**CATHOLIC JUNIOR COLLEGE**
**H2 MATHEMATICS**
**2023 JC2 PRELIM EXAM PAPER 2 SOLUTION**

**Q1**

$$\frac{x^2+2x-5}{x^2-2x}<2$$

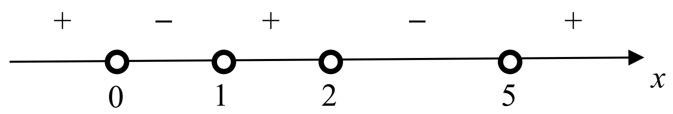
$$\frac{x^2+2x-5}{x^2-2x}-2<0$$

$$\frac{x^2+2x-5-2x^2+4x}{x^2-2x}<0$$

$$\frac{-x^2+6x-5}{x(x-2)}<0$$

$$\frac{x^2-6x+5}{x(x-2)}>0$$

$$\frac{(x-1)(x-5)}{x(x-2)}>0$$

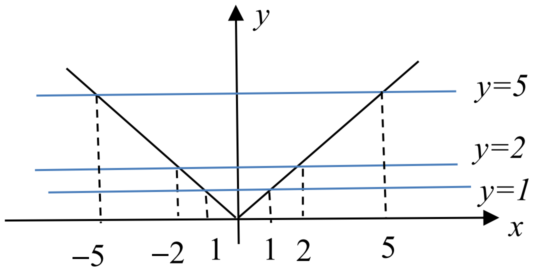



$\therefore\ x<0$ or $1<x<2$ or $x>5$.

Solution of $\frac{x^2+2x-5}{x^2-2x}>2$ is the complementary of solution of $\frac{x^2+2x-5}{x^2-2x}<2$

So, replacing $x$ with $|x|$, solution of $\frac{x^2+2|x|-5}{x^2-2|x|}>2$ will be

$$0<|x|<1 \quad \text{or} \quad 2<|x|<5$$




For $0<|x|<1$, $-1<x<1,\ x\neq 0$

For $2<|x|<5$, $-5<x<-2$ or $2<x<5$

Thus, range of values: $-5<x<-2$ or $2<x<5$ or $-1<x<1,\ x\neq 0$

| Q2 | |
|---|---|
| **(a)** | $y = e^{-x}\sin x + x - 1$ <br> $\frac{dy}{dx} = e^{-x}\cos x - e^{-x}\sin x + 1$ <br> $= e^{-x}(\cos x - \sin x) + 1$ <br> $\frac{d^2y}{dx^2} = e^{-x}(-\sin x - \cos x) - e^{-x}(\cos x - \sin x)$ <br> $= -2e^{-x}\cos x \quad \text{where } k = -2 \text{ (shown)}$ |
| **(b)** | $\frac{d^3y}{dx^3} = -2e^{-x}(-\sin x) - (-2e^{-x})\cos x$ <br> $= 2e^{-x}(\sin x + \cos x)$ <br> When $x = 0: f(0) = -1$ <br> $f'(0) = 2$ <br> $f''(0) = -2$ <br> $f'''(0) = 2$ <br> $y = -1 + 2x - 2\cdot\frac{x^2}{2!} + 2\cdot\frac{x^3}{3!} + \ldots$ <br> $= -1 + 2x - x^2 + \frac{x^3}{3} + \ldots$ |

| | |
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| **(c)** | $$\begin{aligned}\frac{e^{-x}\sin x+x-1}{\cos 2x}&=\frac{-1+2x-x^2+\frac{x^3}{3}+\ldots}{1-\frac{(2x)^2}{2!}+\ldots}\\&=\left(-1+2x-x^2+\frac{x^3}{3}+\ldots\right)\left(1-2x^2\right)^{-1}\\&=\left(-1+2x-x^2+\frac{x^3}{3}+\ldots\right)\left[1+(-1)\left(-2x^2\right)+\ldots\right]\\&=\left(-1+2x-x^2+\frac{x^3}{3}+\ldots\right)\left(1+2x^2+\ldots\right)\\&=-1-2x^2+2x+4x^3-x^2+\frac{x^3}{3}+\ldots\\&=-1+2x-3x^2+\frac{13}{3}x^3+\ldots\end{aligned}$$ |

| Q3 | |
|---|---|
| (a) | **Method 1** |

$$\frac{1}{P(13-2P)}=\frac{M}{P}+\frac{N}{13-2P}$$

$$1=M(13-2P)+NP$$

When $P=0$, $M=\frac{1}{13}$

When $P=\frac{13}{2}$, $N=\frac{2}{13}$

$$\frac{1}{P(13-2P)}=\frac{1}{13P}+\frac{2}{13(13-2P)}$$

$$=\frac{1}{13}\left(\frac{1}{P}+\frac{2}{13-2P}\right)$$

$$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{26}P(13-2P)$$

$$\int\frac{1}{P(13-2P)}\,\mathrm{d}P=\frac{1}{26}\int 1\,\mathrm{d}t$$

$$\frac{1}{13}\int\frac{1}{P}+\frac{2}{(13-2P)}\,\mathrm{d}P=\frac{1}{26}\int 1\,\mathrm{d}t$$

$$\int\frac{1}{P}\,\mathrm{d}P-\int\frac{-2}{(13-2P)}\,\mathrm{d}P=\frac{1}{2}\int 1\,\mathrm{d}t$$

$$\ln|P|-\ln|13-2P|=\frac{1}{2}t+C$$

$$\ln\left|\frac{P}{13-2P}\right|=\frac{1}{2}t+C$$

$$\left|\frac{P}{13-2P}\right|=\mathrm{e}^{\frac{1}{2}t+C}$$

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| | $$\frac{P}{13-2P}=\pm e^{\frac{1}{2}t+C}$$ $$\frac{P}{13-2P}=\pm e^{\frac{1}{2}t}\cdot e^{C}$$ $$\frac{P}{13-2P}=Ae^{\frac{1}{2}t}$$ $$P=13Ae^{\frac{1}{2}t}-2APe^{\frac{1}{2}t}$$ $$P\left(1+2Ae^{\frac{1}{2}t}\right)=13Ae^{\frac{1}{2}t}$$ $$P=\frac{13Ae^{\frac{1}{2}t}}{1+2Ae^{\frac{1}{2}t}}$$ $$P=\frac{13A}{e^{-\frac{1}{2}t}+2A}$$ When $t=0$, $P=2$, $$2=\frac{13A}{1+2A}$$ $$2+4A=13A$$ $$9A=2$$ $$A=\frac{2}{9}$$ |

$$P=\frac{13\left(\frac{2}{9}\right)}{\mathrm{e}^{-\frac{1}{2}t}+2\left(\frac{2}{9}\right)}$$

$$=\frac{26}{9\mathrm{e}^{-\frac{1}{2}t}+4}\ (\text{shown})$$

| | |
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| | **Method 2**<br>$$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{26}P(13-2P)$$<br>$$\int\frac{1}{P(13-2P)}\,\mathrm{d}P=\frac{1}{26}\int 1\,\mathrm{d}t$$<br>$$\int\frac{1}{13P-2P^2}\,\mathrm{d}P=\frac{1}{26}\int 1\,\mathrm{d}t$$<br>$$-\frac{1}{2}\int\frac{1}{\left(P-\frac{13}{4}\right)^2-\left(\frac{13}{4}\right)^2}\,\mathrm{d}P=\frac{1}{26}\int 1\,\mathrm{d}t$$<br>$$-\frac{1}{2}\frac{1}{2\left(\frac{13}{4}\right)}\ln\left\|\frac{P-\frac{13}{4}-\frac{13}{4}}{P-\frac{13}{4}+\frac{13}{4}}\right\|=\frac{1}{26}t+C$$<br>$$\ln\left\|\frac{P-\frac{13}{2}}{P}\right\|=-\frac{1}{2}t+C'$$<br>$$\left\|\frac{P-\frac{13}{2}}{P}\right\|=\mathrm{e}^{-\frac{1}{2}t+C'}$$ |

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| | $$\frac{P-\frac{13}{2}}{P}=\pm e^{-\frac{1}{2}t+C'}$$ $$\frac{P-\frac{13}{2}}{P}=\pm e^{-\frac{1}{2}t}\cdot e^{C'}$$ $$\frac{P-\frac{13}{2}}{P}=Ae^{-\frac{1}{2}t}$$ $$P=\frac{13}{2}+APe^{-\frac{1}{2}t}$$ $$P\left(1-Ae^{-\frac{1}{2}t}\right)=\frac{13}{2}$$ $$P=\frac{13}{2-4Ae^{-\frac{1}{2}t}}$$ When $t=0$, $P=2$, $$2=\frac{13}{2-4A}$$ $$4-8A=13$$ $$A=-\frac{9}{8}$$ $$P=\frac{13}{2-4\left(-\frac{9}{8}\right)e^{-\frac{1}{2}t}}$$ $$=\frac{26}{9e^{-\frac{1}{2}t}+4}\ \text{(shown)}$$ |

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| **(b)** | When $P=4$,<br>$$4=\frac{26}{9e^{-\frac{1}{2}t}+4}$$<br>$$4\left(9e^{-\frac{1}{2}t}+4\right)=26$$<br>$$9e^{-\frac{1}{2}t}=\frac{5}{2}$$<br>$$e^{-\frac{1}{2}t}=\frac{5}{18}$$<br>$$-\frac{1}{2}t=\ln\left(\frac{5}{18}\right)$$<br>$$t=-2\ln\left(\frac{5}{18}\right)$$<br>$$t=2.56$$<br>It takes 2.56 months for the number of people who downloaded Ginseng Impact to double since the launch. |

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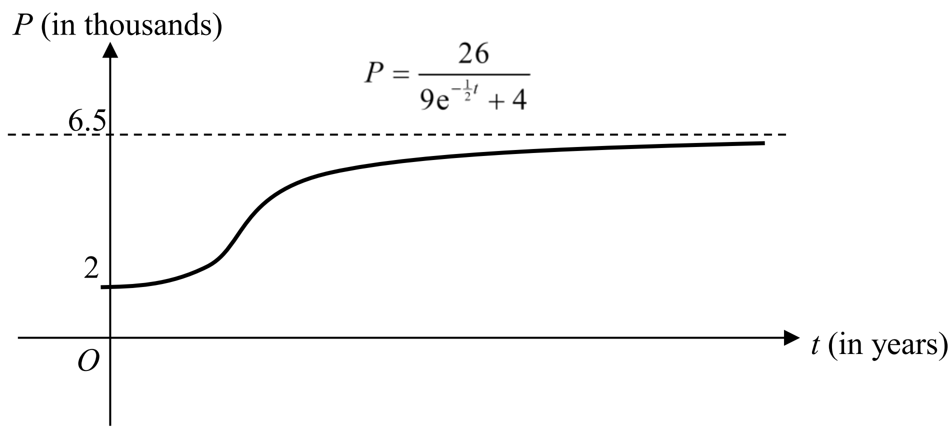
| **(c)** | As $t \to \infty$, $e^{-\frac{1}{2}t} \to 0$ , <br> $$P \to \frac{26}{4}$$ <br> Number of people that downloaded Ginseng Impactful in the long run is $\frac{26}{4}(1000) = 6500$ . |
| **(d)** |   |

| Q4 | |
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| **(a)** | Vector equation of the line $l$ is<br>$$\underset{\sim}{r} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$$<br>Let $\alpha$ be the acute angle between the normal vector of plane $\Pi_1$ and line $l$.<br>$$\alpha = \cos^{-1}\left|\frac{\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}}{\left\|\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}\right\|\left\|\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}\right\|}\right|$$<br>$$= \cos^{-1}\left|\frac{3}{\sqrt{6}\sqrt{14}}\right|$$<br>$= 70.893^\circ$ or $1.2373$<br>$\therefore$ acute angle between the plane $\Pi_1$ and line $l$ is<br>$= 90^\circ - 70.893^\circ$ or $\frac{\pi}{2} - 1.2373$<br>$= 19.1^\circ$ or $0.333$ |
| **(b)** | Substitute equation of line into the equation of plane: |

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| | $\left[\begin{pmatrix}-2\\4\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\1\end{pmatrix}\right]\cdot\begin{pmatrix}2\\1\\-1\end{pmatrix}=3$<br>$-3+3\lambda=3$<br>$\lambda=2$<br>When $\lambda=2$,<br>$\underset{\sim}{r}=\begin{pmatrix}-2\\4\\3\end{pmatrix}+2\begin{pmatrix}3\\-2\\1\end{pmatrix}=\begin{pmatrix}4\\0\\5\end{pmatrix}$<br>Coordinates of the point of intersection is (4, 0, 5). |

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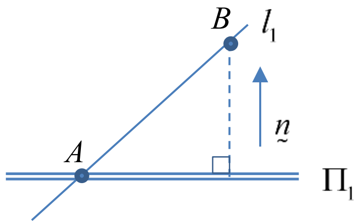
| **(c)** | **<u>Method ①:</u>**<br>Perpendicular distance from $B$ to $\Pi_1$<br>$= \left\lvert \dfrac{\overrightarrow{AB} \cdot \underset{\sim}{n}}{\lvert \underset{\sim}{n} \rvert} \right\rvert$<br>$= \left\lvert \dfrac{\left[\begin{pmatrix}10\\-4\\7\end{pmatrix} - \begin{pmatrix}4\\0\\5\end{pmatrix}\right] \cdot \begin{pmatrix}2\\1\\-1\end{pmatrix}}{\left\lvert \begin{pmatrix}2\\1\\-1\end{pmatrix} \right\rvert} \right\rvert$<br>$= \left\lvert \dfrac{\begin{pmatrix}6\\-4\\2\end{pmatrix} \cdot \begin{pmatrix}2\\1\\-1\end{pmatrix}}{\sqrt{6}} \right\rvert$<br>$= \left\lvert \dfrac{6}{\sqrt{6}} \right\rvert$<br>$= \sqrt{6}$ units<br><br><br><br>**<u>Method ②:</u>**<br>Equation of line that is parallel to normal vector and passing through (10, -4, 7) is:<br>$\underset{\sim}{r} = \begin{pmatrix}10\\-4\\7\end{pmatrix} + \mu \begin{pmatrix}2\\1\\-1\end{pmatrix}, \mu \in \mathbb{R}$<br>At the point of intersection, |

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| | $\left[\begin{pmatrix}10\\-4\\7\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}\right]\cdot\begin{pmatrix}2\\1\\-1\end{pmatrix}=3$<br>$9+6\mu=3$<br>$\mu=-1$<br>When $\mu=1$,<br>$\underset{\sim}{r}=\begin{pmatrix}10\\-4\\7\end{pmatrix}+\begin{pmatrix}-2\\-1\\1\end{pmatrix}=\begin{pmatrix}8\\-5\\8\end{pmatrix}$<br><br>Perpendicular distance from $B$ to $\Pi_1$<br>$=\left\|\begin{pmatrix}8\\-5\\8\end{pmatrix}-\begin{pmatrix}10\\-4\\7\end{pmatrix}\right\|$<br>$=\left\|\begin{pmatrix}-2\\-1\\1\end{pmatrix}\right\|$<br>$=\sqrt{(-2)^2+(-1)^2+1^2}$<br>$=\sqrt{6}$ units |

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| **(d)** | A vector perpendicular to the plane $\Pi_2$ is<br>$$\begin{pmatrix}3\\-2\\1\end{pmatrix}\times\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}1\\5\\7\end{pmatrix}$$<br>Equation of plane $\Pi_2$ is<br>$$\underset{\sim}{r}\cdot\begin{pmatrix}1\\5\\7\end{pmatrix}=\begin{pmatrix}-2\\4\\3\end{pmatrix}\cdot\begin{pmatrix}1\\5\\7\end{pmatrix}$$<br>$x+5y+7z=39$ |
| **(e)** | Note that point $(4, 0, 5)$ lies on both planes $\Pi_1$ and $\Pi_2$.<br><br>A vector parallel to the line of intersection of both planes is<br>$$\begin{pmatrix}2\\1\\-1\end{pmatrix}\times\begin{pmatrix}1\\5\\7\end{pmatrix}=\begin{pmatrix}12\\-15\\9\end{pmatrix}=3\begin{pmatrix}4\\-5\\3\end{pmatrix}$$<br>Vector equation of the line that lies in both planes is<br>$$\underset{\sim}{r}=\begin{pmatrix}4\\0\\5\end{pmatrix}+\mu\begin{pmatrix}4\\-5\\3\end{pmatrix},\mu\in\mathbb{R}$$ |

| Q5 | |
|---|---|
| | Tables of outcomes |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 3 | 3 | 4 | 5 |
| 2 | 3 | 6 | 5 | 6 |
| 3 | 4 | 5 | 9 | 7 |
| 4 | 5 | 6 | 7 | 12 |

$$\mathrm{P}(\text{spin}=1)=\frac{144}{360}=\frac{2}{5}=\frac{4}{10}$$

$$\mathrm{P}(\text{spin}=2)=\frac{108}{360}=\frac{3}{10}$$

$$\mathrm{P}(\text{spin}=3)=\frac{72}{360}=\frac{1}{5}=\frac{2}{10}$$

$$\mathrm{P}(\text{spin}=4)=\frac{36}{360}=\frac{1}{10}$$

**(a)**

$$\mathrm{P}(X=6)$$

$$=\mathrm{P}(\text{spin}_1=2,\text{spin}_2=2)+\mathrm{P}(\text{spin}_1=2,\text{spin}_2=4)+\mathrm{P}(\text{spin}_1=4,\text{spin}_2=2)$$

$$=\left[\left(\frac{3}{10}\right)\left(\frac{3}{10}\right)\right]+\left[\left(\frac{3}{10}\right)\left(\frac{1}{10}\right)\right]+\left[\left(\frac{1}{10}\right)\left(\frac{3}{10}\right)\right]$$

$$=0.15$$

**(b)**

$$\mathrm{P}(X=3)$$

$$=\mathrm{P}(\text{spin}_1=1,\text{spin}_2=1)+\mathrm{P}(\text{spin}_1=1,\text{spin}_2=2)+\mathrm{P}(\text{spin}_1=2,\text{spin}_2=1)$$

$$=\left[\left(\frac{4}{10}\right)\left(\frac{4}{10}\right)\right]+\left[\left(\frac{4}{10}\right)\left(\frac{3}{10}\right)\right]+\left[\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\right]$$

$$=0.4$$

$$\mathrm{P}(X=4)$$
$$=\mathrm{P}(\text{spin}_1=1,\text{spin}_2=3)+\mathrm{P}(\text{spin}_1=3,\text{spin}_2=1)$$
$$=\left[\left(\frac{4}{10}\right)\left(\frac{2}{10}\right)\right]+\left[\left(\frac{2}{10}\right)\left(\frac{4}{10}\right)\right]$$
$$=0.16$$

$$\mathrm{P}(X=5)$$
$$=\mathrm{P}(\text{spin}_1=1,\text{spin}_2=4)+\mathrm{P}(\text{spin}_1=4,\text{spin}_2=1)+\mathrm{P}(\text{spin}_1=2,\text{spin}_2=3)+\mathrm{P}(\text{spin}_1=3,\text{spin}_2=2)$$
$$=\left[\left(\frac{4}{10}\right)\left(\frac{1}{10}\right)\right]+\left[\left(\frac{1}{10}\right)\left(\frac{4}{10}\right)\right]+\left[\left(\frac{3}{10}\right)\left(\frac{2}{10}\right)\right]+\left[\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\right]$$
$$=0.2$$

$$\mathrm{P}(X=7)$$
$$=\mathrm{P}(\text{spin}_1=3,\text{spin}_2=4)+\mathrm{P}(\text{spin}_1=4,\text{spin}_2=3)$$
$$=\left[\left(\frac{2}{10}\right)\left(\frac{1}{10}\right)\right]+\left[\left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\right]$$
$$=0.04$$

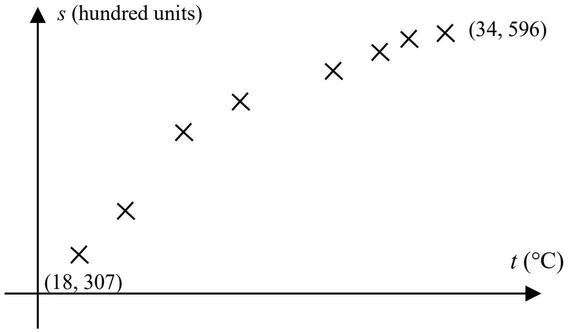
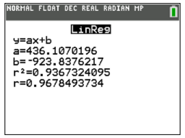
$$\mathrm{P}(X=9)$$
$$=\mathrm{P}(\text{spin}_1=3,\text{spin}_2=3)$$
$$=\left[\left(\frac{2}{10}\right)\left(\frac{2}{10}\right)\right]$$
$$=0.04$$

$$\mathrm{P}(X=12)$$
$$=\mathrm{P}(\text{spin}_1=4,\text{spin}_2=4)$$
$$=0.01$$

**Alternative presentation format (Probability Distribution Table)**

| $x$ | 3 | 4 | 5 | 6 | 7 | 9 | 12 |
|---|---|---|---|---|---|---|---|

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\mathrm{P}(X=x)$ | $\frac{40}{100}$ $=\frac{2}{5}$ $=0.4$ | $\frac{16}{100}$ $=\frac{4}{25}$ $=0.16$ | $\frac{20}{100}$ $=\frac{1}{5}$ $=0.2$ | $\frac{15}{100}$ $=\frac{3}{20}$ $=0.15$ | $\frac{4}{100}$ $=\frac{1}{25}$ $=0.04$ | $\frac{4}{100}$ $=\frac{1}{25}$ $=0.04$ | $\frac{1}{100}$ $=0.01$ |
| **(c)** | $\mathrm{P}(\text{Score} < 10 \mid \text{Customer wins a prize})$ $=\mathrm{P}(X<10 \mid X>6)$ $=\dfrac{\mathrm{P}(X<10 \cap X>6)}{\mathrm{P}(X>6)}$ $=\dfrac{\mathrm{P}(X=7)+\mathrm{P}(X=9)}{\mathrm{P}(X=7)+\mathrm{P}(X=9)+\mathrm{P}(X=12)}$ $=\dfrac{(0.04)+(0.04)}{(0.04)+(0.04)+(0.01)}$ $=\dfrac{8}{9}$ or 0.889 (3 s.f.) | | | | | | | |

<table>
<tr><th colspan="2">Q6</th></tr>
<tr><td>(a)</td><td>
</td></tr>
<tr><td></td><td>$r = 0.9474557$<br>$= 0.947$ (3 s.f.)<br>Although the product moment correlation coefficient $r = 0.947$ is close to +1, which suggests a strong, positive, linear relationship, The scatter diagram indicates that as <u>$t$ increases</u>, <u>$s$ increases at a decreasing rate</u> i.e. the scatter diagram shows the points appear to lie on a curve rather than a straight line, so a linear model may not model the relationship well.</td></tr>
<tr><td>(b)</td><td>For $s = a\ln t + b$,<br>
<br>$a = 436.107$ (3 d.p.)<br>$b = -923.838$ (3 d.p.)</td></tr>
<tr><td>(c)</td><td>$r = 0.967849 = 0.968$ (3 s.f.)</td></tr>
<tr><td>(d)</td><td>$s = a\ln t + b$ is a better model as the $r$-value is <u>closer to 1</u>.</td></tr>
<tr><td>(e)</td><td>When $t = 38$,<br>$s = 436.107\ln(38) - 923.8376$<br>$s = 662.53655$<br><br>Sales = 66,254 units<br><br>Not reliable since 38°C is out of the data range hence extrapolation was performed.</td></tr>
</table>

| Q7 | |
|---|---|
| (a)(i) | No. of ways $= {}^{12}C_3 \times 3! = 1320$ |
| (a)(ii) | **Method ①:**<br>No. of ways $= \underbrace{{}^{12}C_2 \times {}^{6}C_1 \times 3!}_{\text{Select 2 girls from 12 \& 1 boy from 6 followed by arrangement}} + \underbrace{{}^{12}C_1 \times {}^{6}C_2 \times 3!}_{\text{Select 1 girl from 12 \& 2 boys from 6 followed by arrangement}}$<br>$= 3456$<br><br>**Method ②:**<br>No. of ways<br>$= \underbrace{{}^{18}C_3 \times 3!}_{\text{No restriction}} - \underbrace{{}^{12}C_3 \times 3!}_{\text{Select 3 girls from 12 followed by arrangement}} - \underbrace{{}^{6}C_3 \times 3!}_{\text{Select 3 boys from 6 followed by arrangement}}$<br>$= 3456$ |
| (b) | **Method ①:**<br>Required probability $= \dfrac{(15-1)! \times {}^{15}C_3 \times 3!}{(18-1)!}$<br>$= \dfrac{91}{136}$<br><br>**Method ②: [Not Recommended]**<br>Required probability $= 1 - \underbrace{\dfrac{(16-1)! \times 3!}{(18-1)!}}_{\text{chairperson, vice-charperson and secretary together}} - \underbrace{3\left[\dfrac{(15-1)! \times {}^{15}C_2 \times 2! \times 2!}{(18-1)!}\right]}_{\text{any 2 together and 1 separate}}$<br>$= \dfrac{91}{136}$ |
| (c) | **Method ①: [Arrange boys first followed by the girls]** |

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| | Required probability $= \dfrac{(6-1)!\ \times 12!}{(18-1)!}$<br>$= \dfrac{1}{6188}$<br><br>**<u>Method ②:</u> [Arrange girls first followed by the boys]**<br>Required probability $= \dfrac{\left((6-1)! \times \dfrac{{}^{12}C_2 \times {}^{10}C_2 \times {}^{8}C_2 \times {}^{6}C_2 \times {}^{4}C_2 \times {}^{2}C_2}{6!} \times 2^6\right) \times 6!}{(18-1)!}$<br>$= \dfrac{1}{6188}$ |

| Q8 | |
|---|---|
| **(a)** | The probability of an orange being rotten is constant at *p*%.<br><br>The event of an orange being rotten is independent of the event of any other orange being rotten. |
| **(b)(i)** | Let $X$ be the random variable denoting the number of rotten oranges out of 10 oranges. (defined by the question)<br>$X \sim \mathrm{B}(10, 0.2)$<br><br>$\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$<br>$= 0.6241903616$<br>$= 0.624 \quad (\text{to 3 s.f.})$ |

| | |
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| **(b)(ii)** | Expected number of packets of oranges that contains more than 1 rotten orange<br>$=100\left[\mathrm{P}\left(X>1\right)\right]$<br>$=100\left[1-\mathrm{P}\left(X\le 1\right)\right]$<br>$=62.41903616$<br><br>**<u>Method 1</u>**<br>Expected profit when all the packets of oranges are sold $=2\left(100\right)=200$<br><br>For the store manager to have a net profit,<br>$\text{Expected loss} < \text{Expected profit}$<br>$62.41903616d<200$<br>$d<3.204150726$<br>$d<3.20 \quad \left(\text{to 2 d.p.}\right)$<br>$\therefore 0<d<3.20$<br><br>**<u>Method 2</u>**<br>Total profit<br>$=\left(100-62.41903616\right)\left(2\right)+62.41903616\left(2-d\right)$<br>$=200-62.41903616d$<br><br>For the store manager to have a net profit,<br>$\text{Total profit } >0$<br>$200-62.419d>0$<br>$d<3.20$ (to 2 d.p.)<br>$\therefore 0<d<3.20$ |

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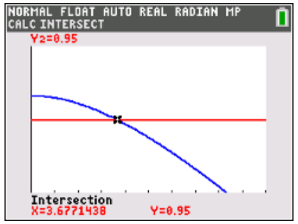
| **(c)** | Let $X$ be the random variable denoting the number of rotten oranges out of 10 oranges. (defined by the question) <br> $X \sim \mathrm{B}(10, 0.01p)$ <br> $\mathrm{P}(X \leq 1) = 0.95$ <br> $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.95$ <br> ${}^{10}\mathrm{C}_0 (0.01p)^0 (1-0.01p)^{10} + {}^{10}\mathrm{C}_1 (0.01p)^1 (1-0.01p)^9 = 0.95$ <br> $(1-0.01p)^{10} + 0.1p(1-0.01p)^9 = 0.95$ <br> Using G.C.,   <br> $p = 3.6771438$ <br> $= 3.68$ (to 3 s.f.) |

| Q9 | |
|---|---|
| **(a)** | Required probability<br>$=\left[\mathrm{P}(A<140)\right]^2\times\mathrm{P}(A>170)\times\frac{3!}{2!}$<br>$=\left[\mathrm{P}(A<140)\right]^2\times\mathrm{P}(A>170)\times 3$<br>$=(0.3341176)^2\times(0.26015833)\times 3$<br>$=0.087128$<br>$=0.0871(3\text{ s.f})$ |
| **(b)** | Let $A$ be the random variable denoting the mass of an apple from the supermarket.<br>Let $G$ be the random variable denoting the mass of a guava from the supermarket.<br>$A\sim N\left(152,28^2\right)$ and $G\sim N\left(268,43^2\right)$<br>$X=A_1+A_2+A_3+A_4+A_5$<br>$X\sim\mathrm{N}\left(5\times152,5\times28^2\right)$<br>$X\sim\mathrm{N}(760,3920)$<br>$Y=G_1+G_2+G_3$<br>$Y\sim\mathrm{N}\left(3\times268,3\times43^2\right)$<br>$Y\sim\mathrm{N}(804,5547)$<br>$X-Y\sim\mathrm{N}(760-804,3920+5547)$<br>$X-Y\sim\mathrm{N}(-44,9467)$<br>$\mathrm{P}(X<Y)=\mathrm{P}(X-Y<0)$<br>$=0.6744435$<br>$=0.674\ (\text{to 3 s.f.})$ |
| **(c)** | $F=A_1+A_2+A_3+G_1+G_2$ |

| | |
|---|---|
| | $$F \sim \mathrm{N}\left(3\times152+2\times268, 3\times28^2+2\times43^2\right)$$ $$F \sim \mathrm{N}(992, 6050)$$ Given $\mathrm{P}(\lvert F-992\rvert < m) = 0.95$ <br> $\mathrm{P}(-m < F-992 < m) = 0.95$ <br> $\mathrm{P}(992-m < F < 992+m) = 0.95$ <br><br> **<u>Method ①:</u> Using right Tail** <br> $992+m = 1144.449$ <br> $m = 152.449$ <br> $m = 153$ (3 s.f.) <br><br> **<u>Method ②:</u> Using left tail** <br> $992-m = 839.551$ <br> $m = 152.449$ <br> $m = 153$ (3 s.f.) |
| **(d)** | **<u>Method ①:</u> Convert weight from grams to kilograms to use selling price in \$/kg given in question** $$F \sim N(992, 6050) \text{ (in g)}$$ $$F' \sim N\left(\frac{992}{1000}, \frac{6050}{1000^2}\right) \text{ (in kg)}$$ $$F' \sim \mathrm{N}(0.992, 0.00605)$$ Let $C$ be the cost of a Family Pack (\$/kg). <br> $C = 5F'$ $$C \sim \mathrm{N}\left(5\times0.992, 5^2\times0.00605\right)$$ $$C \sim \mathrm{N}(4.96, 0.15125)$$ |

| | |
|---|---|
| | $\mathrm{P}(C<5)=0.54096$<br>$=0.541 \text{(to 3 s.f.)}$<br><br>**<u>Method ②:</u> Convert selling price from \$/kg to \$/g**<br>Let $C$ be the cost of a Family Pack in \$/gram and $F$ be the total mass of a Family Pack in grams (from **(iii)**).<br>Selling price of Family pack = \$5/kg = \$0.005/g<br><br>$C'=0.005F$<br>$C'\sim \mathrm{N}\left(0.005\times 992,(0.005)^2\times 6050\right)$<br>$C'\sim \mathrm{N}(4.96,0.15125)$<br>$\mathrm{P}(C<5)=0.54096$<br>$=0.541 \text{(to 3 s.f.)}$<br><br>**<u>Method ③:</u> Convert random variable from price to weight in grams**<br>$F \sim N(992,6050)$ (in g)<br>Let $C$ be the cost of a Family Pack (\$/kg).<br>$C=\frac{5}{1000}F$<br>$\mathrm{P}(C<5)$<br>$=\mathrm{P}\left(\frac{5}{1000}F<5\right)$<br>$=\mathrm{P}(F<1000)$<br>$=0.54096$<br>$=0.541 \text{ (to 3 s.f.)}$ |

| Q10 | |
|---|---|
| **(a)** | Unbiased estimates of the population mean<br>$=\dfrac{\sum(x-650)}{n}+650$<br>$=-\dfrac{34.39}{50}+650$<br>$=649.3122$<br><br>Unbiased estimates of the population variance, $s^2$<br>$=\dfrac{1}{n-1}\left(\sum(x-650)^2-\dfrac{\left(\sum(x-650)\right)^2}{n}\right)$<br>$=\dfrac{1}{49}\left(22769.98-\dfrac{(-34.39)^2}{50}\right)$<br>$=\dfrac{89621}{6087}$ or $464.21$<br>$=464$ (to 3 s.f.) |
| **(b)** | $H_0 : \mu = 650$<br>$H_1 : \mu < 650$, $\mu$ is the population mean travelling distance on a single charge<br>Under $H_0$,<br>Since sample size, $n$ = 50 is sufficiently large,<br>$\overline{X} \sim N\left(650, \dfrac{464.21}{50}\right)$ approximately by CLT |

Distribution of test statistic $Z=\dfrac{\overline{X}-650}{\sqrt{\dfrac{464.21}{50}}}\sim \mathrm{N}(0,1)$

Test statistic, $z=\dfrac{649.3122-650}{\sqrt{\dfrac{464.21}{50}}}=-0.22573\approx -0.226$ (3 s.f.)

| Critical value method | p-value method |
|---|---|
| At 5% level of significance, we reject $\mathrm{H}_0$ if $z_{\text{test}}\le -1.64485$. | At 5% level of significance, we reject $\mathrm{H}_0$ if $p\text{-value}\le 0.05$. |
| | $p$-value $=0.4107056\approx 0.411$ (3 s.f.) |
| Since $z_{\text{test}}\le -1.64485$, we do not reject $\mathrm{H}_0$ and conclude that there is insufficient evidence at the 5% level of significance that the car manufacturer has overstated the travelling distance on a single charge. | Since $p\text{-value}=0.411>0.05$, we do not reject $\mathrm{H}_0$ and conclude that there is insufficient evidence at the 5% level of significance that the car manufacturer has overstated the travelling distance on a single charge. |

**(c)** The MeTube car reviewer **needs** to apply Central Limit Theorem (CLT) because the **distribution of the travelling distance on a single charge is unknown (i.e. not normally distributed).**

With the sample size of 50 (large), CLT can be applied and the <u>mean</u> travelling distance on a single charge is <u>approximately</u> normally distributed.

| | |
|---|---|
| **(d)** | The TokTik car reviewer should use a 2-tail test since travelling distance on a single charge can either be **more than or less than 650 km.** |
| **(e)** | The TokTik car reviewer needs to assume that the travelling distance on a single charge is normally distributed.<br><br>He also needs to assume that the observations of travelling distance on a single charge are independent.<br><br>Accept: Assume that the unbiased estimate of the population variance is a good estimate of the population variance. |