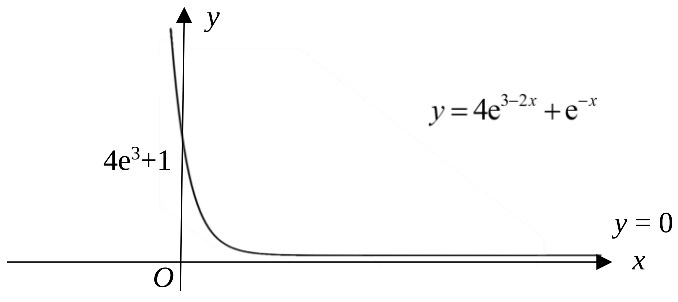
Anglo-Chinese Junior College
2024 JC 2 H1 Mid-Year Assessment Solution

Qn	Solutions
1 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(5+x^2\right)=\frac{2x}{5+x^2}$
1(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{7x-x^2}{\sqrt{x}}\right)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(7x^{\frac{1}{2}}-x^{\frac{3}{2}}\right)$ $=\frac{7}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{\frac{1}{2}}$ or $\frac{7}{2\sqrt{x}}-\frac{3}{2}\sqrt{x}$ or $\frac{7-3x}{2\sqrt{x}}$ or $\frac{7\sqrt{x}-3x\sqrt{x}}{2x}$ Alternative method (quotient rule – not in H1 Syllabus) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{7x-x^2}{\sqrt{x}}\right)$ $=\frac{\sqrt{x}\left(7-2x\right)-\left(7x-x^2\right)\left(\frac{1}{2\sqrt{x}}\right)}{x}$ $=\frac{\sqrt{x}\left(7-2x\right)-\left(7x-x^2\right)\left(\frac{1}{2\sqrt{x}}\right)}{x}$ $=\frac{2x\left(7-2x\right)-\left(7x-x^2\right)}{2x\sqrt{x}}$ $=\frac{14x-4x^2-7x+x^2}{2x\sqrt{x}}$ $=\frac{7x-3x^2}{2x\sqrt{x}}=\frac{7-3x}{2\sqrt{x}}$ or $\frac{7\sqrt{x}-3x\sqrt{x}}{2x}$
1(c)	$\int\left(\frac{1}{4x}+x\right)^2\ \mathrm{d}x$ $=\int\frac{1}{16x^2}+\frac{1}{2}+x^2\ \mathrm{d}x$ $=-\frac{1}{16x}+\frac{1}{2}x+\frac{1}{3}x^3+c$
2(a)	

<table>
<tr><td>2(b)</td><td>$y = 4e^{3-2x} + e^{-x}$
$\frac{dy}{dx} = -8e^{3-2x} - e^{-x}$
When $x = -3$, $y = 4e^{3-2(3)} + e^{-(3)} = 5e^{-3}$
$\frac{dy}{dx} = -8e^{3-2(3)} - e^{-(3)} = -9e^{-3}$
Equation of tangent is
$y - 5e^{-3} = -9e^{-3}\left(x-3\right)$
$y = -9e^{-3}x + 27e^{-3} + 5e^{-3}$
$y = -9e^{-3}x + 32e^{-3}$</td></tr>
<tr><td>3(a)</td><td>$T = \frac{1200}{5}x + 7\left(1200\right) + \frac{1875000}{x^2}$
$T = 240x + 8400 + \frac{1875000}{x^2}$</td></tr>
<tr><td>3(b)</td><td>$\frac{dT}{dx} = 240 - \frac{3750000}{x^3}$
$240 - \frac{3750000}{x^3} = 0$
$x^3 = 15625$
$x = 25$</td></tr>
<tr><td></td><td>$T = 240\left(25\right) + 8400 + \frac{1875000}{25^2} = 17400$</td></tr>
<tr><td></td><td>$\frac{d^2T}{dx^2} = \frac{11250000}{x^4} > 0$
Therefore T is minimum.</td></tr>
<tr><td></td><td>OR
<table>
<tr><td>x</td><td>24.9</td><td>25</td><td>25.1</td></tr>
<tr><td>$\frac{dT}{dx}$</td><td>-2.903</td><td>0</td><td>2.857</td></tr>
<tr><td>Sketch</td><td>\</td><td>–</td><td>/</td></tr>
</table>Therefore T is minimum.</td></tr>
<tr><td>3(c)</td><td>$t = 3.93 (3 \text{ sf})$</td></tr>
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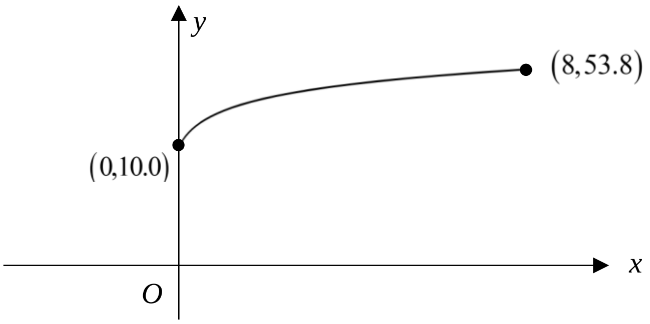
3(d)	
3(e)	$\int_0^5 P \text{ d}t = \int_0^5 1+13\ln(7t+2) \text{ d}t$ $= 185.547$ $= 186 (\text{nearest integer})$ The total profit over the first 5 years is \$186000.
4(a)	$\bar{x} = \frac{24}{80} + 40 = 40.3$
	Unbiased estimate of the population variance $s^2 = \frac{1}{80-1}\left(83.2 - \frac{24^2}{80}\right) = 0.962$
4(b)	An estimator is unbiased if the expected value of the estimator is equal to the population value.
5(a)	Let X be the random variable " Heights of girls, in cm, in TI Junior College" $X \sim N(\mu, \sigma^2)$ $\text{P}(-L \le X - \mu \le L) = 0.4$ $\text{P}\left(-\frac{L}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{L}{\sigma}\right) = 0.4$ $\text{P}\left(-\frac{L}{\sigma} \le Z \le \frac{L}{\sigma}\right) = 0.4$ $\frac{L}{\sigma} = 0.5244005 = 0.524$ (to 3 s.f)
(b)	$\text{P}(X \ge 2L + \mu) = \text{P}\left(\frac{X-\mu}{\sigma} \ge \frac{2L}{\sigma}\right)$ $= \text{P}\left(Z > \frac{2L}{\sigma}\right)$ $= \text{P}(Z > 2(0.5244005))$ $= \text{P}(Z > 1.048801)$ $= 0.14713488$ $= 0.147$ (to 3 s.f)
(c)	$X_1 + X_2 - 2X \sim N(0, 6\sigma^2)$ $\text{P}(X_1 + X_2 \ge 2X) = \text{P}(X_1 + X_2 - 2X \ge 0)$ $= 0.5$

6(a)	$H_0 : \mu = 3.4$ $H_1 : \mu \neq 3.4$ at 10% sig level, where μ represents the population duration
	Under H_0, $\bar{X} \sim N\left(3.4, \frac{0.303}{70}\right)$ approx. by Central Limit Theorem
	p-value: 0.12846 or 0.128 (z-value -1.5202)
	Since p-value = 0.12846> 0.10 ($-1.5202 > -1.64485$), we do not reject H_0.
	There is insufficient evidence at 10% level of significance to reject the claim. Manager's claim should not be rejected.
6(b)	There is no need to assume normal distribution as the distribution can be approximated to normal using Central Limit Theorem since the sample size (*n* = 70) is large.
6(c)	$H_0 : \mu = 3.4$ $H_1 : \mu < 3.4$
	Under H_0, $\bar{X} \sim N\left(3.4, \frac{0.8}{50}\right)$ approx. by Central Limit Theorem Finding the expression z-value= $\frac{3.28 - 3.4}{\sqrt{\frac{0.8}{50}}}$
	For manager's claim to not be supported, we do not reject H_0
	Z-Test μ<3.4 z=-0.9486832981 p=0.1713908408 x̄=3.28 n=50 $\frac{\alpha}{100} <$ p value $\therefore \alpha < 17.1$ (3 sf)
	$\frac{\alpha}{100} <$ p value $\therefore 0 \leq \alpha < 17.1$(3 sf)

7(a) (i)	Let X be the r.v "number of green or yellow light sticks, out of 8 light sticks." $X \sim B(8,\ 0.27)$ Required probability = $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2) = 0.371827 = 0.372$ (3s.f) (shown)
(ii)	Let Y be the r.v "number of light sticks that are not green or yellow, out of 8 light sticks." $Y \sim B(8,\ 0.73)$ Expected number of light sticks that are not green or yellow = $E(Y)$ = 8 (0.73) = 5.84
7b(i)	Let W be the r.v "number of packets of glowing sticks with at least 3 green or yellow light sticks, out of 80 packets of glowing sticks." $W \sim B(80,\ 0.371827)$ Required probability = $\mathrm{P}(W < 30) = \mathrm{P}(W \leq 29) = 0.481269 = 0.481$ (3s.f)
(bii)	$Y \sim B(8,\ 0.73)$ $\mathrm{E}(Y) = 5.84$; $\mathrm{Var}(Y) = 1.5768$ Since n is large, by Central Limit Theorem, $\frac{Y_1 + Y_2 + Y_3 + \ldots + Y_{80}}{80} \sim N\left(5.84,\ \frac{1.5768}{80}\right)$ approx Required prob. = $\mathrm{P}\left(\frac{Y_1 + Y_2 + Y_3 + \ldots + Y_{80}}{80} \leq 5.8\right)$ = 0.388 (3s.f)
8(a) (i)	p (0.7) + (1 – p) (0.3) = 0.4p + 0.3
(ii)	P (Benjamin dives in different direction as the ball is kicked) = 1 – (0.4p + 0.3) = 0.7 – 0.4p Required prob = (0.4p + 0.3) (0.65) + 0.7 – 0.4p = 0.895 – 0.14p
(bi) (a)	If Benjamin is the goalkeeper: ${}^6C_4\,{}^5C_4\,{}^5C_2 = 750$ If Benjamin is the forward: ${}^6C_4\,{}^5C_4\,{}^5C_1 = 375$ Number of ways required = 1125
(bi) (b)	If no sibling selected: ${}^6C_4\,{}^5C_4\,{}^2C_2 = 75$ If 1 of the siblings included: ${}^6C_4\,{}^5C_4\,{}^3C_1\,{}^2C_1 = 450$ Number of ways required = 525
(bii)	Number of ways required = ${}^4C_3\, 3!\, 5!\, 3!\, 6!\, 2!$ = 24883200
(bii)	**<u>Alternatively,</u>** Number of ways required = ${}^4C_3\, 3!\, 5!\, 3!\, {}^6C_3\, (3!)^2\, 2!$ = 24883200