RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Tutorial 8B: Applications of Integration

1 Evaluate $\int_{-a}^{a} \left| x^2 + 1 - e^x \right| dx$ where $a > 0$, giving your answer in terms of a. **[3]**

[$-2 + e^{-a} + e^{a}$]

2 **(i)** Sketch, on the same diagram, the graphs of $x^2 + y^2 = 9$ and $y = e^{\frac{1}{4}x^2}$. **[2]**

(ii) The finite region in the first quadrant bounded by the curves $x^2 + y^2 = 9$, $y = e^{\frac{1}{4}x^2}$, the x–axis and the y–axis is denoted by R.

(a) Shade the region R. **[1]**

(b) Find the volume of the solid of revolution formed when R is rotated through 2π radians about the x-axis. **[4]**

[(ii)(b) 22.7]

3 It is given that

$$f(x) = \begin{cases} 7 - x^2 & \text{for } 0 < x \le 2, \\ 2x - 1 & \text{for } 2 < x \le 4, \end{cases}$$

and that $f(x) = f(x+4)$ for all real values of x.

(i) Evaluate f(27) + f(45). [2]

(ii) Sketch the graph of $y = f(x)$ for $-7 \le x \le 10$. [3]

(iii) Find $\int_{-4}^{3} f(x)\, dx$. [3]

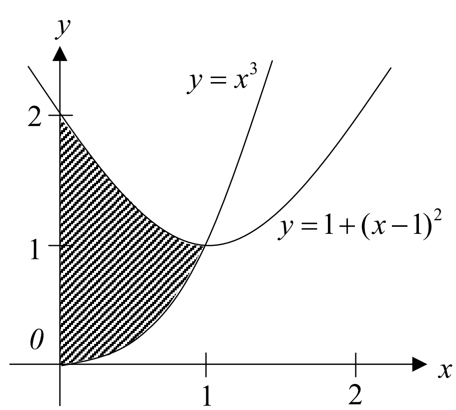
[**(i)** 11 **(iii)** $\dfrac{110}{3}$]

4 With reference to the given figure,

(i) find the exact area of the shaded region.

(ii) find the exact volume generated by the shaded region, when it is rotated through 4 right angles about the y-axis.

[**(i)** $\dfrac{13}{12}$ **(ii)** $\dfrac{23}{30}\pi$]



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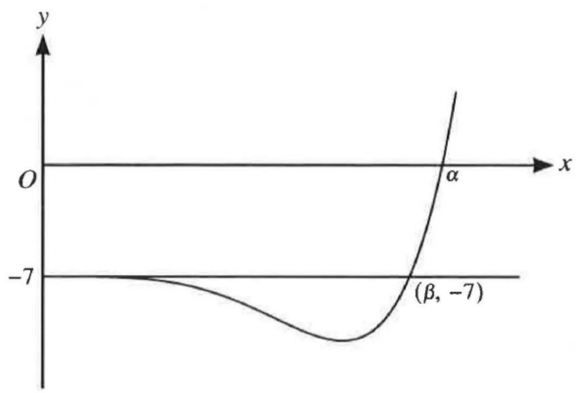

It is given that $\mathrm{f}(x) = x^6 - 3x^4 - 7$. The diagram shows the curve with equation $y = \mathrm{f}(x)$ and the line with equation $y = -7$, for $x \geq 0$. The curve crosses the positive $x-$axis at $x = \alpha$, and the curve and the line meet where $x = 0$ and $x = \beta$.

(i) Find the value of α, giving your answer correct to 3 decimal places, and find the exact value of β. [2]

(ii) Evaluate $\int_{\beta}^{\alpha} \mathrm{f}(x)\,\mathrm{d}x$, giving your answer correct to 3 decimal places. [2]

(iii) Find, in terms of $\sqrt{3}$, the area of the finite region bounded by the curve and the line, for $x \geq 0$. [3]

(iv) Show that $\mathrm{f}(x) = \mathrm{f}(-x)$. What can be said about the six roots of the equation $\mathrm{f}(x) = 0$? [4]

[**(i)** $\alpha = 1.885$, $\beta = \sqrt{3}$ **(ii)** -0.597 **(iii)** $\frac{54}{35}\sqrt{3}$]

6 O is the origin and A is the point on the curve $y = \tan x$ where $x = \frac{\pi}{3}$.

(i) Calculate the area of the region R enclosed by the arc OA, the $x-$axis, and the line $x = \frac{\pi}{3}$, giving your answer in an exact form. [3]

(ii) The region S is enclosed by the arc OA, the y-axis, and the line $y = \sqrt{3}$. Find the volume of the solid of revolution formed where S is rotated 360° about the $x-$axis, giving your answer in an exact form. [6]

(iii) Find $\int_0^{\sqrt{3}} \tan^{-1} y\,\mathrm{d}y$, giving your answer in an exact form. [3]

[**(i)** ln2 **(ii)** $\frac{4\pi^2}{3} - \pi\sqrt{3}$ **(iii)** $\frac{\pi\sqrt{3}}{3} - \ln 2$]

7 The curve C has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = xe^{-x^2}$.

(i) Sketch the curve C. [2]

(ii) Find the exact coordinates of the turning points on the curve. [4]

(iii) Use the substitution $u = x^2$ to find $\int_0^n \mathrm{f}(x)\,\mathrm{d}x$, for $n > 0$. Hence find the area of the region between the curve and the positive x-axis. [4]

(iv) Find the exact value of $\int_{-2}^{2} \left|\mathrm{f}(x)\right| \mathrm{d}x$. [2]

(v) Find the volume of revolution when the region bounded by the curve, the lines $x = 0$, $x = 1$ and the x-axis is rotated completely about the x-axis. Give your answer correct to 3 significant figures. [2]

[**(ii)** $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2e}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2e}}\right)$ **(iii)** $\frac{1}{2}(1 - e^{-n^2}); \frac{1}{2}$ **(iv)** $1 - e^{-4}$ **(v)** 0.363]

8 The region R is bounded by the curve C with equation $y = \sqrt{x} + \dfrac{2}{\sqrt{x}}$ and the line $y = 3$.

(i) Calculate the exact area of R. [4]

(ii) Write down the equation of the curve obtained when C is translated 3 units in the negative y–direction. [2]

(iii) Hence, show that the volume of the solid formed when R is rotated completely about the line $y = 3$ is given by $\pi \int_1^4 \left(x - 6\sqrt{x} + 13 - \dfrac{12}{\sqrt{x}} + \dfrac{4}{x} \right) \mathrm{d}x$. [4]

(iv) Determine the exact volume of the solid. [2]

[**(i)** $\frac{1}{3}$ **(iv)** $\pi(8\ln 2 - 5\frac{1}{2})$]

9 **(i)** Given that f is a continuous function, explain, with the aid of a sketch, why the value of

$$\lim_{n \to \infty} \frac{1}{n} \left\{ \mathrm{f}\left(\frac{1}{n}\right) + \mathrm{f}\left(\frac{2}{n}\right) + \ldots + \mathrm{f}\left(\frac{n}{n}\right) \right\}$$

is $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$. [2]

(ii) Hence evaluate $\lim_{n \to \infty} \frac{1}{n} \left(\dfrac{\sqrt[3]{1} + \sqrt[3]{2} + \ldots + \sqrt[3]{n}}{\sqrt[3]{n}} \right)$. [3]

[**(ii)** $\frac{3}{4}$]