Class	Register No.

Candidate Name ____________________________________

PEIRCE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4049/02**
Paper 2 **14 Sep 2021**
2 hours 15 minutes

Additional Materials:
Plain Paper (for rough work)

INSTRUCTIONS TO CANDIDATES
Candidates answer on the Question Paper.

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

PARENT'S SIGNATURE

For Examiner's Use	
Total	

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(i)** Factorise $(x-1)^3+64$ completely.

[2]

(ii) Hence solve the equation $(x-1)^3+64=16(x+3)$.

[3]

2 **(i)** Express $\frac{7x^2-2}{x^2(2x-1)}$ in partial fractions.

[5]

(ii) Hence, evaluate $\int_1^2 \frac{7x^2-2}{x^2(2x-1)}\,dx$.

[4]

3 **(i)** State the range of values of x for $\log_x(3x-2)$ to be defined. [1]

(ii) Solve the equation $4\log_y 3 - \log_2 \frac{1}{16} = 3\log_3 y$. [5]

(iii) Given $2\log_8 k = \log_2 \sqrt{z}$, express *z* in terms of *k*.

[4]

4 **(a)** Without using a calculator, given that $\cos\left(A+\frac{\pi}{3}\right)=4\sin\left(A+\frac{\pi}{2}\right)$, find the **exact value** of tan *A*.

[4]

(b) **(i)** Find cos 105° in the form $\frac{\sqrt{p}+\sqrt{q}}{4}$, where *p* and *q* are integers.
[3]

(ii) Hence, calculate the exact value of sec 105°.
[3]

5 **(i)** Express $2\cos^2 x + 5\sin x\cos x$ in the form $p\sin 2x + \cos 2x + q$, where p and q are constants to be found.

[3]

(ii) Hence, or otherwise, find the values of x between 0° and 360° for which

$$\cos x\,(2\cos x + 5\sin x) = 1.$$

[5]

6 Sand is poured onto a flat surface at a rate of 96π cm^3/s and formed a right circular cone. The height of the cone is always three times its radius. [Volume of circular cone = $\frac{1}{3}\pi r^2 h$]

(i) Find the rate of change of the radius 4 seconds after the start of pouring. [4]

(ii) Showing your working clearly, determine whether this rate will increase or decrease as *t* increases.

[2]

7 (a) The curve $y=(k-8)x^2-6x+k$ cuts the *x*-axis at two points and has a maximum point. Find the range of values of *k*.

[5]

(b) The equation of a curve is $y = p^2 + 2 - px + x^2$, where p is a constant.

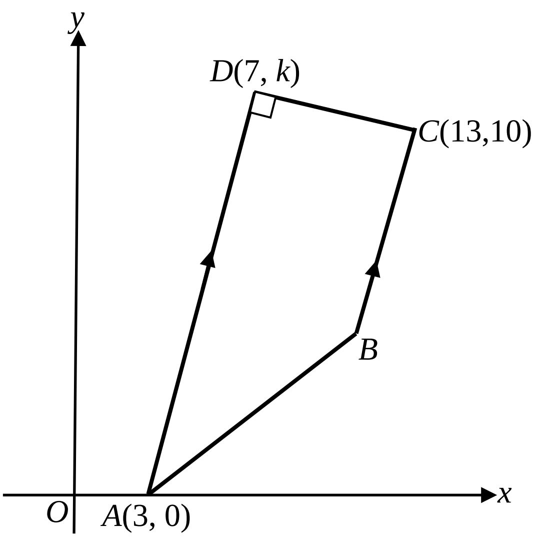
(i) Determine the nature of roots of the equation for all real values of p.

[3]

(ii) Find the values of p for which the curve is a tangent to the line $y = 5$ for all real values of x.

[3]

8 The diagram not drawn to scale, shows a trapezium *ABCD* with vertices *A*(3, 0), *C*(13, 10), *D*(7, *k*) and angle *ADC* is $90°$. The line *AD* is parallel to the line *BC*. The equation of the line *AB* is $2y - x + 3 = 0$.



(i) Show that the value of *k* is 12.

[3]

(ii) Find the equation of *BC*.

[2]

(iii) Find the coordinates of *B*.

[3]

(iv) Calculate the area of trapezium *ABCD*.

[2]

(v) Given that *ADCE* is a rectangle, calculate the coordinates of *E*.

[2]

9 It is known that x and y are connected by the equation $y = ax^2 + b\sqrt{x}$, where a and b are constants.

x	1	2	3	4	5
y	24.0	44.3	70.6	104.0	144.7

(i) On the grid on page 17, draw the graph of $\frac{y}{\sqrt{x}}$ plotted against $x\sqrt{x}$ for the given data. [3]

Use your graph to

(ii) estimate the value of a and of b,

[3]

(iii) find the value of x when $\frac{\sqrt{x}}{y} = \frac{1}{50}$.

[3]

Mary said she used the same data to plot a straight line, but her vertical axis was $\frac{y}{x^2}$.

(iv) Write down an algebraic expression for horizontal axis.

[1]

(v) What do the constants *a* and *b* represent now?

[2]

10 (a) The graph of $y = \log_a x$ passes through the points with coordinates (2, *b*), (*c*, 0) and (8, 1.5).

(i) Determine the value of each of the constants *a*, *b* and *c*.

[3]

(ii) Sketch the graph of $y = \log_a x$.

[2]

(b) Sketch the graph of $y = e^x$.

[2]

END OF PAPER

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