中正中学 义顺
CHUNG CHENG HIGH SCHOOL YISHUN

Mid-Year Examination (2019)
Secondary 4 Express / 5 Normal (Academic)

Candidate **Name**	**Register No**	**Class**

Additional Mathematics Paper 2
4047 / 2

Date : 14 May 2019

Duration : 2 hours 30 minutes

Additional Materials : Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question. The total marks for this paper is 100.

Setter : Ang Wee Hoon Fyn

This paper consists of **20** printed pages, INCLUDING the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\dbinom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\cdots(n-r+1)}{r!}$

2. Trigonometry

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{area of } \triangle ABC = \frac{1}{2} ab \sin C$$

1. The table below shows the experimental values of two variables x and y.

x	0.5	1.0	1.5	2.0	2.5	3.0
y	1.20	1.00	0.86	0.74	0.66	0.59

It is known that x and y are related by an equation of the form $y = \dfrac{a}{x+b}$, where a and b are constants.

(i)   Using a scale of 4cm to 1 unit on the x – axis, and 4cm to 0.5units on the $\dfrac{1}{y}$ – axis,   [3]

draw a straight line graph.

(ii)   Use your graph to estimate the value of a and of b.   [2]

(iii)   Find the value of x where $y = \dfrac{10}{9}$.   [2]

From another set of experimental data, it is found that x and y are related by the equation $7xy = 28 - 25y$.

(iv)   By drawing a suitable straight line on the axes in (i) , find the value of x which is consistent in the two experiments.   [3]

2. (a) (i) Show that $x^2 - 2x + 5$ is always positive for all real values of x. [2]

(ii) Hence, find the largest integer value of k for $\dfrac{3x^2 + kx + 7}{x^2 - 2x + 5} > 1$ to be true for all real values of x. [4]

(b) Simplify $\dfrac{45 \times 3^{x-2} - 9 \times 3^{x-3}}{3^{2(1-x)} \times 27^{x-1}}$. [3]

3. The roots of a quadratic equation $2x^2 - 2x + 1 = 0$ are α and β.

(i) Show that $\dfrac{2}{\alpha^3} + \dfrac{2}{\beta^3} = -8$, [4]

(ii) Find the equation whose roots are $\dfrac{2}{\alpha^3}$ and $\dfrac{2}{\beta^3}$. [2]

4. A student learns a new topic from a teacher. After learning the topic for t days, the percentage, P %, of the topic that the student remembers can be modelled by $P = 80\mathrm{e}^{-kt} + 20$, where k is a constant.

    (i)   Explain with clear working steps why the student is able to remember 100% of what he learnt from the teacher on the day it was taught.   [1]

    (ii)   Find the value of k if the student can only remember 40% of what he had learnt from the teacher a week ago.   [2]

(iii)   How many days would have elapsed when a student is only able to retain at most half   [2]
of what he learnt from the teacher?

(iv)   Lester claimed that in the long run, a student would not be able to remember what he   [1]
learnt from the teacher entirely. Do you agree with Lester? Support your decision with
clear working steps.

5. (i) In the answer space below, sketch the graph of $y = 1 - 3x^{-\frac{1}{2}}$ for $x > 0$. [1]
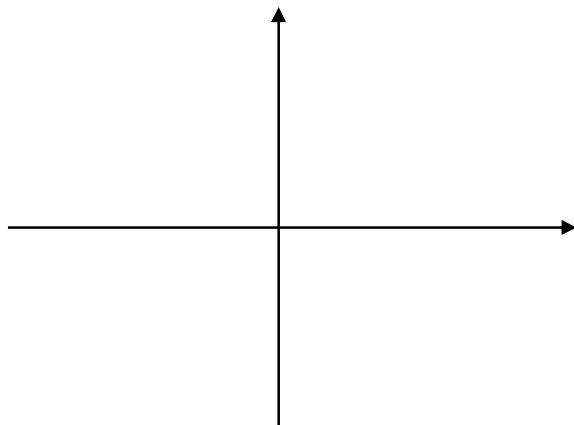
(ii) On the same diagram, sketch the graph $y = 1 - 3x^{\frac{1}{2}}$ for $x \geq 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with clear working steps, whether the tangents to the graphs at the point of intersection are perpendicular. [3]

6. (i) In the binomial expansion of $\left(x - \dfrac{3}{x}\right)^n$, where n is a positive integer, the coefficient of [4]
   the third term is 135. Find the value of n.

(ii)   Using the value of n found in part **(i)**, find the coefficient of x^4 in the expansion of   [4]

$$(1+x^2)\left(x-\frac{3}{x}\right)^n.$$

7.
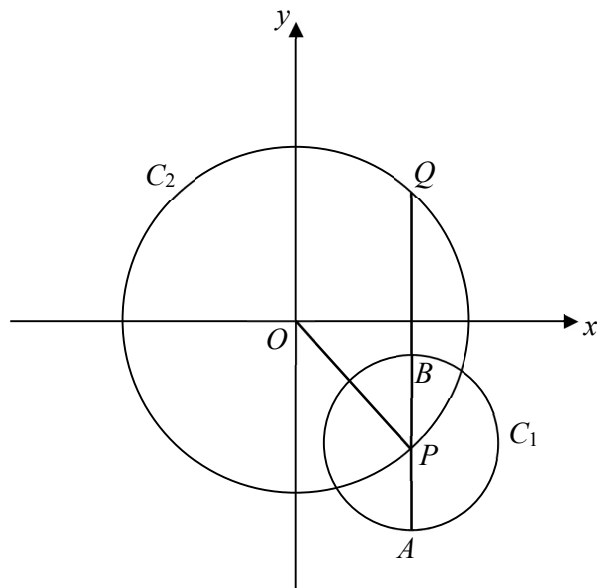
The diagram shows two circles C_1 and C_2. Circle C_1 has its centre at P. Circle C_2 passes through P and has its centre at the origin O.

(i)   Given that the equation of C_1 is $x^2 + y^2 - 4x + 4y + 6 = 0$, find

(a)   the coordinates of P and the radius of C_1,   [3]

(b)   the equation of C_2.   [2]

APB is a diameter of C_1 and parallel to the *y*-axis. *APB* produced meets C_2 at *Q*.

(ii)   Show that the *y*-coordinates of *A* and *B* are $a + b\sqrt{2}$ and $a - b\sqrt{2}$ respectively, where   [3]
       a and *b* are integers to be found.

(iii)   Find the coordinates of *Q*.   [2]

8. A curve has a gradient of -4 at the point $P(1, 4)$ and is such that $\dfrac{d^2 y}{dx^2} = 6x - 8$.

(i) Find $\dfrac{dy}{dx}$. [3]

(ii) Find the equation of the curve. [3]

(iii) Find the equation of the normal to the curve at P. [2]

9. (a) (i) Prove the identity of $\dfrac{\sin A}{\sin 2A} + \dfrac{\cos A}{1 + \cos 2A} = \sec A$. [2]

(ii) Hence, find all the angles between $-180° < A < 180°$ which satisfy [4]
$\dfrac{\sin(2A+10°)}{\sin 2(2A+10°)} + \dfrac{\cos(2A+10°)}{1 + \cos 2(2A+10°)} = -2$.

(b)   Given that A and B are acute angles such that $\tan A = \dfrac{1}{7}$ and $\cos B = \dfrac{3}{\sqrt{10}}$, find the value of each of the following without using a calculator. Rationalize your answers where applicable.
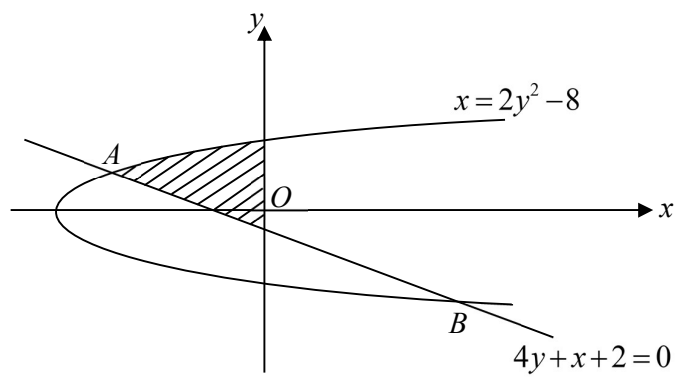
(i) $\cot B$,   [2]

(ii) $\sin(A+B)$,   [2]

(iii) $\tan 2B$.   [1]

(c)   Show that $A + 2B = 45°$.   [2]

10. In the diagram, the curve $x = 2y^2 - 8$ intersects the line $4y + x + 2 = 0$ at points A and B.

(i)   Find the coordinates of A and of B.   [3]

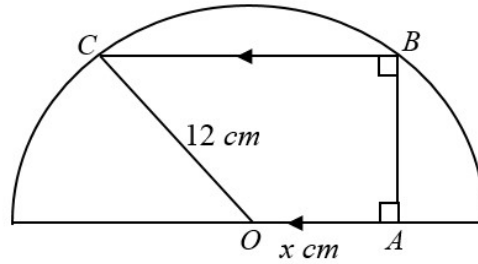(ii)   Find the area of the shaded region.   [6]

11. A curve has the equation $y = 2(x-1)e^{2x}$.

(i) Show that $\dfrac{dy}{dx} = kxe^{2x} - 2e^{2x}$, where k is a constant. [2]

(ii) Use your answer to part (i), find the exact value of $\displaystyle\int_0^1 xe^{2x}\, dx$. [3]

12. The diagram shows a trapezium $OABC$ inscribed in a semi-circle, centre O and radius 12 cm. CB is parallel to the diameter, and AB is perpendicular to both CB and OA.

(i) Given that $OA = x$ cm, explain why $BC = 2x$. [1]

(ii) Show that the area, A cm^2, of the trapezium is given by $A = \dfrac{3x}{2}\left(\sqrt{144 - x^2}\right)$. [2]

(iii)   Given that x can vary, find the value of x for which A has a stationary value. Hence,   [6]
determine whether this value of A is a maximum or a minimum.