2022 JPJC J2 H1 Maths Preliminary Examinations Solutions:

1 $kx^2 + x^2 - 4x + k - 1 > 0$

$(k+1)x^2 - 4x + (k-1) > 0$

Discriminant < 0 and $k+1>0$

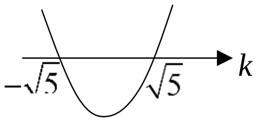
$(-4)^2 - 4(k+1)(k-1) < 0$ $k > -1$

$16 - 4\left(k^2 - 1\right) < 0$

$20 - 4k^2 < 0$

$k^2 - 5 > 0$

$\left(k+\sqrt{5}\right)\left(k-\sqrt{5}\right) > 0$



$k < -\sqrt{5}$ or $k > \sqrt{5}$

Since $k > -1$, therefore $k > \sqrt{5}$

$\{x \in \mathbb{R} : x > \sqrt{5}\}$

2(i) $\displaystyle\int \frac{2}{3\sqrt{1-4x}}\,\mathrm{d}x$

$$= \frac{2}{3}\int \left((1-4x)^{-\frac{1}{2}}\right)\mathrm{d}x$$

$$= \frac{2}{3}\left(\frac{\left((1-4x)^{\frac{1}{2}}\right)}{\frac{1}{2}(-4)}\right) + C, \textit{where } C \textit{ is an arbitrary constant}$$

$$= \frac{2}{3}\left(\frac{\left((1-4x)^{\frac{1}{2}}\right)}{-2}\right) + C$$

$$= -\frac{1}{3}(1-4x)^{\frac{1}{2}} + C$$

$$= -\frac{1}{3}\sqrt{1-4x} + C$$

2(ii) $\left(3e^{2x}-\frac{5}{e^{2x}}\right)^2=9e^{4x}-30+\frac{25}{e^{4x}}$

$=9e^{4x}-30+25e^{-4x}$

$\frac{d}{dx}\left(\left(3e^{2x}-\frac{5}{e^{2x}}\right)^2\right)$

$=9\left(4e^{4x}\right)+25\left(-4e^{-4x}\right)$

$=36e^{4x}-100e^{-4x}$

Hence, $p=36, q=-100$

Alternative method:

$\frac{d}{dx}\left(3e^{2x}-\frac{5}{e^{2x}}\right)^2$

$=\frac{d}{dx}\left(3e^{2x}-5e^{-2x}\right)^2$

$=2\left(3e^{2x}-5e^{-2x}\right)\left(6e^{2x}+10e^{-2x}\right)$

$=2\left(18e^{4x}+30-30-50e^{-4x}\right)$

$=36e^{4x}-100e^{-4x}$

Hence, $p=36, q=-100$

3(i) Let the original prices of Set Meals *A*, *B* and *C* be *a*, *b* and *c* respectively

$a+b-c=11$

$a+b+c=51.80$

$\frac{1}{2}a+\frac{3}{2}b+2c=73.40$

Using GC, $a=14.50,\quad b=16.90,\quad c=20.40$

Therefore, the original prices of Set Meals *A*, *B* and *C* are \$14.50, \$16.90 and \$20.40 respectively.

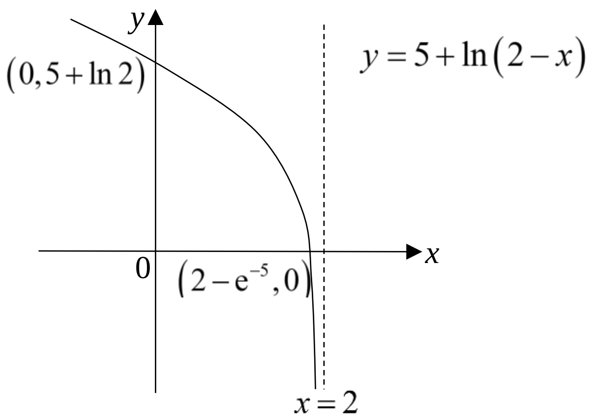
3(ii) Total amount they will spend if they use the membership discount to pay

$=0.80\left(14.50+2\times16.90+2\times20.40\right)$

$=71.28$

As \$71.28 < \$73.40, the total amount that they need to pay using membership discount is lesser than using the sales promotion. Hence, they should pay for the food that they intend to order using the membership discount.

4(i)



When $x=0$, $y=5+\ln 2$

When $y=0$,

$5+\ln\left(2-x\right)=0$

$\ln\left(2-x\right)=-5$

$2-x=e^{-5}$

$x=2-e^{-5}$

4(ii) $y=5+\ln\left(2-x\right)$

$\frac{dy}{dx}=-\frac{1}{2-x}$

When $x=-1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2-(-1)}=-\frac{1}{3}$, $y=5+\ln(2-x)=5+\ln 3$

Equation of tangent: $y=-\frac{1}{3}x+c$, where C is an arbitrary constant

Substitute $(-1, 5+\ln 3)$:

$5+\ln 3=-\frac{1}{3}(-1)+c$

$c=\frac{14}{3}+\ln 3$

Therefore, equation of tangent is $y=-\frac{1}{3}x+\frac{14}{3}+\ln 3$, where $m=-\frac{1}{3}$ and $c=\frac{14}{3}+\ln 3$

4(iii) Area = $\int_{-3}^{0} 5+\ln(2-x)\,\mathrm{d}x$ = 18.7 unit^2

5(i) $P=\frac{1}{2}\left(a+\mathrm{e}^{bt}\right)$

When $t=0$, $P=2$: $2=\frac{1}{2}\left(a+\mathrm{e}^{0}\right) \Rightarrow a=3$

When $t=1$, $P=1.75$: $1.75=\frac{1}{2}\left(3+\mathrm{e}^{b}\right)$

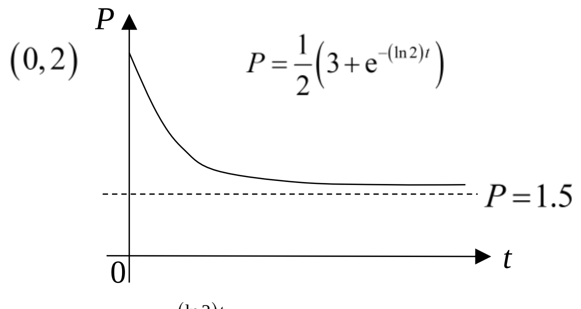
$\mathrm{e}^{b}=\frac{1}{2}$

$b=\ln\frac{1}{2}=-\ln 2$

5(ii) Using GC, at $t=5$, $\frac{\mathrm{d}P}{\mathrm{d}t}=-0.0108$

The value means that the population of the endangered birds in the 5th year is decreasing at 0.0108 thousand per year.

5(iii)



5(iv) As $t\to\infty$, $\mathrm{e}^{-(\ln 2)t}\to 0$, $P\to 1.5$

As observed from the equation and graph, the population decreases to (or approaches) 1.5 thousand in the long run.

5(v) $Q=\frac{1}{10}t\left(1+\frac{5}{t}+\frac{5}{t^3}\right)=\frac{1}{10}t+\frac{1}{2}+\frac{1}{2t^2}$

$\frac{\mathrm{d}Q}{\mathrm{d}t}=\frac{1}{10}-\frac{1}{t^3}$

At minimum point,

$\frac{\mathrm{d}Q}{\mathrm{d}t}=\frac{1}{10}-\frac{1}{t^3}=0$

$\frac{1}{t^3}=\frac{1}{10}$

$t^3=10$

$t=2.1544\approx 2.15$

$Q=\frac{1}{10}(2.1544)+\frac{1}{2}+\frac{1}{2(2.1544)^2}=0.82317$

t	2	2.1544	3
$\frac{\mathrm{d}Q}{\mathrm{d}t}$	–0.025	0	0.0630
Outline			

Therefore $(2.15, 0.823)$ is a minimum point.

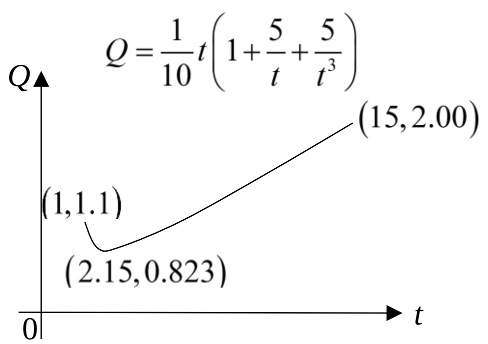
OR

$\frac{\mathrm{d}^2Q}{\mathrm{d}t^2}=\frac{3}{t^4}$

When $t=2.1544$, $\frac{\mathrm{d}^2Q}{\mathrm{d}t^2}=\frac{3}{t^4}=0.13926>0$

Therefore $(2.15, 0.823)$ is a minimum point.

5(vi)



5(vii) From GC, $t=9.95\approx 10$

6(i) Case 1 (2 vowels): Number of ways = ${}^6C_3\times{}^3C_2=60$

Case 2 (1 vowel) : Number of ways = ${}^6C_4\times{}^3C_1=45$

Case 3 (0 vowel) : Number of ways = ${}^6C_5\times{}^3C_0=6$

Hence, total number of ways = 60 + 45 + 6 = 111

6(ii) Method 1

Choose 4 letters from the remaining 7 letters to be in between S and T: ${}^{7}C_{4}$ ways
Arrange the 4 letters in between S and T: 4! ways
S and T and be in either order: 2 ways
Arrange 3 remaining letter and group of S, T and 4 letters: 4! Ways
Hence, total number of ways = ${}^{7}C_{4} \times 4! \times 2 \times 4! = 40320$

Method 2

Case 1: S_ _ _ _ T _ _ _ or T_ _ _ _ S _ _ _ : $7! \times 2 = 10080$

Case 2: _ S_ _ _ _ T _ _ or _ T_ _ _ _ S _ _ : $7! \times 2 = 10080$

Case 3: _ _ S_ _ _ _ T_ or _ _ T_ _ _ _ S_ : $7! \times 2 = 10080$

Case 4: _ _ _ S_ _ _ _ T_ or_ _ _ T_ _ _ _ S : $7! \times 2 = 10080$

Hence, total number of ways = 40320

7 Let C ~ number of office workers who wish to pick up coding skills, out of 12 office workers
C ~ B (12, 0.45)

(a) (i) $\mathrm{P}(C < 6) = \mathrm{P}(C \le 5) = 0.52693 \approx 0.527$

(ii) $\mathrm{P}(C \ge 4) = 1 - \mathrm{P}(C \le 3) = 0.86553 \approx 0.866$

(b) Method 1

Let X ~ number of groups with less than four office workers who wish to pick up coding skills, out of 6 groups
X ~ B (6, $1 - 0.52693$)
$\mathrm{P}(X = 6) = 0.0112086 \approx 0.01121$ (4sf)

Method 2

Required probability = $(1 - 0.52693)^6 =$ $0.0112086 \approx 0.01121$ (4sf)

8(i) Required probability = $\dfrac{1325}{1500} = \dfrac{53}{60}$ or 0.883(3sf)

8(ii) Required probability = $\dfrac{525}{1148} = \dfrac{75}{164}$ or 0.457(3sf)

8(iii) Let F and G be the events 'the student is a foreigner' and 'the student is in Year 2 respectively

$$\mathrm{P}(F) = \frac{114}{1500} = \frac{19}{250}$$

$$\mathrm{P}(G) = \frac{700}{1500} = \frac{7}{15}$$

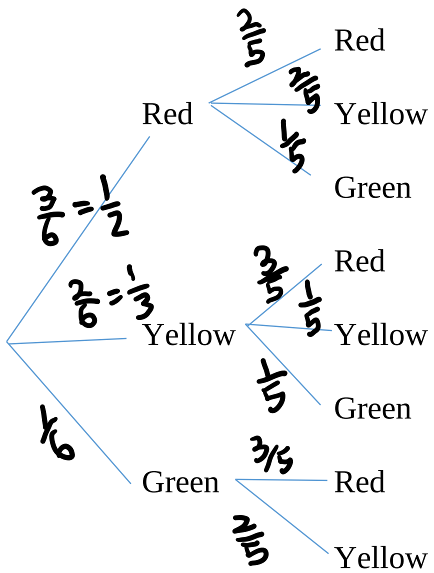
$$\mathrm{P}(F) \times \mathrm{P}(G) = \frac{19}{250} \times \frac{7}{15} = \frac{13}{3750} \quad \text{or} \quad 0.0355$$

$$\mathrm{P}(F \cap G) = \frac{52}{1500} = \frac{13}{375} \quad \text{or} \quad 0.0347$$

Since $P(F) \times P(G) \neq P(F \cap G)$, F and G are not independent.

8(iv) Required probability = $\dfrac{238}{1500} \times \dfrac{237}{1499} \times \dfrac{1262}{1498} \times 3 = 0.06340(5dp)$

9(i)

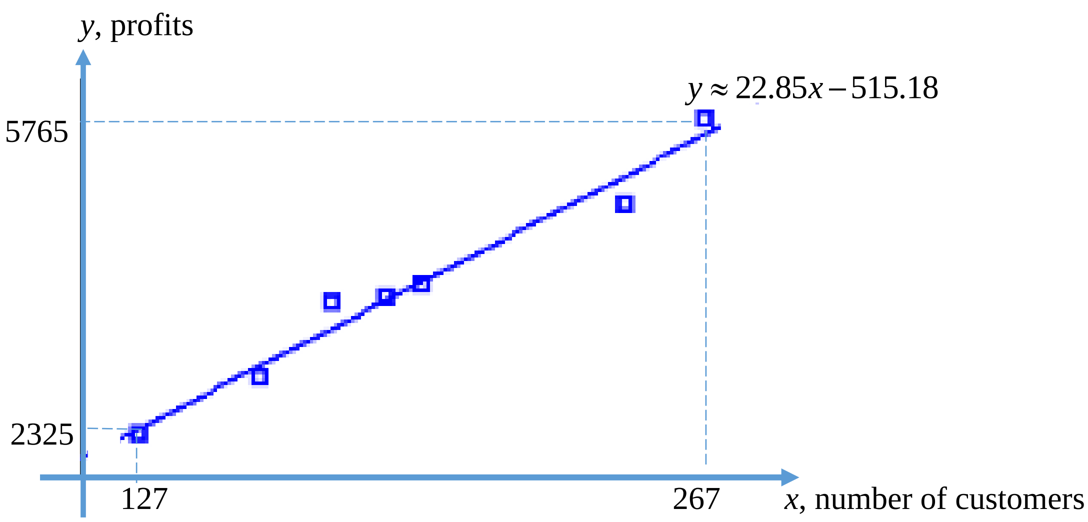


(ii) P(red, red) + P (yellow, yellow) = $\left(\dfrac{3}{6} \times \dfrac{2}{5}\right) + \left(\dfrac{2}{6} \times \dfrac{1}{5}\right) = \dfrac{4}{15}$ or 0.267

(iii) P(red, red, not red) + P(red, not red, red) + P(not red, red, red)

$= 3 \times \left(\dfrac{3}{6} \times \dfrac{2}{5} \times \dfrac{3}{4}\right) = \dfrac{9}{20}$ or 0.45

10(i)



(ii) $r = 0.98588 \approx 0.986$. Since r = 0.986 is close to +1, there is strong positive linear correlation between number of customers (x) and profits generated (y).

(iii) $y = 22.8493x - 515.1844 \approx 22.85x - 515.18$ (2dp)

(iv) m represents the rate of change of profits with respect to number of customers. For every increase in 1 customer, the average revenue increases by \$22.85.
c represents the amount of profit when there are no customers. Hence, when there are no customers, the restaurant is making an average loss of \$515.18

(v) $y = 22.8493(200) - 515.1844 = 4055$ (nearest dollar)
The estimate is reliable as:
(1) r is near to 1
(2) this estimate is an interpolation.

11(i) Unbiased estimate of population mean, $\bar{x} = \dfrac{-264}{30} + 1200 = 1191.2$

Unbiased estimate of population variance, $s^2 = \dfrac{1}{30-1}\left(18462 - \dfrac{(-264)^2}{30}\right) = 556.51$

Let X = mass of a randomly chosen chicken
$H_0 : \mu = 1200$ $H_1 : \mu < 1200$
where μ is the mean mass of chicken

Since $n = 30$ is large, by Central Limit Theorem, $\overline{X} \sim N\left(1200, \dfrac{556.51}{30}\right)$ approximately

Level of significance: 5%
Critical region: $z < -1.6449$

Test statistics value, $z = \dfrac{1191.2 - 1200}{\sqrt{\dfrac{556.51}{30}}} = -2.0432$ < -1.6449
From GC, p-value = 0.020517 $\approx$ 0.0205 < 0.05

Since p-value is less than level of significance (or test statistics is within critical region), we reject H_0 . There is sufficient evidence, at the 5% level of significance to indicate that the mean mass of chicken is less than 1200 g. Hence, farmer’s claim is not supported.

(ii) Since the sample size $n = 30$ is large, the sample mean mass of chicken can be approximated to a normal distribution by Central Limit Theorem. Hence it is not necessary to assume that the masses of chicken are distributed normally for this test to be valid.

(iii) Let Y = mass of a randomly chosen duck
$H_0 : \mu = 1495$ $H_1 : \mu \neq 1495$
where μ is the mean mass of ducks

Since $n = 40$ is large, by Central Limit Theorem, $\overline{Y} \sim N\left(1495, \dfrac{200}{40}\right)$ approximately

Level of significance: α %
p-value = 0.025347
In order for H_0 to be rejected,

level of significance > p-value

$$\frac{\alpha}{100} > 0.025347$$

$$\alpha > 2.53$$

12 Let A, B, C ~ journey times of Bus Service A, B and C respectively.

$A \sim \mathrm{N}\left(27, 10^2\right), B \sim \mathrm{N}\left(35, 6^2\right),\ C \sim \mathrm{N}\left(32, \sigma^2\right)$

(i) $\mathrm{P}\left(C > 36\right) = 0.24$

$$\mathrm{P}\left(Z > \frac{36-32}{\sigma}\right) = 0.24, \quad \mathrm{Z \sim N(0,1)}$$

$$\mathrm{P}\left(Z > \frac{4}{\sigma}\right) = 0.24$$

$$\frac{4}{\sigma} = 0.70630$$

$$\sigma = 5.6633 \approx 5.66$$

(ii) Required probability = $\mathrm{P}\left(A > 30\right) \times \mathrm{P}\left(A > 30\right) = 0.14599 \approx 0.146$

(iii) **(ii)** is a subset of p. Eg, p also include the case where one journey might not be more than 30 minutes but the total journey time is still more than 60 minutes.

(iv) $D = \left(A_1 + ... + A_5\right) - \left(B_1 + ... + B_4\right)$

$$E(D) = 5(27) - 4\left(35\right) = -5$$

$$Var(D) = 5(10^2) + 4\left(6^2\right) = 644$$

$D \sim \mathrm{N}\left(-5, 644\right)$

$\mathrm{P}\left(D < 10\right) = 0.72277 \approx 0.723$

(v) $T = 1.10\left(A_1 + ... + A_6\right) + 0.9\left(B_1 + ... + B_4\right)$

$$E(T) = (1.1)(6)(27) + (0.9)(4)\left(35\right) = 304.2$$

$$Var(T) = (1.1^2)(6)(10^2) + (0.9^2)(4)\left(6^2\right) = 842.64$$

$T \sim \mathrm{N}\left(304.2, 842.64\right)$

5 hours = 300 minutes

$\mathrm{P}\left(T \geq 300\right) = 0.55752 \approx 0.558$