

## NATIONAL JUNIOR COLLEGE SENIOR HIGH 2 PRELIMINARY EXAMINATION Higher 2

**MATHEMATICS** 

Paper 2

# 9740/02

15 September 2015

3 hours

Additional Materials: Answer Paper List of Formulae (MF15) Cover Sheet

## **READ THESE INSTRUCTIONS FIRST**

Write your name, registration number, subject tutorial group, on all the work you hand in. Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in the brackets [] at the end of each question or part question.

This document consists of 6 printed pages.



National Junior College

#### Section A: Pure Mathematics [40 marks]

1 (a) Use the substitution 
$$x = 3 \tan \theta$$
 to find the exact value of  $\int_{\sqrt{3}}^{3} \frac{1}{x^2 \sqrt{x^2 + 9}} dx$ . [4]

- (b) Using integration by parts, find  $\int \ln(x^2 + 4) dx$ . [4]
- 2 (a) Joanne begins a monthly savings plan. In the first month, she puts \$1000 into her savings, and for each subsequent month she puts 5% more than in the previous month.

How many months would it take for Joanne's total savings to first exceed \$20000? [3]

Jim starts a monthly savings plan two months later than Joanne with an initial savings of \$2000, and for each subsequent month he puts \$100 more than in the previous month. The table below shows the person whose total savings exceeds the other person's total savings in the *N*th month since Joanne started saving.

N	Person with more total savings in <i>N</i> th month					
1	Joanne					
2	Joanne					
3	Joanne					
4	Joanne					
5	Jim					
6	Jim					
•						
<i>n</i> – 1	Jim					
n	Joanne					
•						

Find the value of *n*.

(b) Suppose instead that Joanne puts a fixed amount of \$1000 into her bank account on the first day of every month. The interest rate is r% per month, so that on the last day of each month the amount in the account on that day is increased by r%.

At the end of 2 years, the total amount in her account will be at least \$30000. Find the smallest value of r, correct to 1 decimal place. [3]

[4]

- 3 The planes  $p_1$  and  $p_2$  have equations  $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$  and  $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = 6$  respectively.
  - (i) Find a vector equation of l, the line of intersection between  $p_1$  and  $p_2$ . [2]

Another line, 
$$l_1$$
, has equation  $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$ , where  $\mu$  is a real parameter.

(ii) Use the result in part (i) to show that  $p_1$ ,  $p_2$  and  $l_1$  have exactly one common point of intersection. (You are not allowed to use any calculator for this part of the question.) [3]

A third plane,  $p_3$ , has equation  $\mathbf{r} \cdot \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = d$ . If  $p_1$ ,  $p_2$  and  $p_3$  have a common line of intersection, show that b = -1. [1]

- (iii) If it is further given that  $p_2$  bisects the acute angle between  $p_1$  and  $p_3$ , find the equation for  $p_3$  completely, in scalar-product form. [5]
- 4 The complex number *z* satisfies both the relations

$$|z+3-3i| \le 5\sqrt{2}$$
 and  $0 \le \arg(z-6) \le \frac{3}{4}\pi$ .

- (i) On an Argand diagram, shade the region in which the points representing z can lie. [3]
- (ii) Label the point(s) that correspond to the maximum value of |z-4-10i| on your diagram with the letter *P*. (You do not have to find the coordinates of the point(s).) [2]
- (iii) Express the smallest value of |z-4-10i| in the form  $m\sqrt{2}$ , where *m* is an integervalued constant to be determined. Show your working clearly. [2]

It is given that the complex number w satisfies the relation  $|w+3-3i| \le 5\sqrt{2}$  only.

(iv) Find the minimum value of  $|\arg(w-6)|$ , giving your answer in radians, correct to 3 decimal places. [4]

### Section B: Statistics [60 marks]

5 A junior college comprises a total of 2500 students. A canteen operator conducts a survey on the students' preferences on the types of food. A sample of 500 students is to be selected to take part in the survey. Describe briefly how this sample can be obtained via

(a)	random sampling,	[2]
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- (b) systematic sampling. [2]
- 6 In a certain school, students are allowed to wear either their school uniforms, PE T-shirts or House T-shirts on Fridays. It was found that on a particular Friday, 78% of the students wore their school uniforms, 8% wore PE T-shirts and the rest of the students wore their House Tshirts. (You may assume that the school population size is very large.)
  - (i) A random sample of 20 students was taken. Find the probability that exactly 3 students from this sample wore their house T-shirts on that day. [2]
  - (ii) Another random sample of 60 students was taken. Using a suitable approximation, find the probability that more than 45 students wore their school uniforms on that day. [4]
- 7 A box contains 6 orange, 6 red, 6 green, 6 blue and 6 yellow balls. Balls of the same colour are considered to be indistinguishable.

Find the number of ways to select

(i)	4 distinct balls,	[1]
(ii)	4 balls, of which exactly 3 are identical,	[2]
(iii)	4 balls without any restriction.	[3]

- 8 A hotel has 34 single-bed rooms and 60 double-bed rooms, each of which can be booked for a day at a time.
  - (i) State, in context, two assumptions needed for the number of demands for a single-bed room for a particular day to be well modelled by a Poisson distribution. [2]

Assume now that these assumptions in fact hold, and also that the number of demands per day for a single-bed room and the number of demands per day for a double-bed room follow independent Poisson distributions with means 5.8 and 37.1 respectively.

- (ii) On a particular day, 24 of the single-bed rooms are undergoing renovation and not available for booking. Find the probability that all the remaining single-bed rooms are booked for this day.
- (iii) Show that there is a probability of 0.0927 that, for a given day, there are at least 36 demands for a double-bed room and at least 1 demand for a single-bed room, given that there is a total of 38 demands for that day. [3]

**9** In this question, you should state clearly the values of the parameters of any normal distributions you use.

The weight, in grams, of a handphone is a random variable with the distribution  $N(\mu, 5^2)$ , where  $\mu$  is a constant. The weight, in grams, of a tablet is a random variable with the independent distribution  $N(330, 6^2)$ .

- (a) Suppose  $\mu = 130$ . Calculate the probability that the total weight of 5 randomly selected handphones is less than twice the weight of one randomly selected tablet. [3]
- (b) Suppose instead that  $\mu$  is unknown. A random sample of *n* handphones is taken. There is a probability of at most 0.04 that the mean weight of these handphones differs from the population mean weight of the handphones by more than 1 gram. Calculate the least value of *n*. [4]
- 10 In a particular country, cultured milk are manufactured, in bottles, by two companies, Yacoat and Vitergent. 60% of all cultured milk in that country are manufactured by Yacoat while the remaining 40% are manufactured by Vitergent. Both companies manufacture the cultured milk in three different flavours, "Apple", "Grape" and "Orange", and the percentage of each flavor manufactured by each of the two companies is as follows,

	Apple	Grape	Orange
Yacoat	50%	30%	20%
Vitergent	100 <i>p</i> %	10%	100 <i>q</i> %

where p and q are real constants. A bottle of cultured milk is randomly selected.

- (a) Find the probability that
  - (i) it is grape-flavoured. [2]
  - (ii) it was manufactured by Yacoat, given that it is grape-flavoured. [1]
- (b) Suppose that the probability that it is apple-flavoured and is manufactured by Vitergent is 0.1 less than the probability that it is orange-flavoured and manufactured by Vitergent. By setting up two equations involving p and q, find the values of p and q. [3]

Assume now that p = 0.4 and q = 0.5, and that two bottles of cultured milk are randomly selected.

(c) Events *A* and *B* are defined as follows:

*A* : One of them is manufactured by Yacoat and the other by Vitergent. *B* : Both bottles are apple-flavoured.

Find 
$$P(A \cup B)$$
.

[4]

11 The manufacturer of Best Chocolates claims that the mean weight of his products is 40 grams. A consumer suspects that the manufacturer is overstating the mean weight of the chocolates and decides to determine the validity of the manufacturer's claim. He gathers a random sample of 40 chocolates and weighs each of the 40 chocolates. The results are summarised by

$$\sum (x-30) = 360$$
 and  $\sum (x-30)^2 = 4050$ .

- (i) Find unbiased estimates of the population mean and the variance. [2]
- (ii) Using a 4% level of significance, test whether the manufacturer is overstating the mean weight of his product. Explain the meaning of 4% level of significance in the context of the question.
- (iii) A rival manufacturer, Godeva Chocolates, claims that the mean weight of its chocolates is  $\mu_0$  grams. An investigator collects a random sample of 20 chocolates with weights that have a mean and variance of 41 grams and 0.25 grams<sup>2</sup> respectively. It can be assumed that the weights of Godeva chocolates are normally distributed. If there is insufficient evidence at 5% significance level to claim that Godeva Chocolates has overstated the mean weight of its products, find the range of values of  $\mu_0$ . [3]
- 12 A medical officer wishes to investigate a patient's walking speed s km/h and his heart-beat rate h beats per minute (bpm). The data is shown below:

<i>s</i> (km/h)	1	1.5	2	2.5	3	3.5	4	4.5	5
h (bpm)	60	63	66	75	86	99	150	110	130

- (i) Sketch a scatter plot of the above data.
- (ii) One of the values of *h* appears to be incorrect. Indicate the corresponding point on your diagram by labelling it *P*. [1]

Omit *P* for the remainder of this question.

(iii) Calculate the product moment correlation coefficient for this set of data. Use the equation of an appropriate regression line to predict the value of s when h = 100, justifying your choice of regression line. [4]

It is suggested to use one of the following two models instead:

Model (I): 
$$h = a + bs^2$$
,  
Model (II):  $h = a + be^s$ ,

where *a* and *b* are real constants.

- (iv) Determine which of the two models is a better choice, giving a reason for your answer.
- (v) Suppose a new data pair  $(\bar{s}, \bar{h})$  is added to the table above, where  $\bar{s}$  and  $\bar{h}$  are the patient's sample mean walking speed (in km/h) and his sample mean heart-beat rate (in bpm) respectively, based on the data above. Without any calculations, explain whether the equation of the regression line you have obtained in part (iii) would change. [2]

[1]

[2]