2. Equations and Inequalities

(I) Equations

1 SRJC/2015Prelim/I/1

A landscape gardener is tasked to design a garden with a total area of 140 m^2.
Part of the garden will be decking, part will be flowers and the rest will be grass. Let the area of decking, the area of flowers and the area of grass, be denoted by d, f and g respectively, all measured in m^2.

It is required that the area of grass is 20 m^2 more than the total area of flowers and decking.

Including labour, each square metre of grass, decking and flowers costs \$10, \$21 and \$42 respectively. The landscape gardener has been instructed to come up with a design that will cost \$2900.

Find the values of d, f and g that the landscape gardener should use. [4]

2 YJC/2015Prelim/I/1

A confectionery bakes 3 types of cupcakes; blueberry, strawberry and chocolate. Each type is prepared in trays of 12. The time taken to prepare a tray of cupcakes, the mass of flour used for each tray and the selling price of each cupcake are given in the following table.

Type of cupcake	Preparation Time (min) for each tray	Mass of flour (g) for each tray	Selling price of a cupcake
Blueberry	8	600	\$1.00
Strawberry	7	600	\$0.90
Chocolate	6	800	\$0.80

On a particular day, 17 hours of preparation time was spent and 96 kg of flour was used to make the cupcakes. All the cupcakes were sold and the total amount collected was \$1572. By forming a system of linear equations, determine the number of trays of each type of cupcakes made on that day. [4]

3 VJC/2009Prelim/I/2

Mr Spongebob went to the supermarket on 3 separate occasions to buy crabs, lobsters and bamboo clams. He observed that while the price of crabs and bamboo clams remained constant, the price of lobsters consistently increased by 10% compared to the immediate previous visit. The amount of crabs, lobsters and bamboo clams that he bought by weight for each visit as well as the total amount spent is shown in the table below.

	1st visit	2nd visit	3rd visit
Crab (kg)	3.20	5.60	4.50
Lobster (kg)	1.50	1.20	2
Bamboo Clam (kg)	7	6.50	6.50
Total amount paid in $	277.50	347.00	395.18

What is the price per kilogram for crab, lobster and bamboo clam during Mr Spongebob's third visit to the supermarket? [5]

4 JJC/2009Prelim/I/6(a)

On a farm there are 3 different types of animals: chickens, horses, and sheep. The farmer confirms that the number of sheep is twice the number of chickens. He also counted a total of 1250 animal legs. Due to his handwriting, he was not sure if there were 250 animals or 350 animals in total.

Find the correct number of chickens, horses and sheep. [4]

5 CJC/2015Prelim/II/1

A Singaporean tourist is visiting three countries, Denmark, England and Russia and plans to prepare souvenir packs for her loved ones consisting of 1kg cheese, 1kg chocolate and 1kg candy, where the three items are from the same country. She plans to spend SGD$84 on cheese, SGD$85 on chocolate, and SGD$77 on candy.

The exchange rates to the three countries are as follows:

SGD$1 – 5 Danish Krone
SGD$1 – 0.5 British Pound
SGD$1 – 35 Russian Ruble

The prices of the commodities in the various countries' currency are given in the following table.

	Cheese/kg	Chocolate/kg	Candy/kg
Price in Danish Krone	20	30	30
Price in British Pound	4	5	2
Price in Russian Ruble	280	175	245

Find the total number of souvenir packs she should buy so that she spends all her money. **[4]**

6 VJC/2015Prelim/I/2

The function h is given by

$$\mathrm{h}: x \mapsto ax^3 + bx^2 + cx + d, \quad x \in \mathbb{R},$$

where a, b, c and d are real constants.

The graph of $y = \mathrm{h}(x)$ passes through the point $(1,1)$. Given that $(2,2)$ is a stationary point, find three linear equations involving a, b, c and d. [3]

By writing each of a, b and c in terms of d, find the exact set of values of d such that $\frac{ab}{c} \leqslant 0$. [3]

7 TJC/2015Prelim/I/1

The equation of a circle M is given by $x^2 + y^2 + Ax + By + C = 0$ where A, B and C are real constants. The line $y = -2(x + 1)$ passes through the centre of M and the graph of $y = |x|$ intersects M at the points where $x = -2$ and $x = -8$. Find the equation of M. [4]

8 NJC 2009/II/2

A sumo wrestler would like to have fish fillet, salad and fries for breakfast. As he is on a special diet, he must make sure that his intake (in grams) of protein, carbohydrates and fats per meal is in the ratio of 4:8:3. The table below shows the nutritional breakdown for one serving of each item.

	Protein (in grams)	**Carbohydrates (in grams)**	**Fats (in grams)**
Fish Fillet	150	60	25
Salad with dressing	15	30	5
Fries	5	250	110

Calculate the ratio of the servings of fish fillet, salad and fries that the wrestler should take. [4]

9 NYJC/2020Promo/2

(i) The first four terms of a sequence u_n are given by $u_1 = 32.1$, $u_2 = 17$, $u_3 = 0.7$ and $u_4 = -7.8$. Given that u_n is a cubic polynomial in n, find u_n in terms of n. [3]

(ii) Find the least value of n for which u_n is greater than 555. [2]

(II) Inequalities

10 MJC/2009Promo/4

Solve the inequality $\frac{x+3}{x-1} < x < \frac{1}{2}$ exactly. [5]

11 NYJC/2015Promo/1 (modified)

(i) Without using a calculator, solve the inequality $\dfrac{(2x+1)^2}{4x-x^2-2} \geq 0$. [4]

(ii) Hence solve $\dfrac{\left(2\sqrt{x}+1\right)^2}{4\sqrt{x}-x-2} \geq 0$, giving the answer correct to 3 significant figures. [2]

12 DHS/2009Promo/1

Using an algebraic method, solve the inequality $\dfrac{6}{x^2} \leq \dfrac{x+1}{x}$.

Hence solve $\dfrac{6}{(2-x)^2} \leq \dfrac{x-1}{x-2}$. [6]

13 TJC/2009Promo/5 (modified)

Find the solution set of the inequality $\dfrac{5}{x-2} \leq x+2$ using an algebraic method. [3]

Hence or otherwise, find the exact solution set of the inequality $\dfrac{5}{x^2-2} \geq x^2+2$. [3]

14 NJC/2008Promo/4

Solve the inequality $x(x+4)(x-1) \geq 4x(x+2)$. [7]

Hence deduce the solution for

(i) $|x|(|x|+4)(|x|-1) \geq 4|x|(|x|+2)$;

(ii) $x(x+8)(x-2) \geq 8x(x+4)$. [7]

15 PJC/2012Promo/2 (modified)

Without the use of graphic calculator, solve the inequality

$$\frac{x^2-2x+15}{x^2-6x+6} \geq 0.$$

[3]

Hence solve the inequality $\dfrac{x^2-2|x|+15}{x^2-6|x|+6} \geq 0.$ [3]

16 HCI/2008Promo/3

Solve the inequality $\sqrt{\frac{x}{2}} > \ln x$. Deduce the solution of $x > 2\sqrt{2}\ln x$. [5]

17 TJC/2011Prelim/I/2

Given that a is a positive constant, solve the inequality $|x-2a| < 2x+a$. [4]

Hence solve the inequality $|x+4| < 2-2x$. [2]

18 IJC/2009Promo/6

On the same diagram, sketch the graphs of $y = |x^2 - 5x + 6|$ and $3y + 5x = 15$. [3]

Hence, or otherwise, solve the inequality $3|x-3| \le \frac{15-5x}{|x-2|}$. [4]

19 NJC/2009Promo/1

(a) **(i)** Show that $x^2 + 4x + 5$ is always positive. [1]

(ii) Hence, without using a calculator, solve $\frac{(2x-5)}{(x^2+4x+5)(x-1)} \le 0$. [2]

(b) Solve the inequality $(x+2)(x-3) > |2x-1|$. [3]

20 NYJC/2009Promo/5

Without the use of a graphing calculator, solve the inequality $\frac{(x+1)(4-x)}{(3x+1)^2} \ge 0$. [3]

Hence deduce the solution to the inequality $\frac{(\sqrt{x}+1)(4-\sqrt{x})}{(3\sqrt{x}+1)^2} \ge 0$. [3]

21 SAJC/2009Promo/10

Solve the inequality $\frac{x^2}{x+2} \le 1$. [3]

Hence, deduce the solution set of

(i) $\frac{x^2}{|x|+2} \le 1$ [3]

(ii) $\frac{e^{2x}}{2-e^x} \le 1$ [3]

22 CJC/2010Promo/6

Solve the inequality $\frac{2}{x} < 6 - x$, leaving your answers in its exact form. [4]

Hence find the range of values of x for which $2\,\text{cosec}\, x < 6 - \sin x$ for $0 \le x \le 2\pi$. [4]

23 MI/2010Promo/12 (part)

(a) Solve the inequality $e^{2x} < 1 - e^{x}$. [2]

(b) Show that $2x^2 - 4x + 3$ can be expressed as $2(x+a)^2 + b$, where a and b are constants to be determined. [2]

Hence find the range of values of x that satisfies $\frac{x^2}{x-3} < 1 - x$. [3]

24 RI/2009Promo/2

(a) Using the method of completing the square, show that $4x^2 - 4x + 3$ is always positive for all real values of x. [1]

Without using a graphic calculator, solve the inequality $\frac{32x - 243}{x^2 + 7x - 60} > 4$. [4]

(b) Sketch, on the same diagram, the graphs of $y = |x-5| - |2-3x|$ and $y = 7 - 4x$.

Hence or otherwise solve the inequality $\ln\left(|x-5| - |2-3x|\right) \le \ln(7-4x)$. [4]

Answers

(I) Equations

1. $d = 20, f = 40, g = 80$
2. $x = 50,\ y = 50,\ z = 45$
3. Price per kilogram for crab, lobster and bamboo clam are \$36.20, \$96.90, \$5.95 respectively.
4. Number of chickens, horses and sheep are 75, 125, 150 respect
5. Packs = 13
6. $a = -\frac{1}{2} - \frac{1}{4}d \quad b = \frac{3}{2} + \frac{5}{4}d \quad c = -2d \quad \{d \in \mathbb{R} : d \le -2 \text{ or } -\frac{6}{5} \le d < 0\}$
7. $M: x^2 + y^2 + 8x - 12y + 32 = 0$
8. Ratio is 1:3:1
9. $u_n = 1.5n^3 - 9.6n^2 + 3.2n + 37$, 10.

(II) Inequalities

10. $-1 < x < \frac{1}{2}$

11. $2-\sqrt{2} < x < 2+\sqrt{2}$ or $x = -\frac{1}{2}$; $0.343 < x < 11.7$ or $\sqrt{x} = -\frac{1}{2}$ (rej)

12. $x \le -3$ or $x \ge 2$; $x \le -1$ or $x \ge 4$

13. $\{x \in \mathbb{R} : x \ge 3$ or $-3 \le x < 2\}$; $\sqrt{2} < x \le \sqrt{3}$ or $-\sqrt{3} \le x < -\sqrt{2}$

14. $-3 \le x \le 0$ or $x \ge 4$ (i) $x = 0$ or $x \ge 4$ or $x \le -4$. (ii) $-6 \le x \le 0$ or $x \ge 8$

15. $x < 3-\sqrt{3}$ or $x > 3+\sqrt{3}$; $-3+\sqrt{3} < x < 3-\sqrt{3}$ or $x < -3-\sqrt{3}$ or $x > 3+\sqrt{3}$

16. $0 < x < 4.43$ or $x > 13.7$; $0 < x < 2.10$ or $x > 3.70$

17. $x > \frac{a}{3}$; $-x > \frac{2}{3} \Rightarrow x < -\frac{2}{3}$

18. $\frac{1}{3} \le x < 2$ or $2 < x \le 3$ (OR $\frac{1}{3} \le x \le 3,\ x \ne 2$)

19. $1 < x \le \frac{5}{2}$; $x < -3.19$ or $x > 4.19$

20. $-1 \le x \le 4, x \ne -\frac{1}{3}$; $0 \le x \le 16$

21. $\{ x < -2$ or $-1 \le x \le 2 \}$ (i) $\{ x \in \mathbb{R} : -2 \le x \le 2 \}$ (ii) $\{x \in \mathbb{R} : x \le 0$ or $x > \ln 2\}$

22. $\therefore x < 0,\quad 0.354 < x < 5.65$; $0.362 < x < 2.78,\quad \pi < x < 2\pi.$

23. (a) $x < -0.481$; (b) $x < 3$

24. $-12 < x < 5$, $-\frac{3}{2} < x < \frac{7}{4}$