CANDIDATE NAME	CLASS	INDEX NUMBER

SEMBAWANG SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS — 4049/01

Paper 1

26 Aug 2021
1050 – 1305
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of the marks for this paper is 90.

FOR EXAMINER'S USE	
TOTAL	90

PARENT'S SIGNATURE

Setter: Ms Siew Yan Yee

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0,$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}.$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The equation of a curve is $y = mx^2 + 9x + k$, where k is a constant. In the case where $m = k$, find the set of values of k for which the curve lies completely above the x-axis. [3]

2 Given that $\sqrt{343^x} = \dfrac{7^{1-x}}{49}$, find the value of $\sqrt{343^x}$. [4]

3 Given that $y = \dfrac{e^{2x}}{x-1}$ for $x > 1$, find the range of values of *x* for which *y* is a decreasing function of *x*. [4]

4 A rectangular block has a square base of side $(\sqrt{10}+\sqrt{2})$ cm and a height of *h* cm. The volume of the rectangular block is $(44+4\sqrt{5})$ cm^3. **Without using a calculator**, show that *h* can be expressed as $a+b\sqrt{5}$, where *a* and *b* are integers. [5]

5 The term containing the highest power of x in the polynomial f(x) is $4x^4$. Given that f(x) $= 0$ has two roots –2 and 3, and that $2x^2 + 3x + 2$ is a quadratic factor of f(x),

(i) express f(x) as a polynomial in x with integer coefficients, [3]

(ii) explain why f(x) $= 0$ has exactly two real roots. [2]

6 It is given that $f(x) = a^2x^3 + 7x + b^2$ and $g(x) = -18 + 2abx^3$.

(a) Show that there are no real values of *a* and *b* for which f(*x*) and g(*x*) leave the same remainder when divided by $x - 1$. [4]

(b) Given that $3x - 2$ is a factor of g(*x*), express *a* in terms of *b*. [2]

[Turn over

7 Winston believes that the depth of water, d metres, at the end of a jetty, t hours after low tide, can be modelled by the equation $d = a + b\cos kt,$ where a, b and k are constants.

(i) Assuming that low tides occur every 12 hours, show that $k = \frac{\pi}{6}$. [2]

(ii) Winston measures the depth of water at low tide and high tide to be 2 metres and 6 metres respectively. Calculate the value of a and of b. [2]

(iii) Winston requires the depth of water at the end of the jetty to be at least 3 metres to sail his boat.

Given that the low tide on a particular day was at 0830, find the earliest time that Winston could sail his boat. [3]

8 A curve is such that $y = 2x + \dfrac{8x}{x-1}$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Obtain an expression for $\dfrac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of each stationary point. [3]

9 In a factory, a machine part P, which is connected to a food cutter, moves in a straight line from a fixed point O so that t seconds after passing O, its velocity, v m s^{-1}, is given by $v = 2\cos\frac{t}{2} + 1,$ where $0 \le t \le 2\pi.$

(i) Find the value of t when P first comes to an instantaneous rest, giving your answer in terms of π. [3]

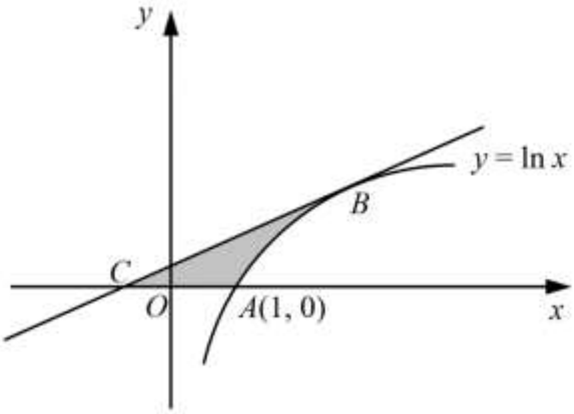
(ii) Find the total distance travelled by *P* from $t = 0$ to $t = 2\pi$. [4]

(iii) Find the acceleration of *P* when $t = \pi$. [2]

10 **(i)** Express $\dfrac{52x^2-20x-6}{(x-2)(4x+1)^2}$ as the sum of 3 partial fractions. [6]

(ii) Hence, evaluate $\int_3^4 \frac{52x^2-20x-6}{(x-2)(4x+1)^2}\,\mathrm{d}x.$ [4]

11



The diagram shows part of the curve $y = \ln x$, meeting the x-axis at the point $A(1, 0)$. The tangent to the curve at B meets the x-axis at the point C.

(i) Given that the gradient of the tangent at B is $\frac{1}{e^2}$, show that the coordinates of B are $(e^2, 2)$. [3]

(ii) By finding the equation of the tangent to the curve at *B*, determine the coordinates of *C*. [3]

(iii) Using the result $\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x - x) = \ln x,$ find the area of the shaded region, leaving your answer in exact form. [4]

12 The equation of a circle C_1 is $x^2 + y^2 - 2x - 8y - 33 = 0$.

(i) Find the coordinates of the centre and the radius of C_1. [3]

(ii) The circle C_1 intersects the x-axis at the points P and Q.
By finding the x-coordinates of P and of Q, determine the coordinates of the midpoint of PQ. [3]

(iii) Show that the equation of the tangent to the circle C_1 at the point P(6, –1) is $y = x - 7$. [3]

(iv) Find the equation of another circle C_2 which has the same centre as C_1 and passes through (1, –5). [1]

13 **(a)** **(i)** Without the use of a calculator, show that $\cos 75° = \frac{\sqrt{6} - \sqrt{2}}{4}$. [3]

(ii) Hence, evaluate $\cos 150°$, leaving your answer in the form $-\frac{\sqrt{a}}{b}$. [3]

(b) Prove that $\left(\frac{1}{\tan x}+\frac{1}{\sin x}\right)^2=\frac{1+\cos x}{1-\cos x}$. [4]

End of Paper

[Turn over