A LEVEL
H2 MATHEMATICS
APPLICATIONS OF INTEGRATION

CHAPTER ANALYSIS

MASTERY

- Approximation By Rectangles
- Area Under Curve
- Volume of Solid of Revolution

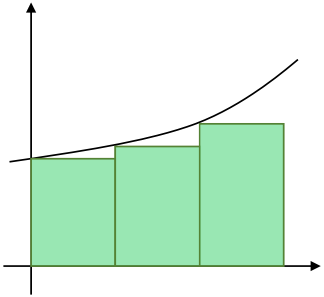
EXAM

- Differentiation and integration techniques should be strong to do well in this chapter
- Chapter is often tested with parametric curves

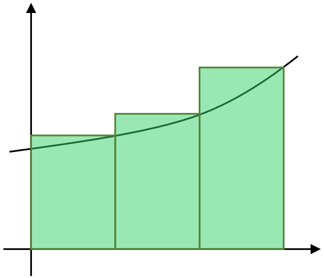
WEIGHTAGE

- Usually appears as 1 questions in the exam, tested with integration techniques and commonly parametric curves
- Chapter itself (finding area/volume) is low weightage; a part of a bigger question, around 5 marks

Approximation By Rectangles

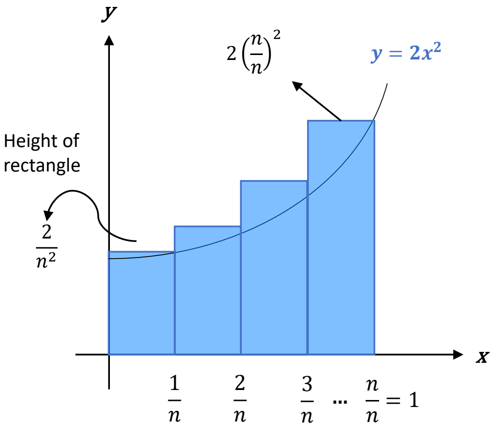


Underestimation



Overestimation

Use Summation to Approximate Area



Approximated Area

$$= \left(\frac{1}{n}\right)\left(\frac{2}{n^2}\right) + \left(\frac{1}{n}\right)\left(\frac{8}{n^2}\right) + \cdots + \left(\frac{1}{n}\right)\left(2\left(\frac{n}{n}\right)^2\right)$$

$$= \frac{2}{n^3}\sum_{r=1}^{n} r^2 = \frac{2}{n^3} \times \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{2n^2} = \frac{2n^2+3n+1}{2n^2}$$

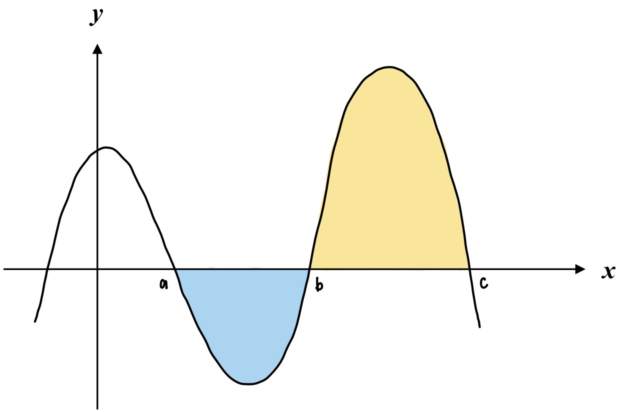
$$= 1 + \frac{3}{2n} + \frac{1}{2n^2}$$

As $n \to \infty, \frac{3}{2n} \to 0$ and $\frac{1}{2n^2} \to 0$

Therefore, $Area \to 1$ from $x = 0$ to $x = 1$

Evaluating Area Bound By Curves & Axes

X-AXIS



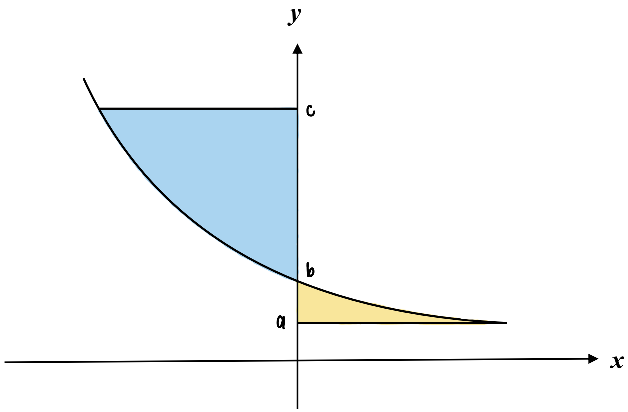
Negative Area

$$= -\int_a^b f(x)\,dx = \int_a^b |f(x)|\,dx$$

Positive Area

$$= \int_b^c f(x)\,dx$$

Y-AXIS



Negative Area

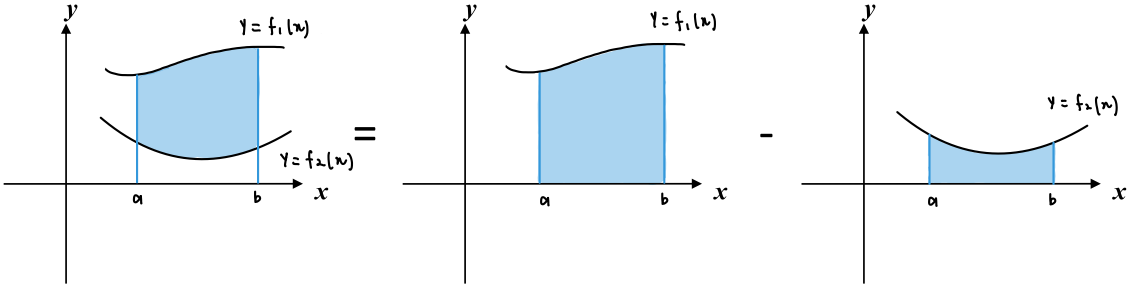
$$= -\int_b^c f(y)\,dy = \int_b^c |f(y)|\,dy$$

Positive Area

$$= \int_a^b f(y)\,dy$$

Evaluating Area Bound Between Curves

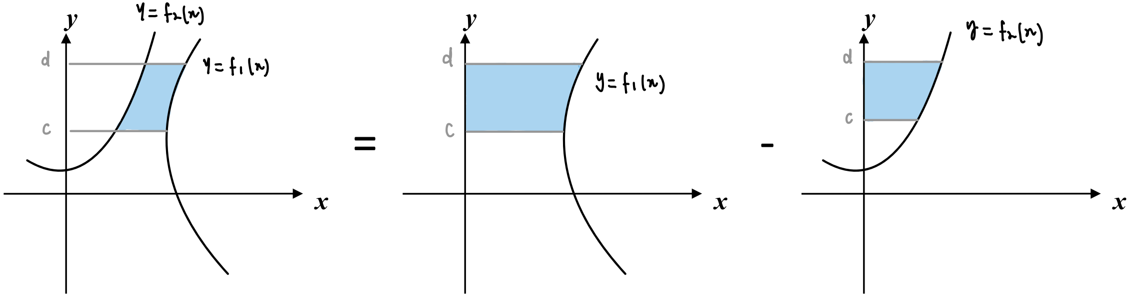
X-AXIS



Area

$$= \int_a^b |f_1(x) - f_2(x)|\, dx$$

Y-AXIS

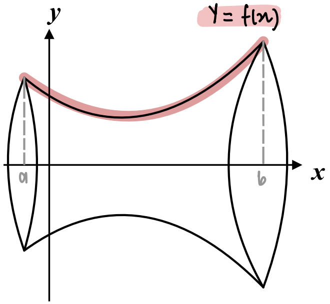


Area

$$= \int_c^d |f_1(y) - f_2(y)|\, dy$$

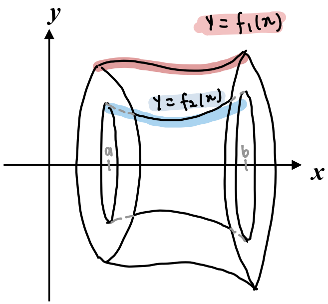
Volume Of Solid Of Revolution

Rotation About X-AXIS



Volume

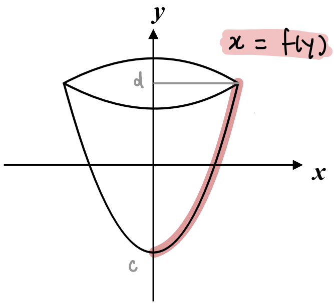
$$= \pi \int_a^b [f(x)]^2 \, dx$$



Volume

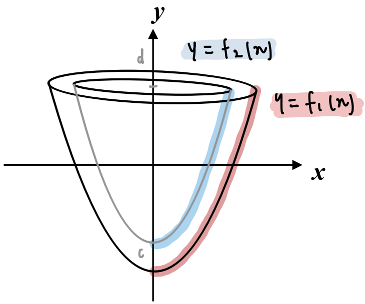
$$= \pi \int_a^b [f_1(x)]^2 - [f_2(x)]^2 \, dx$$

Rotation About Y-AXIS



Volume

$$= \pi \int_c^d [f(y)]^2 \, dy$$



Volume

$$= \pi \int_c^d [f_1(y)]^2 - [f_2(y)]^2 \, dy$$