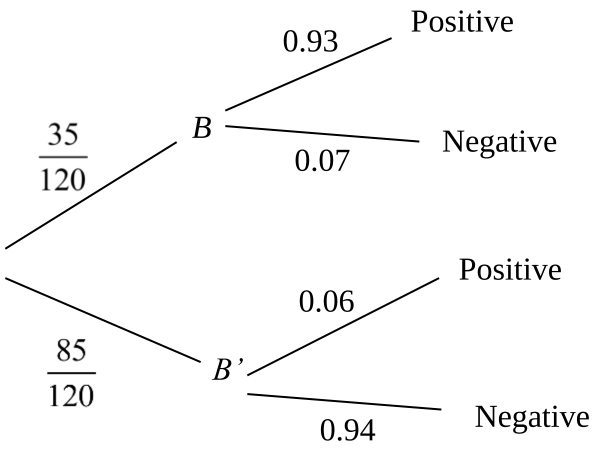
Therefore A and B are not independent events.

(iv) Required probability

$=\frac{35}{120}\times\frac{85}{119}\times 2$

$=\frac{5}{12}$ or 0.417

(va)



(vb) Required probability

$= \frac{35}{120} \times 0.93 + \frac{85}{120} \times 0.06$

$= \frac{251}{800}$ or 0.31375 (exact)

(vc) $\mathrm{P}(\text{has flu} \mid \text{Positive})$

$= \frac{\mathrm{P}(\text{has flu} \cap \text{Positive})}{\mathrm{P}(\text{Positive})}$

$= \frac{\frac{35}{120} \times 0.93}{\frac{251}{800}}$

$= \frac{217}{251}$ or 0.865

5.	ACJC JC2 Prelim 8865/2019/Q6

Balls are drawn one at a time, without replacement, from a bag containing 10 red balls, 10 blue balls and 10 yellow balls. Balls of each colour are numbered 1 to 10 and points are scored according to the numbers on the respective balls. Find the probabilities that

(i) the first ball drawn scores at least 9 points, [1]

(ii) the first two balls drawn each score at least 9 points, [2]

(iii) the first two balls drawn score at least 18 points altogether, [2]

(iv) the first two balls drawn each score at least 9 points given that they score at least 18 points altogether. [2]

Answer: (i) $\frac{1}{5}$ (ii) $\frac{1}{29}$ (iii) $\frac{8}{145}$ (iv) $\frac{5}{8}$

5.	ACJC JC2 Prelim 8865/2019/Q6 (Solutions)
(i)	$\mathrm{P}(1\text{st ball} = 9,10) = \frac{6}{30} = \frac{1}{5}$
(ii)	Let A be the event that the first 2 balls drawn each score at least 9 points.

	$P(A)=P(\text{1st 2 balls}=9,10)=\frac{6}{30}\times\frac{5}{29}=\frac{1}{29}$ <u>OR</u> $P(9\&9,9\&10,10\&9,10\&10)=\frac{3\times2}{30\times29}+2\left(\frac{3\times3}{30\times29}\right)+\frac{3\times2}{30\times29}$ $=\frac{30}{870}=\frac{1}{29}$
(iii)	Let B be the event that the first 2 balls drawn score at least 18 points altogether. $P(B)=P(A)+P(8\&10,10\&8)$ $=\frac{1}{29}+2\left(\frac{3\times3}{30\times29}\right)$ $=\frac{8}{145}$
(iv)	$P(A\mid B)=\frac{P(A\cap B)}{P(B)}$ $=\frac{1}{29}\div\frac{8}{145}$ $=\frac{5}{8}$

6. NYJC JC2 Prelim 8865/2019/Q6

A code consists of 4 digits. The digits are chosen from {1, 2, 3, 4, 5, 6}. Suppose that repetitions are allowed, find the probability that a code chosen at random

(i) has the digits arranged in a decreasing order from the left to the right and no two consecutive numbers are the same, [2]

(ii) ends with an even digit, [1]

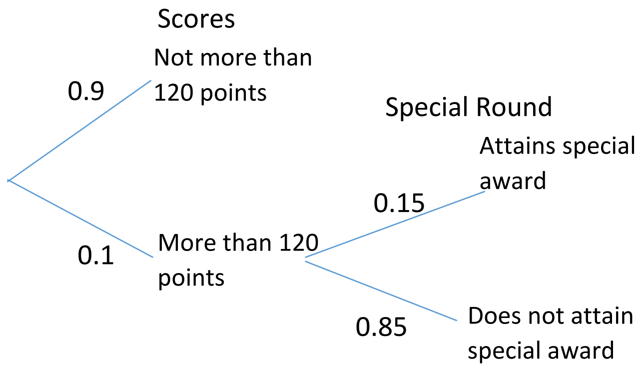
(iii) contains the digit 2 exactly once or ends with an even digit, but not both. [4]

Asnwer: (i) $\frac{5}{432}$ (ii) $\frac{1}{2}$ (iii) $\frac{299}{648}$

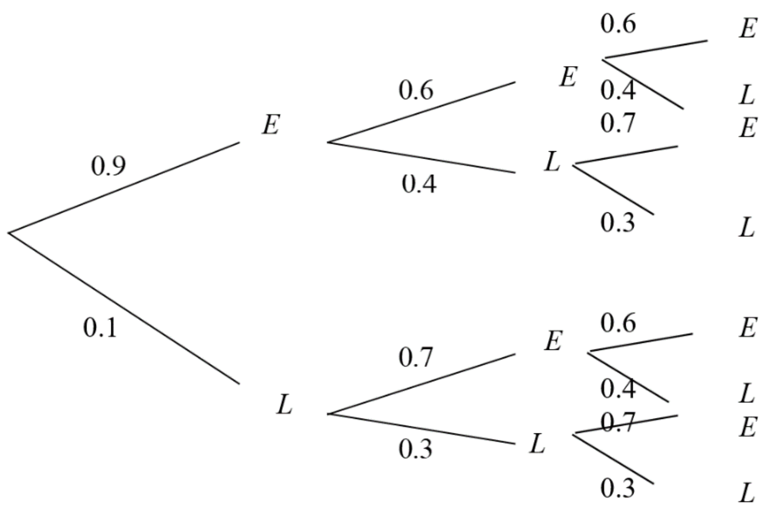
6.	NYJC JC2 Prelim 8865/2019/Q6 (Solutions)
(i)	$\frac{{}^6C_4}{6^4}=\frac{5}{432}=0.0116$
(ii)	$\frac{(6^3)\,{}^3C_1}{6^4}=\frac{1}{2}$
(iii)	Let A be the event of code containing the digit 2 exactly once, B be the event of code ending with even number. $P(A\cup B)-P(A\cap B)$ $=P(A)+P(B)-2P(A\cap B)$

	$= \dfrac{5^3({}^4C_1)+3(6^3)-2(5^3+{}^2C_1({}^3C_1)(5^2))}{6^4}$ $= \dfrac{299}{648} = 0.461$

7.	NJC JC2 Prelim 8865/2019/Q6
	A group of students take part in a Mathematics competition. A student who scores above 120 points is invited to take part in the Special Round. For a randomly selected student, the probability of scoring not more than 120 points is 0.9, and the probability of attaining special award in the Special Round given that the student is invited in the Special Round is 0.15. (i) Find the probability that a randomly selected student takes part in the Math competition and does not attain the special award. [2] (ii) Three students taking the Mathematics competition are chosen at random. Find the probability that one of them is invited to the Special Round and attains the special award, and the other two are not invited in the Special Round. [2] Asnwer: (i) 0.985 (ii) 0.03645

7.	NJC JC2 Prelim 8865/2019/Q6 (Solutions)
(i)	Using a probability tree diagram,  P (one student takes part in the Math competition and does not attain the special award) $= 0.9 + (0.1 \times 0.85)$ $= 0.985$
	OR P (one student takes part in the Math competition and does not attain the special award) = 1 – P (one student takes part in the Math competition and attains the special award) $= 1 - (0.1 \times 0.15)$ $= 0.985$
(ii)	P(one student invited to the Special Round and attained the special award) = $0.1 \times 0.15 = 0.015$ Required probability $= 0.015 \times 0.9 \times 0.9 \times \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $= 0.03645$

8.	MI PU2 Prelim 8865/2019/Q7
	In light of recent thefts, food delivery company Grub decides to employ drones in replacement of deliverymen. The probability that the drone is early on the first delivery is 0.9. The probability that the drone is early following an early delivery is 0.6 and the probability that the drone is early following a late delivery is 0.7. A total of three deliveries are made. (i) Draw a tree diagram to represent this information. [2] (ii) Find the probability that the drone is early on the first delivery and late on the third delivery. [2] (iii) Given that the drone is late on the third delivery, find the probability that the drone is early on the first delivery. [3] Answer: (ii) 0.324 (iii) $\frac{324}{361}$

8.	MI PU2 Prelim 8865/2019/Q7 (Solutions)
(i)	Let E be the event that the drone made an early delivery Let L be the event that the drone made a late delivery 
(ii)	Probability the drone will be early on 1st day and late on 3rd day = P(EEL) + P(ELL) $= 0.9 \times 0.6 \times 0.4 + 0.9 \times 0.4 \times 0.3$ $= 0.216 + 0.108$ $= 0.324$
(iii)	P(drone early on 1st day | drone late on 3rd day)

	$= \dfrac{P\left(\text{drone early on 1st delivery} \cap \text{drone late on 3rd delivery}\right)}{P\left(\text{drone late on 3rd delivery}\right)}$ $= \dfrac{0.324}{P(LLL) + P(LEL) + P(ELL) + P(EEL)}$ $= \dfrac{0.324}{0.009 + 0.028 + 0.108 + 0.216}$ $= \dfrac{324}{361}$ $= 0.89750$ $= 0.898$ (3 s.f.)

9. JPJC JC2 Prelim 8865/2019/Q10

Bag A contains 4 red, 5 blue and 1 green ball. Bag B contains 3 red, 3 blue and 2 green balls. Bag C contains 2 red, 1 blue and 1 green ball. One of the bags is selected by throwing a fair six-sided die. If the score is 1, 2 or 3, bag A is selected, if the score is 4, bag B is selected, otherwise bag C is selected.

One ball is selected at random from the selected bag and the colour is noted

(i) Draw a tree diagram to represent the possible outcomes. [2]

Events X and Y are defined as follows:

Event X: A ball is selected from bag A.

Event Y : A red ball is selected.

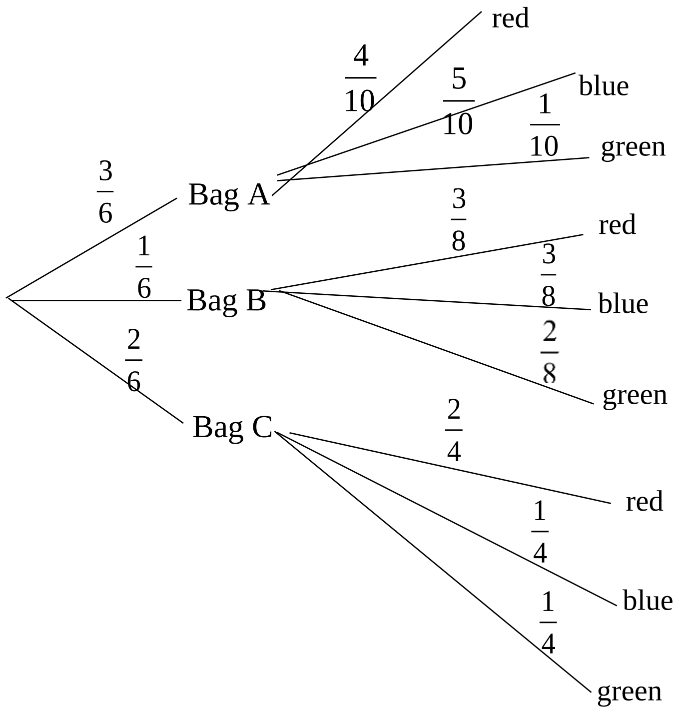
(ii) Find $P(Y)$. [2]

(iii) Find $P(X \cup Y)$. [2]

(iv) Explain, in the context of this question, what is meant by $P(X \mid Y')$, and find its value.

[3]

Answer: (ii) $\dfrac{103}{240}$ (iii) $\dfrac{35}{48}$ (iv) $\dfrac{72}{137}$

9.	JPJC JC2 Prelim 8865/2019/Q10 (Solutions)
(i)	
(ii)	$\mathrm{P}(Y)=\left(\frac{3}{6}\times\frac{4}{10}\right)+\left(\frac{1}{6}\times\frac{3}{8}\right)+\left(\frac{2}{6}\times\frac{2}{4}\right)=\frac{103}{240}$ or 0.429 (3sf)
(iii)	$\mathrm{P}(X\cup Y)=\mathrm{P}(X)+\mathrm{P}(Y)-\mathrm{P}(X\cap Y)$ $=\frac{3}{6}+\frac{103}{240}-\left(\frac{3}{6}\times\frac{4}{10}\right)$ $=\frac{35}{48}$
	Alternatively, $\mathrm{P}(X\cup Y)=\mathrm{P}(X)+\mathrm{P}(X'\cap Y)$ $=\frac{3}{6}+\left(\frac{1}{6}\times\frac{3}{8}\right)+\left(\frac{2}{6}\times\frac{2}{4}\right)$ $=\frac{35}{48}$
(iv)	$\mathrm{P}(X\mid Y')=\frac{\mathrm{P}(X\cap Y')}{\mathrm{P}(Y')}=\frac{\frac{3}{6}\times\left(1-\frac{4}{10}\right)}{1-\frac{103}{240}}=\frac{72}{137}$ or 0.526
	It is the probability of selecting the ball from bag A given that the ball is not red.

10. DHS JC2 Prelim 8865/2019/Q7

Five hundred male and female employees of different nationalities in a particular international company are shown in the following table. Each employee can only be of one nationality.

	Chinese	French	Indian	Total
Male	152	84	122	358
Female	28	64	50	142
Total	180	148	172	500

An employee is chosen at random from the company.

(i) Find the probability that the employee is a French male. [1]

(ii) Find the probability that the employee is a male or an Indian. [2]

(iii) Find the probability that the employee is a female who is not Chinese. [1]

On another occasion, 2 of these employees are chosen at random, without replacement.

(iv) Find the probability that exactly one is Chinese. [2]

(v) Find the probability that both are males, given that exactly one is Chinese. [3]

Answer: (i) $\frac{21}{125}$ (ii) $\frac{102}{125}$ (iii) $\frac{57}{250}$ (iv) 0.462 (v) 0.544

10. DHS JC2 Prelim 8865/2019/Q7 (Solutions)

(i)	P(french and male) $= \frac{84}{500} = \frac{21}{125}$ or 0.168
(ii)	P(male or Indian) $= \frac{358+172-122}{500} = \frac{102}{125}$ or 0.816
(iii)	P(female and not Chinese) $= \frac{64+50}{500} = \frac{57}{250}$ or 0.228
(iv)	P(exactly one out of two is Chinese) $= \frac{{}^{180}C_1 \times {}^{148+172}C_1}{{}^{500}C_2}$ $= \frac{1152}{2495}$ or $0.46172 = 0.462$ (3s.f.) Alternative: P(exactly one out of two is Chinese) $= 2! \times \frac{180}{500} \times \frac{320}{499}$ $= 0.46172 = 0.462$ (3s.f.)

(v)	P(both are males | exactly one is Chinese) $= \dfrac{\text{P(Chinese male and non Chinese male)}}{\text{P(exactly one Chinese)}}$ $= \dfrac{2! \times \frac{152}{500} \times \frac{84+122}{499}}{\text{(iv)}}$ or $\dfrac{{}^{152}C_1 \times {}^{84+122}C_1}{{}^{500}C_2} \div \text{(iv)}$ $= \dfrac{0.250998}{0.46172}$ $= 0.54362 = 0.544$ (3s.f.)

11. RVHS JC2 Prelim 8865/2019/Q9

During a survey, the number of adults and children speaking English, Mother Tongue and 3rd language at home are as shown in the table below. Each person only speaks exactly one of the three languages at home.

	Adult	Child	Total
English	48	22	70
Mother Tongue	18	12	30
3^{rd} Language	6	14	20
Total	72	48	120

A person is chosen at random from these 120 people. Let

C be the event that the person is a child,
E be the event that the person speaks English,
M be the event that the person speaks Mother Tongue,
L be the event that the person speaks 3^{rd} langauge.

(i) Find the following probabilities

(a) P(L') [1]
(b) P($E \cap C$) [1]
(c) P($E \cup C$) [1]
(d) P($C \mid L'$) [1]

(ii) Determine whether the events *C* and *M* are independent. [2]

(iii) An adult and a child are chosen at random. Find the probability that only one of them uses Mother Tongue at home. [3]

Answer: (ia) $\frac{5}{6}$ (ib) $\frac{11}{60}$ (ic) $\frac{4}{5}$ (id) $\frac{17}{50}$ (iii) $\frac{3}{8}$

11. RVHS JC2 Prelim 8865/2019/Q9 (Solutions)

(a) $P(L') = \dfrac{100}{120} = \dfrac{5}{6}$

(b) $P(E \cap C) = \dfrac{22}{120} = \dfrac{11}{60}$

(c) $P(E \cup C) = \frac{96}{120} = \frac{4}{5}$

(d) $P(C \mid L') = \frac{34}{100} = \frac{17}{50}$

(ii) $P(M).P(C) = \frac{30}{120} \cdot \frac{48}{120} = \frac{1}{10}$

$P(M \cap C) = \frac{12}{120} = \frac{1}{10} = P(M).P(C)$

Thus M & C are independent.

(iii) P(only 1 speaks MT| 1 adult & 1 child are chosen)

$$= \frac{P(\text{only 1 speaks MT \& 1A1C chosen})}{P(\text{1A1C chosen})}$$

$$= \frac{P(\text{only adult speaks MT}) + P(\text{only child speaks MT})}{P(\text{1A1C chosen})}$$

$$= \frac{\dfrac{\binom{18}{1}\binom{36}{1}}{\binom{120}{2}} + \dfrac{\binom{12}{1}\binom{54}{1}}{\binom{120}{2}}}{\dfrac{\binom{72}{1}\binom{48}{1}}{\binom{120}{2}}} = \frac{1296}{3486} = \frac{3}{8}$$