Solutions to Statistics Tutorial 2: Probability

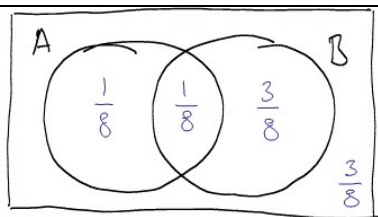## Basic Mastery Questions

1.   The events A and B are such that $P(A) = 0.43$, $P(B) = 0.48$, and $P(A \cup B) = 0.78$. Show that the events are neither mutually exclusive nor independent.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.43 + 0.48 - 0.78 = 0.13 \neq 0 \Rightarrow A$ and B are not mutually exclusive.
$P(A) \times P(B) = 0.43 \times 0.48 = 0.2064 \neq 0.13 = P(A \cap B) \Rightarrow A$ and B are not
           independent.

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2.   Two events A and B are such that $P(A) = \dfrac{1}{4}, P(B) = \dfrac{1}{2}$ and $P(A \cap B) = \dfrac{1}{8}$. Find

   (a) $P(A \cup B)$ (b) $P(A \cup B')$ (c) $P(A \mid B)$

(a) $P(A \cup B) = \dfrac{5}{8}$

(b) $P(A \cup B') = \dfrac{5}{8}$

(c) $P(A \mid B) = \dfrac{1/8}{1/2} = \dfrac{1}{4}$

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3.   A bag contains three green balls and five yellow balls. One ball is chosen and its colour noted before being replaced in the bag. A second ball is selected and its colour is also noted. By drawing a probability tree. work out:
   (i)   the probability that two green balls are chosen;
   (ii)   the probability that the two balls are of different colours.
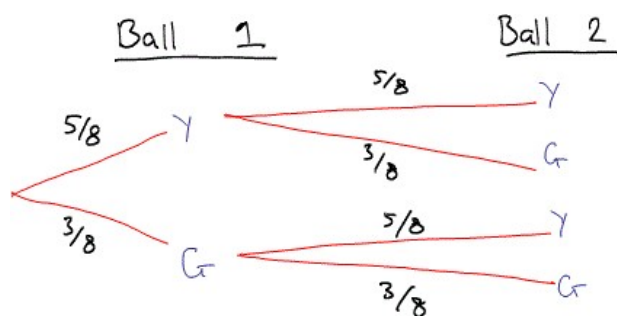
(i) $P(GG)$
   $= \dfrac{3}{8} \times \dfrac{3}{8} = \dfrac{9}{64}$

(ii) $P(GY) + P(YG)$
   $= 2 \times \dfrac{3}{8} \times \dfrac{5}{8} = \dfrac{15}{32}$
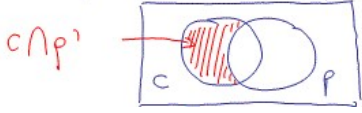
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4. In a group of 60 students, 30 study either chemistry or physics. If 20 study chemistry and 17 study physics, what is the probability that a student chosen at random studies:
    (a)   both physics and chemistry,
    (b)   neither physics nor chemistry,
    (c)   chemistry but not physics?

Let p be the event the student studies physics, and c be the event the student studies chemistry.

(a) $P(p \cap c) = P(p) + P(c) - P(p \cup c)$

$= \frac{17}{60} + \frac{20}{60} - \frac{20}{60} = \frac{7}{60}$

(b) $P(p' \cap c') = 1 - P(p \cup c) = 1 - \frac{30}{60} = \frac{30}{60}$

(c) $P(c \cap p') = P(c \cup p) - P(p) = \frac{13}{60}$

$c \cap p'$

5. The number of boys and girls in a class who travel to school by bus and car are given in the following table.

	Bus	Car
Boys	x	$20 - x$
Girls	6	4

If two of the 10 girls are selected at random, calculate the probability that
    (i)   both travel to school by bus,
    (ii)   both travel to school by different means of transportation,
    (iii) at least one travels to school by bus.

(i) $(6/10)(5/9) = 1/3$ OR $\dfrac{\binom{6}{2}\binom{4}{0}}{\binom{10}{2}} = \dfrac{15}{45} = \dfrac{1}{3}$

(ii) $(6/10)(4/9) + (4/10)(6/9) = 8/15$ OR $\dfrac{\binom{6}{1}\binom{4}{1}}{\binom{10}{2}} = \dfrac{8}{15}$

(iii) $1 - P(\text{both travel by car}) = 1 - (4/10)(3/9) = 13/15$ OR $1 - \dfrac{\binom{6}{0}\binom{4}{2}}{\binom{10}{2}} = \dfrac{13}{15}$