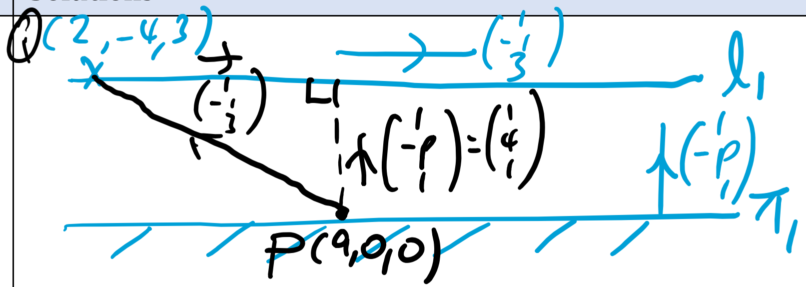
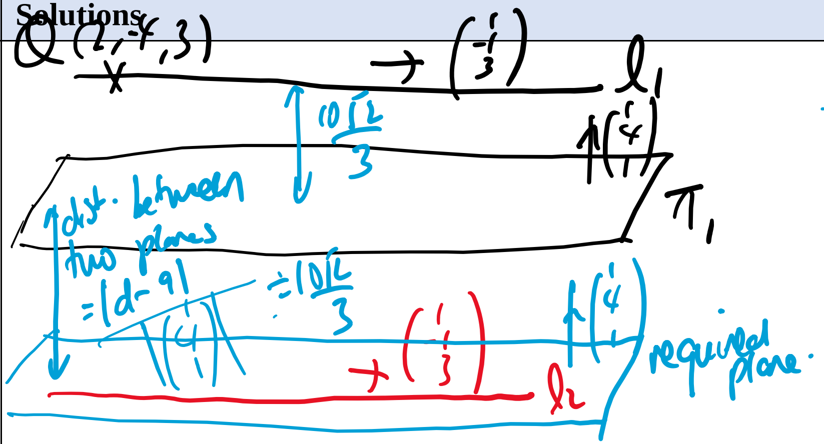
St Andrew's Junior College
2022 Preliminary Examination
H2 Mathematics Paper 2 (9758/02)
Section A: Pure Mathematics

Q	Solutions	Mark Scheme
1(i)	$\sum_{n=2}^{N} \frac{1-n}{n!} = \sum_{n=2}^{N} \left(u_n - u_{n-1}\right)$ $= \begin{bmatrix} u_2 - u_1 \\ +u_3 - u_2 \\ +u_4 - u_3 \\ \dots \\ +u_{N-1} - u_{N-2} \\ +u_N - u_{N-1} \end{bmatrix}$ $= u_N - u_1$ $= \frac{1}{N!} - \frac{1}{1!}$ $= \frac{1}{N!} - 1$	

Q	Solutions	Mark Scheme
(ii)	As $N \to \infty$, $\frac{1}{N!} \to 0$ $\sum_{n=2}^{N} \frac{1-n}{n!} \to -1$ which is finite, hence $\sum_{n=2}^{N} \frac{1-n}{n!}$ converges and the sum to infinity is –1.	
(iii)	$\sum_{n=8}^{N+5} \frac{2-n}{(n-1)!} = \sum_{n=7}^{N+4} \frac{1-n}{n!}$ $\left(\text{Replace } n \text{ by } n+1\right)$ $= \sum_{n=2}^{N+4} \frac{1-n}{n!} - \sum_{n=2}^{6} \frac{1-n}{n!}$ $= \frac{1}{(N+4)!} - 1 - \left(\frac{1}{6!} - 1\right)$ $= \frac{1}{(N+4)!} - \frac{1}{720}$	

Q	Solutions	Mark Scheme
2(i)	 l_1 is parallel to Π_1, $\begin{pmatrix}1\\-1\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-p\\1\end{pmatrix}=0 \Rightarrow p=-4$ Let point P be the point (9, 0, 0) which lies on Π_1 and let point Q be (2, −4, 3). Distance between l_1 and Π_1 $=\dfrac{\left\lvert \overrightarrow{PQ}\bullet\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}=\dfrac{\left\lvert\begin{pmatrix}-7\\-4\\3\end{pmatrix}\bullet\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\sqrt{18}}=\dfrac{10\sqrt{2}}{3}$ units	
(ii)	Let the required Cartesian equation by $x+4y+z=d$	

<table>
<tr><th>Q</th><th>Solutions</th><th>Mark Scheme</th></tr>
<tr><td></td><td>

Distance between the two planes = $\dfrac{|d-9|}{\left\|\begin{pmatrix}1\\4\\1\end{pmatrix}\right\|}$

$\dfrac{|d-9|}{3\sqrt{2}} = \dfrac{10\sqrt{2}}{3} \Rightarrow d = 29 \text{ or } -11$

Since (2, –4, 3) lies on $x+4y+z=-11$, the required Cartesian equation of plane is $x+4y+z=29$</td><td></td></tr>
<tr><td>(iii)</td><td>Let $y = 0$, $\left.\begin{matrix}3x-2z=10\\ x-z=5\end{matrix}\right\} x=0, z=-5$. So a common point between the planes is (0, 0, –5).</td><td></td></tr>
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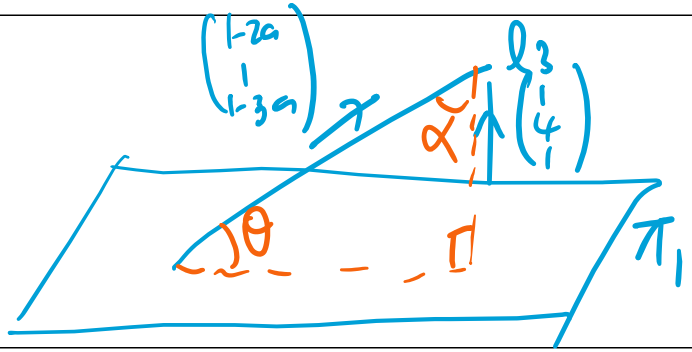
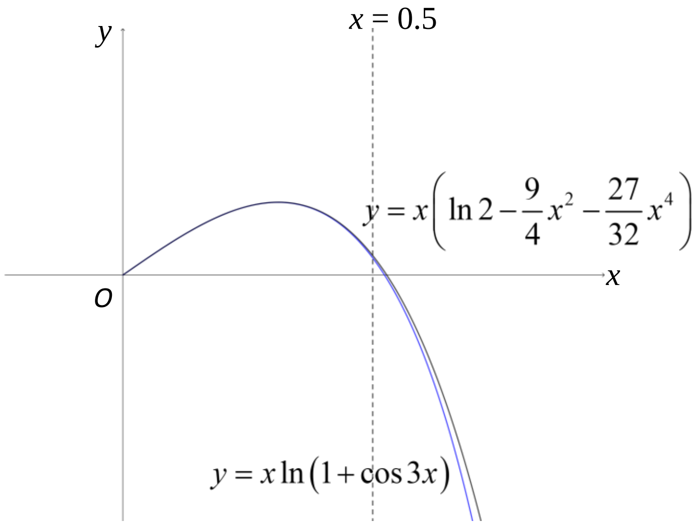
Q	Solutions	Mark Scheme
	A vector parallel to the l_3 = $\begin{pmatrix}3\\-1\\-2\end{pmatrix}\times\begin{pmatrix}1\\-a\\-1\end{pmatrix}=\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}$ $l_3:\mathbf{r}=\begin{pmatrix}0\\0\\-5\end{pmatrix}+\lambda\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix},\lambda\in\mathbb{R}$ **<u>Alternative Solution 1</u>** Let x = 0, $\left.\begin{matrix}-y-2z=10\\-ay-z=5\end{matrix}\right\}y=0,z=-5$ $l_3:\mathbf{r}=\begin{pmatrix}0\\0\\-5\end{pmatrix}+\lambda\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix},\lambda\in\mathbb{R}$ **<u>Alternative Solution 2</u>** Let z = 0, $\left.\begin{matrix}3x-y=10\\x-ay=5\end{matrix}\right\}x=\dfrac{5-10a}{1-3a},y=\dfrac{5}{1-3a}$ $l_3:\mathbf{r}=\dfrac{5}{1-3a}\begin{pmatrix}1-2a\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix},\lambda\in\mathbb{R}$ **<u>Alternative Solution 3</u>**	

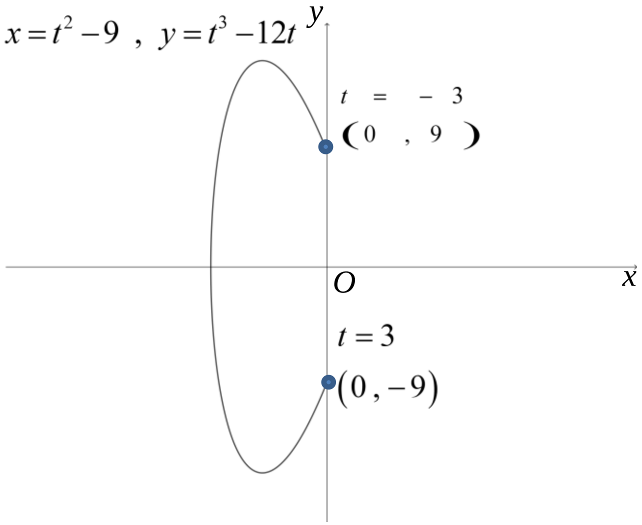
Q	Solutions	Mark Scheme
	$3x-y-2z=10 \Rightarrow 3x-y=10+2z$ --- (1) $x-ay-z=5 \Rightarrow x-ay=5+z$ --- (2) Solving (1) and (2), $3x-y=2x-2ay \Rightarrow x=(1-2a)y$ From (2): $y=\dfrac{1}{1-3a}(5+z), x=\dfrac{1-2a}{1-3a}(5+z)$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dfrac{1}{1-3a}\begin{pmatrix} 5-10a+z \\ 5+z \\ (1-3a)z \end{pmatrix}$ $l_3 : \mathbf{r} = \dfrac{5}{1-3a}\begin{pmatrix} 1-2a \\ 1 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 1-2a \\ 1 \\ 1-3a \end{pmatrix}, \lambda \in \mathbb{R}$	
(iv)		

Q	Solutions	Mark Scheme
	$\left(\theta = \frac{\pi}{2} - \alpha\right)$ Given θ be the acute angle between l_3 and Π_1, $$\frac{\left\lvert \begin{pmatrix} 1-2a \\ 1 \\ 1-3a \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \right\rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}\sqrt{18}} = \sin\theta$$ $$\frac{\left\lvert \begin{pmatrix} 1-2a \\ 1 \\ 1-3a \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \right\rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}\sqrt{18}} = \frac{\sqrt{3}}{18} \text{ (given)}$$ $$\frac{\lvert (1-2a)+4+(1-3a) \rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}\sqrt{18}} = \frac{\sqrt{3}}{18}$$ $$\frac{\lvert 6-5a \rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}} = \frac{\sqrt{54}}{18}$$ Squaring both sides, $$\frac{\lvert 6-5a \rvert^2}{(1-2a)^2+1+(1-3a)^2} = \frac{54}{324} = \frac{1}{6}$$	

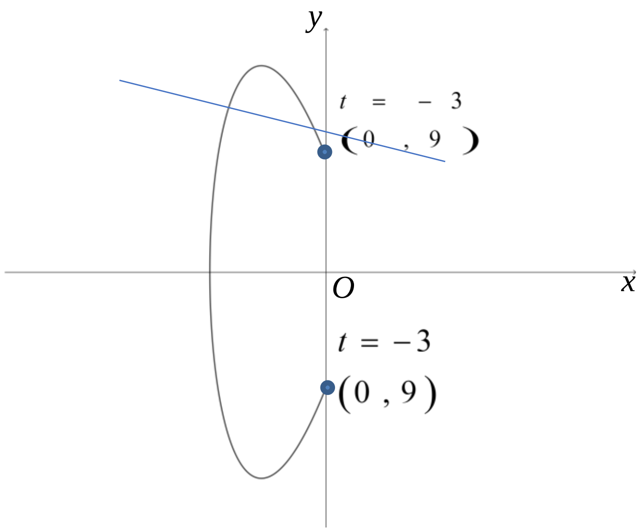
Q	Solutions	Mark Scheme
	$\dfrac{(6-5a)^2}{1-4a+4a^2+1+1-6a+9a^2}=\dfrac{1}{6}$ $\dfrac{(6-5a)^2}{13a^2-10a+3}=\dfrac{1}{6}$ $6(6-5a)^2=13a^2-10a+3$ $6(36-60a+25a^2)-13a^2+10a-3=0$ $137a^2-350a-213=0$ $(a-1)(137a-213)=0$ $\Rightarrow a=1$ or $a=\dfrac{213}{137}$	
3(i)	$\cos 3x=1-\dfrac{(3x)^2}{2!}+\dfrac{(3x)^4}{4!}+\ldots$ $=1-\dfrac{9}{2}x^2+\dfrac{27}{8}x^4+\ldots$	

Q	Solutions	Mark Scheme
	$\ln(1+\cos 3x) = \ln\left(1+\left(1-\frac{9}{2}x^2+\frac{27}{8}x^4+\ldots\right)\right)$ $= \ln 2\left(1-\frac{9}{4}x^2+\frac{27}{16}x^4+\ldots\right)$ $= \ln 2+\ln\left(1-\frac{9}{4}x^2+\frac{27}{16}x^4+\ldots\right)$ $= \ln 2+\left(-\frac{9}{4}x^2+\frac{27}{16}x^4+\ldots\right)-\frac{\left(-\frac{9}{4}x^2+\frac{27}{16}x^4+\ldots\right)^2}{2}+\ldots$ $= \ln 2-\frac{9}{4}x^2+\frac{27}{16}x^4-\frac{81}{32}x^4+\ldots$ $= \ln 2-\frac{9}{4}x^2-\frac{27}{32}x^4+\ldots$	
(ii)	$\int_0^{0.5} x\ln(1+\cos 3x)\mathrm{d}x = \int_0^{0.5} x\left(\ln 2-\frac{9}{4}x^2-\frac{27}{32}x^4+\ldots\right)\mathrm{d}x$ $= \int_0^{0.5}\left((\ln 2)x-\frac{9}{4}x^3-\frac{27}{32}x^5+\ldots\right)\mathrm{d}x$ $= \left[(\ln 2)\frac{x^2}{2}-\frac{9}{16}x^4-\frac{9}{64}x^6+\ldots\right]_0^{0.5}$ ≈ 0.04929 (5 d.p.)	
(iii)	Using GC, $\int_0^{0.5} x\ln(1+\cos 3x)\mathrm{d}x = 0.04900$ (5 d.p.)	

Q	Solutions	Mark Scheme
(iv)	 From the diagram, it can be seen that the graphs of $y = x\ln(1+\cos 3x)$ and $y = x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4\right)$ are close to each other mostly from $x = 0$ to $x = 0.5$. Hence, the approximated value of $\int_0^{0.5} x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4 + ...\right)\mathrm{d}x$ from (ii) is approximately equal to the actual value of $\int_0^{0.5} x\ln(1+\cos 3x)\mathrm{d}x = 0.04900$ (5 d.p.) in (iii).	

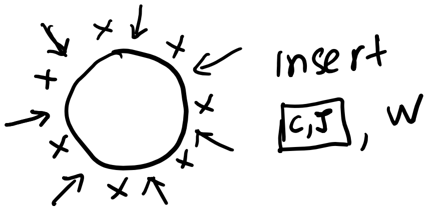
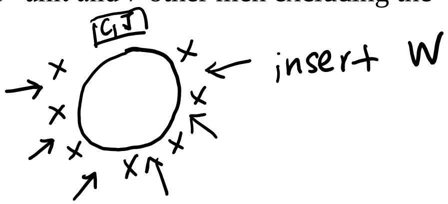
Q	Solutions	Mark Scheme
4(i)	$x=t^2-9$, $y=t^3-12t$ 	
(ii)	$x=t^2-9$, $y=t^3-12t$ $\frac{\mathrm{d}x}{\mathrm{d}t}=2t$, $\frac{\mathrm{d}y}{\mathrm{d}t}=3t^2-12$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{1}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{3t^2-12}{2t}$ $l:9y+2x+83=0$	

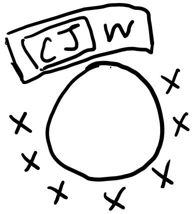
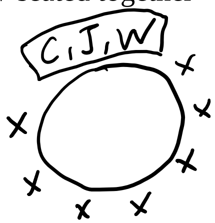
Q	Solutions	Mark Scheme
	$\frac{3t^2-12}{2t}=\frac{9}{2}$ $t^2-3t-4=0$ $(t-4)(t+1)=0$ $t=-1$ or $t=4$ Since $t\in[-3,3]$, $t=-1$.	
(iii)	Since the curve C intersects the line l at $Q\left(q^2-9,\, q^3-12q\right)$ $9\left(q^3-12q\right)+2\left(q^2-9\right)=83$ $9q^3-108q+2q^2-18-83=0$ $9q^3+2q^2-108q-101=0$ (Shown) Using G.C., $q=-2.9836$ or -1 or 3.7613 Since $-3\le q\le 3$ and $q\ne -1$, $q=-2.9836$. Hence, $x=(-2.9836)^2-9=-0.09813$, $y=(-2.9836)^3-12(-2.9836)=9.2436$ $Q(-0.0981,\, 9.24)$	

Q	Solutions	Mark Scheme
(iv)	 Required area	

Q	Solutions	Mark Scheme
	$=\int_{-8}^{-0.0981} y\,\mathrm{d}x-\frac{1}{2}\left[-0.09813-\left(-8\right)\right]\left[9.2436+11\right]$ $=\int_{-1}^{-2.9836}\left(t^3-12t\right)\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\mathrm{d}t-79.981$ $=\int_{-1}^{-2.9836}\left(t^3-12t\right)\left(2t\right)\mathrm{d}t-79.981$ $=30.324\text{ units}^2$ $=30.3\ \text{ units}^2\text{ (to 3 sf)}$	

Section B: Probability and Statistics

Q	Solutions	Mark Scheme
5(i)	Let C be Caleb, J be James and W be the woman. Number of ways to arrange the 7 people (exclude C, J and W) = $(7-1)!$  Number of ways to insert the pair C & J together and the woman = ${}^{7}C_{2}\times 2!$ Number of ways to arrange C and J within the pair = $2!$ Total number of ways $=(7-1)!\times {}^{7}C_{2}\times 2!\times 2!=60480$ Alternative mtd 1 : No. of ways to arrange the ‘CJ’ unit and 7 other men excluding the woman = $(8-1)!\times 2!$  No. of ways to insert W such that she is not next to the ‘CJ’ unit = 6 or ${}^{6}C_{1}$ Total number of ways $=(8-1)!\times 2!\times 6=60480$ Alternative mtd 2 (Complementary mtd 1): No. of arrangements where C and J are grouped together = $(9-1)!\times 2!$ No. of ways to arrange the ‘CJ’ unit and W = 2!	

	No. of arrangements where C and J group together and the ‘CJ’ unit is also next to W = $(8-1)!\times 2!\times 2!$  Note : 2! to arrange C and J and 2! to arrange the ‘C,J’ unit and W Total number of ways = No. of arrangements where C and J are grouped together - No. of arrangements where C and J group together and the ‘CJ’ unit is also next to W = $(9-1)!\times 2!-(8-1)!\times 2!\times 2!=60480$ Alternative mtd 3 (Complementary mtd 2): No. of arrangements where C and J are separated = $(8-1)!\times {}^{8}C_{2}\times 2!$ (using slot in mtd) Total number of ways = No. of arrangements with no restrictions - No. of arrangements where C and J are separated - No. of arrangements where C and J group together and the ‘CJ’ unit is also next to W = $(10-1)!-(8-1)!\times {}^{8}C_{2}\times 2!-(8-1)!\times 2!\times 2!=60480$	
(ii)	Number of ways (with identical seats) = number of ways without restriction – number of ways in which C, J and W seated together = $(10-1)!-(8-1)!3!$ = 332640  Number of ways (with different seats) =	

Number of ways (with identical seats) × number of seats

$=\left[(10-1)!-(8-1)!3!\right]\times 10$

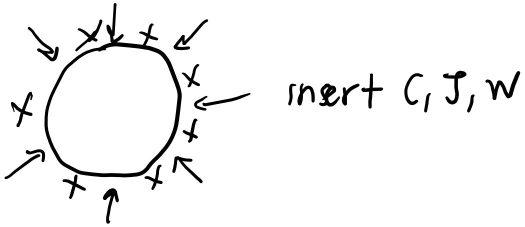
= 332640 × 10

= 3326400

Alternative mtd :

Case 1 : C, J, W are all separated

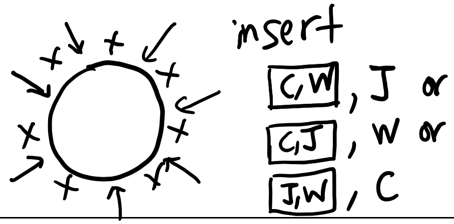
No. of arrangements (using slot in mtd) = $(7-1)!\times {}^{7}C_{3}\times 3!=151200$



Case 2 : Any two of C, J, W are group together but separated from the 3rd person

No. of cases = ${}^{3}C_{2}$ (any two of C, J, W)

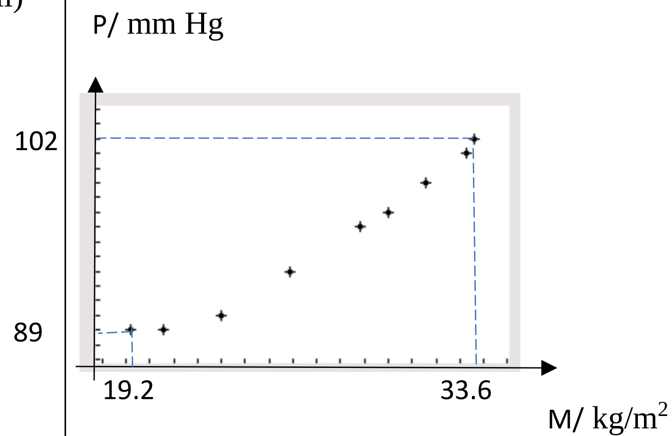
For each case, no. of arrangements (using slot in mtd) =
$(7-1)!\times {}^{7}C_{2}\times 2!\times 2!$



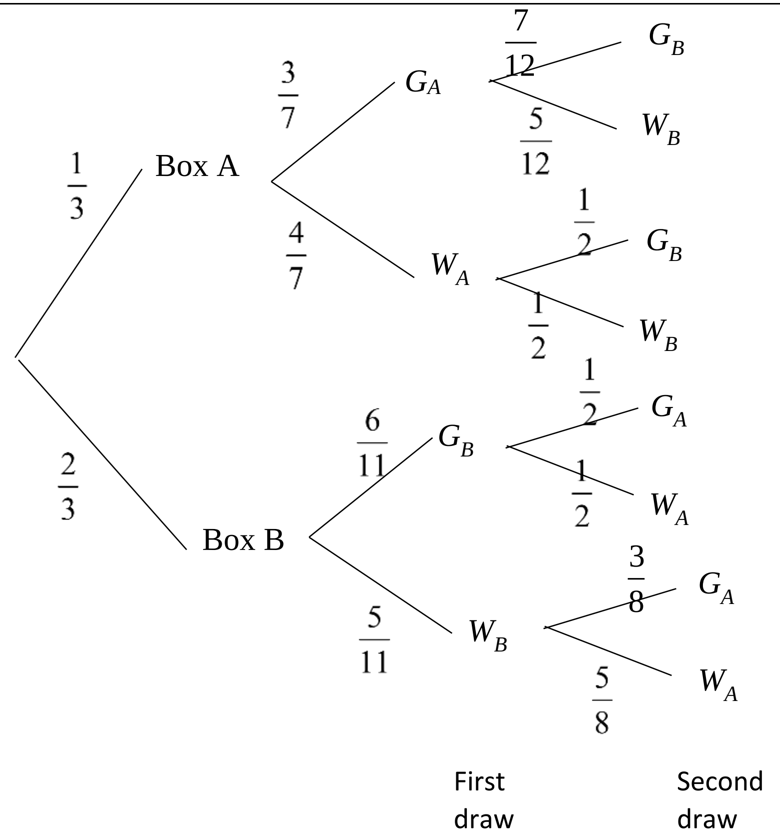
	No. of arrangements for Case 2 = ${}^{3}C_2 \times (7-1)! \times {}^{7}C_2 \times 2! \times 2! = 181440$ Total no. of ways(with different seats) = $(151200+181440)\times 10 = 3326400$	
6(i)	Let X be the random variable "no of students out of 30 students who could do the Differentiation question" $X \sim \mathrm{B}(30, 0.3)$ $\mathrm{P}(X \geq 6) = 1 - \mathrm{P}(X \leq 5)$ $= 0.92341 \approx 0.923$ (3 sig. fig.)	
(ii)	Let S be the random variable "no of students out of 8 who could do the Differentiation question " Let T be the random variable "no of students out of 22 who could do the Differentiation question " $S \sim \mathrm{B}(8, 0.3)$ $T \sim \mathrm{B}(22, 0.3)$	

	P(only 2 among first 8 could do that question$\mid X \geq 6$) $= \dfrac{\text{P(only 2 among first 8 could do that question} \cap X \geq 6)}{\text{P}(X \geq 6)}$ $= \dfrac{\text{P}\begin{pmatrix}\text{only 2 among first 8 could do that question} \\ \cap \text{at least 4 among the next 22 could do the question}\end{pmatrix}}{\text{P}(X \geq 6)}$ $= \dfrac{\text{P}(S = 2 \cap T \geq 4)}{\text{P}(X \geq 6)}$ $= \dfrac{\text{P}(S=2)\text{P}(T \geq 4)}{\text{P}(X \geq 6)}$ $= \dfrac{\text{P}(S = 2)[1 - \text{P}(T \leq 3)]}{\text{P}(X \geq 6)}$ $= \dfrac{0.296475 \times 0.931937}{0.92341} = \dfrac{0.276297}{0.92341}$ $= 0.29921 \approx 0.299$ (to 3 sig. fig.)	
(iii)	Let Y be the random variable "no of students out of n who could do the Differentiation question " $Y \sim \text{B}(n, 0.3)$ $\text{P}(Y \leq 5) > 0.9$ From G.C,	

n	$\text{P}(Y \leq 5)$
10	0.95265 < 0.9
11	0.92178 < 0.9
12	0.88215 > 0.9

	Therefore, the largest possible value of n is 11.	
7(i)	$P = 0.92588M + 69.804$ $\dfrac{763+k}{9} = 0.92588(27.333) + 69.804$ $763 + k = 9(95.111)$ $k = 855.999 - 763$ $k = 92.999 = 93.0 \quad (3\text{sf}) \quad (\text{Shown})$	
(ii)		
(iii)	$\ln P = aM + b$ $\Rightarrow P = e^{aM+b}$	

	From the scatter diagram, as M increases, P increases at an increasing rate, hence a is positive. [From the scatter diagram, as M increases, P increases. Hence, ln P will also increase. Therefore a is positive] Product moment correlation coefficient between M and ln P = 0.988128=0.988 (to 3 sf)	
(iv)	From (ii), as M increases, P increases at an increasing rate instead of constant rate. From (iii), the PMCC between M and ln P is closer to 1 as compared to the PMCC between M and P. Hence $\ln P = aM + b$ is the better model.	
(v)	It may not be reliable to estimate John's DBP using the model as John's BMI is outside the given BMI data range of 19.2 to 33.6 and extrapolation is needed.	
8(i)	Let G_A and G_B represent the event that the ball drawn is green from Box A and Box B respectively. Let W_A and W_B represent the event that the ball drawn is white from Box A and Box B respectively.	


Box A
Box B
$\frac{1}{3}$
$\frac{2}{3}$
$\frac{3}{7}$
$\frac{4}{7}$
$\frac{6}{11}$
$\frac{5}{11}$
G_A
W_A
G_B
W_B
$\frac{7}{12}$
$\frac{5}{12}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{3}{8}$
$\frac{5}{8}$
G_B
W_B
G_B
W_B
G_A
W_A
G_A
W_A
First draw
Second draw

(ii)	P(wins the game) = P(second ball is white) $= \mathrm{P}(G_A, W_B) + \mathrm{P}(W_A, W_B) + \mathrm{P}(G_B, W_A) + \mathrm{P}(W_B, W_A)$ $= \frac{1}{3}\left[\left(\frac{3}{7}\right)\left(\frac{5}{12}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)\right] + \frac{2}{3}\left[\left(\frac{6}{11}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{11}\right)\left(\frac{5}{8}\right)\right]$ $= \frac{81}{154}$ P(first ball from Box *A*| Player wins the game) $= \frac{\text{P(first ball from } A \cap \text{player wins the game)}}{\text{P(player wins the game)}}$ $= \frac{\mathrm{P}\begin{pmatrix} \text{score of die is less than 3 followed by} \\ \text{drawing a ball from A} \\ \text{and the second ball drawn is white} \end{pmatrix}}{\mathrm{P}(\text{player wins the game})}$ $= \frac{\frac{1}{3}\left[\left(\frac{3}{7}\right)\left(\frac{5}{12}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)\right]}{\frac{81}{154}}$ $= \frac{143}{486}$	

9(i)	Given that X is a discrete random variable, $\sum_{\text{all } x} \mathrm{P}(X=x)=1$. $2p+2q=1$ ---(1) Also $\mathrm{E}\left(X^2\right)=\sum_{\text{all } x} x^2 \mathrm{P}(X=x)=13.3$ $4p+9q+16q+25p=13.3$ $\Rightarrow 29p+25q=13.3$ ---(2) Solving (1) and (2), by GC, $p=\frac{1}{5}, q=\frac{3}{10}$ By symmetry, E(X) = 3.5 $\mathrm{Var}(X)=\mathrm{E}\left(X^2\right)-[\mathrm{E}(X)]^2$ $=13.3-3.5^2$ $=1.05$	
(ii)	Since n = 30 is sufficiently large, by Central Limit Theorem	

	$\overline{X} \sim N\left(3.5, \frac{1.05}{30}\right)$ approximately $P\left(\overline{X} > 3.8\right) = 0.054405 = 0.0544$ (3 s.f.)	

Qn	Solutions	Mark Scheme
10(i)	Let H be the mass of a honeydew in kg. $H \sim \mathrm{N}(1.5,\ 0.2^2)$ Let W be the mass of a watermelon in kg. $W \sim \mathrm{N}(8.5,\ 0.3^2)$ Required probability $=3\times \mathrm{P}(H<1.8)^2\mathrm{P}(H>1.8)$ $=0.17454$ $=0.175$	
(ii)	Let $T = H_1 + H_2 + H_3 + H_4 + H_5 - W$ $\mathrm{E}(T) = \mathrm{E}(H_1 + H_2 + H_3 + H_4 + H_5 - W)$ $= 5\mathrm{E}(H) - \mathrm{E}(W)$ $= 5(1.5) - 8.5 = -1$ $\mathrm{Var}(T) = \mathrm{Var}(H_1 + H_2 + H_3 + H_4 + H_5 - W)$ $= 5\mathrm{Var}(H) + \mathrm{Var}(W)$ $= 5(0.2)^2 + 0.3^2 = 0.29$ $T \sim \mathrm{N}(-1, 0.29)$ $\mathrm{P}(T<0)$ $=0.96834$ $=0.968$	
(iii)	Let $F = H + W$	

	$\mathrm{E}(F)=\mathrm{E}(H+W)$ $\quad$ $\mathrm{Var}(F)=\mathrm{Var}(H+W)$ $=\mathrm{E}(H)+\mathrm{E}(W)$ $\quad$ $=\mathrm{Var}(H)+\mathrm{Var}(W)$ $=1.5+8.5=10$ $\quad$ $=0.2^2+0.3^2=0.13$ $F\sim \mathrm{N}(10, 0.13)$ $\mathrm{P}(\lvert F-10\rvert<m)=0.9$ $\mathrm{P}(10-m<F<10+m)=0.9$ $10+m=10.593$ $\Rightarrow \quad m=0.593$	
(iv)	Let $C=3.5H+0.7W$ $\mathrm{E}(F)=\mathrm{E}(3.5H+0.7W)$ $=3.5\mathrm{E}(H)+0.7\mathrm{E}(W)$ $=3.5(1.5)+0.7(8.5)$ $=11.2$ $\mathrm{Var}(F)=\mathrm{Var}(3.5H+0.7W)$ $=3.5^2\,\mathrm{Var}(H)+0.7^2\,\mathrm{Var}(W)$ $=(3.5)^2(0.2^2)+(0.7)^2(0.3^2)$ $=0.5341$ $C\sim \mathrm{N}(11.2, 0.5341)$ $\mathrm{P}(C\le 10)=0.050296$=0.0503 (to 3 sf)	

(v)	The event that a honeydew and watermelon cost at most \$5 **each** is a subset of the event that the total cost of one honeydew and one watermelon is \$10.	

11 (i)	The branch manager obtains a sampling frame consisting of all the customers of the branch, **numbering all the customers with a distinct number from 1 to *N*.** **Randomly select 80** of these customers by **generating 80 distinct random numbers (using a random number generator)** and **select the corresponding** customers.	
(ii)	Let T be the random variable denoting the waiting time of a customer at the branch in minutes and μ be the population mean waiting time. Unbiased estimate of population mean, $\bar{t}$ $=\dfrac{\sum(t-15)}{50}+15$ $=\dfrac{-60}{50}+15=13.8$ (Exact) Unbiased estimate of population variance, s^2 $=\dfrac{1}{n-1}\left[\sum(t-15)^2-\dfrac{\left(\sum(t-15)\right)^2}{n}\right]$ $=\dfrac{1}{50-1}\left(1168-\dfrac{(-60)^2}{50}\right)$ $=22.367$ $=22.4$ (to 3 sf)	
(iii)	Test $\quad H_0:\mu=15$ against $\quad H_1:\mu<15$ at 5% significance level.	

	Under H_0, since $n = 50 > 30$ is large, $\bar{T} \sim N\left(15, \frac{22.367}{50}\right)$ approximately by Central Limit Theorem. Using a 1-tailed z-test, The test statistic value $\bar{t} = 13.8$ gives z_{calc}= -1.7942 and p-value = 0.036393=0.0364 ≤ 0.05 Since p-value = $0.0364 \leq 0.05$, we reject H_0 and conclude that at the 5% level of significance, there is sufficient evidence to conclude that the mean waiting time is less than 15 minutes.	
(iv)	The p-value of **0.0364** is the **probability that sample mean waiting time of a customer** in the branch is **at most 13.8 minutes** when the **(population) mean waiting time of a customer in the branch is actually 15 minutes**. *Note:*	
(v)	Test $\mathrm{H}_0 : \mu = k$ against $\mathrm{H}_1 : \mu \neq k$ at 2% significance level. Under H_0, since $n = 50 > 30$ is large, $\bar{T} \sim N\left(k, \frac{22.367}{50}\right)$ approximately by Central Limit Theorem.	

Using a 2-tailed z-test, the test statistic $Z = \dfrac{\overline{T} - k}{\sqrt{\dfrac{22.367}{50}}} \sim N(0, 1)$

Given that there is insufficient evidence to reject H_0,

$$-2.3263 < \frac{13.8 - k}{\sqrt{\dfrac{22.367}{50}}} < 2.3263$$

$$-1.5559 < 13.8 - k < 1.5559$$

$$-15.3559 < -k < -12.2441$$

$$12.2441 < k < 15.3559$$

$$12.2 < k < 15.4$$