Topic 4
Forces

Content
- Types of force
- Centre of gravity
- Turning effects of forces
- Equilibrium of forces
- Upthrust

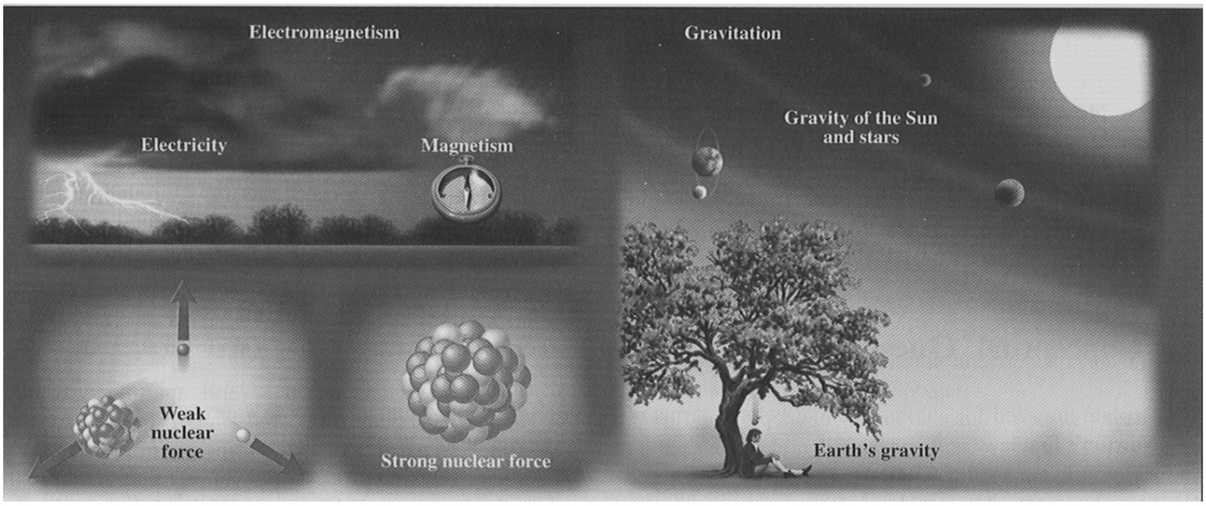
Learning Outcomes
Candidates should be able to:

(a) recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems.
(b) describe the forces on mass, charge and current in gravitational, electric and magnetic fields as appropriate.
(c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance. (No treatment of the coefficients of friction and viscosity is required).
(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.
(e) define and apply the moment of a force and the torque of a couple.
(f) show an understanding that a couple is a pair of forces which tends to produce rotation only.
(g) apply the principle of moments to new situations or to solve related problems.
(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.
(i) use a vector triangle to represent forces in equilibrium.
(j)* derive, from definitions of pressure and density, the equation $p = \rho gh$.
(k)* solve problems using the equation $p = \rho gh$.
(l)* show an understanding of the origin of the upthrust acting on a body in a fluid.
(m)* state that upthrust is equal to the weight of the fluid displaced by a submerged or floating object.
(n)* calculate the upthrust in terms of the weight of the displaced fluid.
(o)* recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal to the weight of the object to new situations or to solve related problems.

* Not required for 8867 H1 Physics

4.0 Introduction

One of the main goals of Physics has been to understand the immense variety of forces in the universe in terms of the fewest number of fundamental laws. Today, all forces are understood in terms of just four fundamental interactions as follows:



Force	Effects	Range/m
Gravitational Force	Weakest. Acts on all masses	infinite
Electromagnetic Force	Acts on electric charges	infinite
Strong Nuclear Force	Holds protons and neutrons together in a nucleus	10^{-15}
Weak Nuclear Force	Causes radioactive decay processes.	10^{-17}

A force is a push or pull exerted by one body on another. It is an interaction between two bodies or between a body and its environment. Force is a vector quantity and hence it is quantified by having both a magnitude and a direction.

The unit of force is newton (N). 1 N is defined as the magnitude of a force that accelerates a mass of 1 kg at a rate of 1 m s^{-2} in the direction of the force.

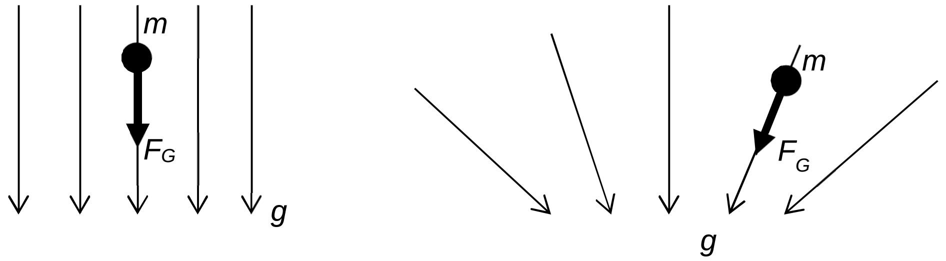
4.1 Types of Forces

4.1.1 Force on a Mass in a Gravitational Field

When a body of mass m is placed in a gravitational field of gravitational field strength $\vec{g}$, it will experience a gravitational force $\vec{F}_G$ with a magnitude given by

$$\boxed{\vec{F}_G = m\vec{g}}$$

and $\vec{F}_G$ acts in the direction of the gravitational field strength $\vec{g}$ at that point



If $\vec{g}$ is set up by a massive body, $\vec{F}_G$ is also called the *weight* of a body of mass m. The weight can be taken to act at a single point known as the *centre of gravity* of the body.

Conceptual Question

Does the centre of mass and the centre of gravity of a body always coincide?

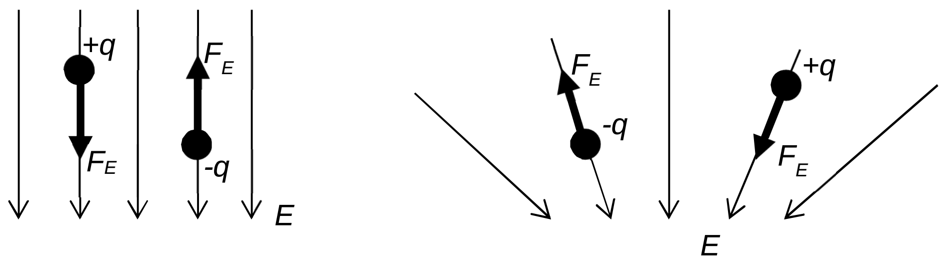
No. The two points will only coincide when the body is place in a region of uniform gravitational field.

4.1.2 Force on a Charge in an Electric Field

When a charge q is placed in an electric field of electric field strength $\vec{E}$, it will experience an electric force $\vec{F}_E$ where

$$\boxed{\vec{F}_E = q\vec{E}}$$

If q is positive, $\vec{F}_E$ is in the direction of $\vec{E}$. If q is negative, $\vec{F}_E$ is in the opposite direction to $\vec{E}$.



4.1.3 Force on a Moving Charge in a Magnetic Force

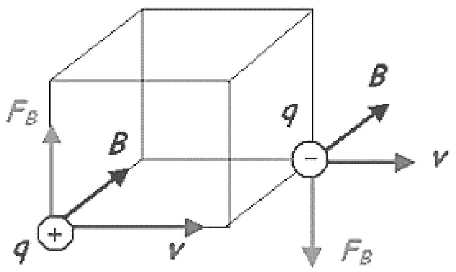
When a charge q is moving with velocity $\vec{v}$ at an angle θ to the magnetic field $\vec{B}$, it will experience a magnetic force $\vec{F}_B$ where

$$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$

Magnitude of $\left|\vec{F}_B\right| = q|\vec{v}|\left|\vec{B}\right| Sin\theta$

The direction of $\vec{v}$ follows that of the movement of a positive charges.

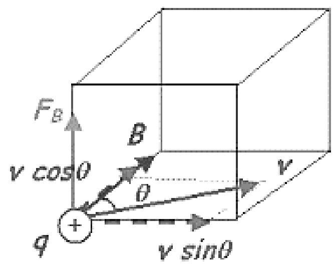
If $\theta = 90^0$



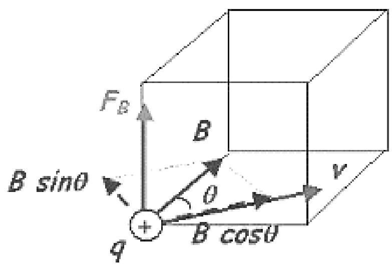
NB: 1. Direction of F_B is predicted using Fleming's Left Hand Rule.

2. F_B causes q to trace a circular path whose plane is perpendicular to B.

If $\theta \neq 90^0$



Resolving $\vec{v}$ perpendicular to $\vec{B}$ and parallel to $\vec{B}$

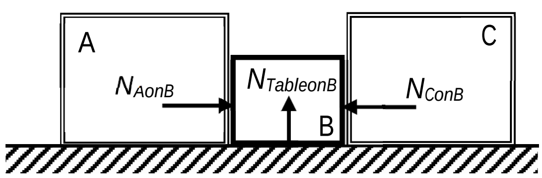


Resolving $\vec{B}$ perpendicular to $\vec{v}$ and parallel to $\vec{v}$

4.1.4 Normal Contact Force

Normal contact force is the force that the surface of one body exerts (or pushes) on the surface of another body it is in contact with. Normal contact force is always perpendicular (or normal) to the surfaces in contact with each other.

The diagram below shows the normal contact forces exerted by the table, block A and block C on block B.



4.1.5 Tensile and Compressive Forces

Tension and compression are one dimensional forces exerted along the axis of a body. These forces cause changes to the linear dimension of the body.

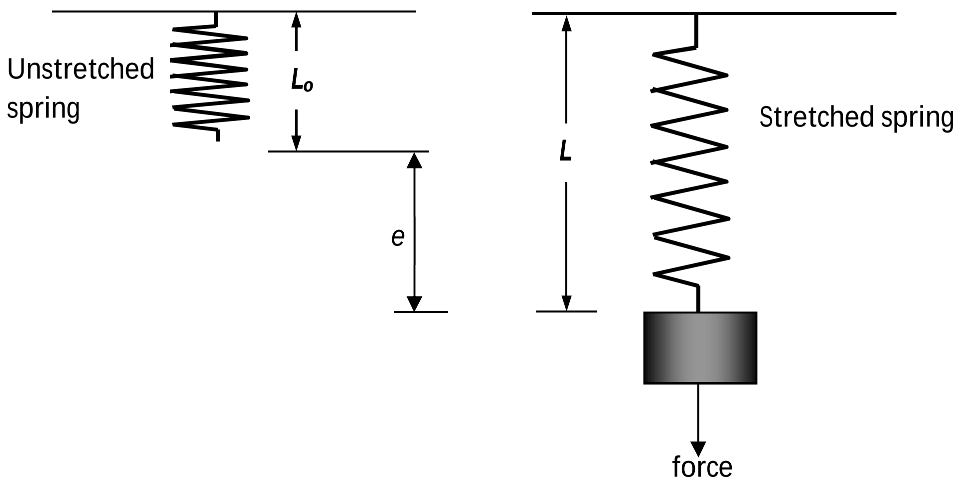
A material is in tension when its ends are pulled apart along its axis by equal and opposite external forces.

A material in compression when its ends are pushed together along its axis by equal and opposite external forces.

> *Conceptual Question*
>
> A tensile force is exerted at the lower end of a vertical rod of negligible mass along its axis. Is the tension the same throughout the rod?
>
> Yes. If not, it will mean that there will be a resultant force acting various portions of the rod which will in turn cause these portions to experience an acceleration.

4.1.5.1 Hooke's Law

Hooke's law states that <u>the change in length *x* of a material is directly proportional to the force *F* applied on it, provided that the limit of proportionality is not exceeded</u>.



$$\boxed{F = kx}$$

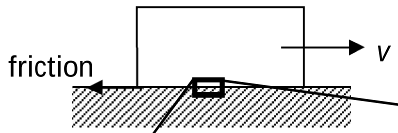
where
- F is the force applied to the material,
- L_o is the unstretched length of material
- L is final length of material
- x is the extension / compression of the material ($L - L_o$) or ($L_o - L$)
- k is the proportionality (force) constant

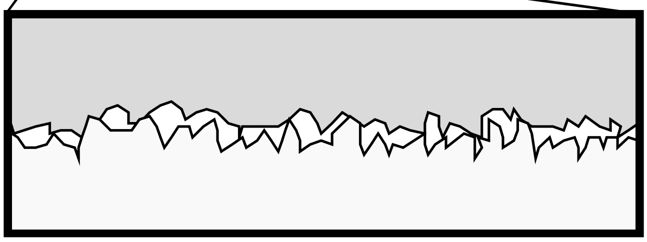
4.1.6 Friction and Viscous Forces

Friction and viscous forces are known as *dissipative forces*. Some mechanical energy of the object experiencing these forces is dissipated (as heat) to the surroundings.

4.1.6.1 Friction

Friction acts along the surface between two objects whenever one moves or tries to move over the other and in the direction so as to oppose relative motion (or impending relative motion) of the surfaces.



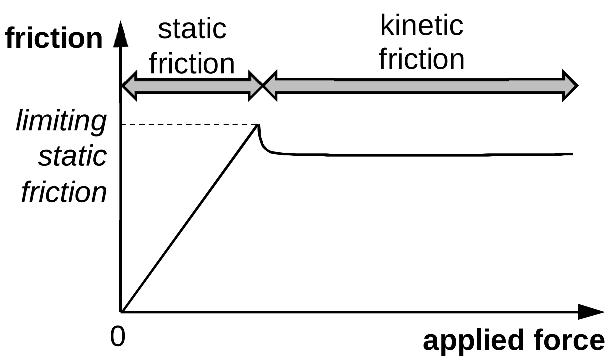


Close examination of the flattest and most highly polished surface reveals hollows and humps more than one hundred atoms high. When one solid is placed on another, contact occurs only at a few places of small areas. The pressure at the points of contact is extremely high and causes the humps to flatten out until the increased area of contact enables the upper solid to be supported. At the points of contact, small, cold-welded 'joints' are formed by the strong adhesive forces between molecules which are very close together. These joints have to be broken before one surface can move over the other.

The value of frictional force is dependent on the types of surfaces in contact with each other, as well as the magnitude of the normal contact force exerted by one surface on the other.

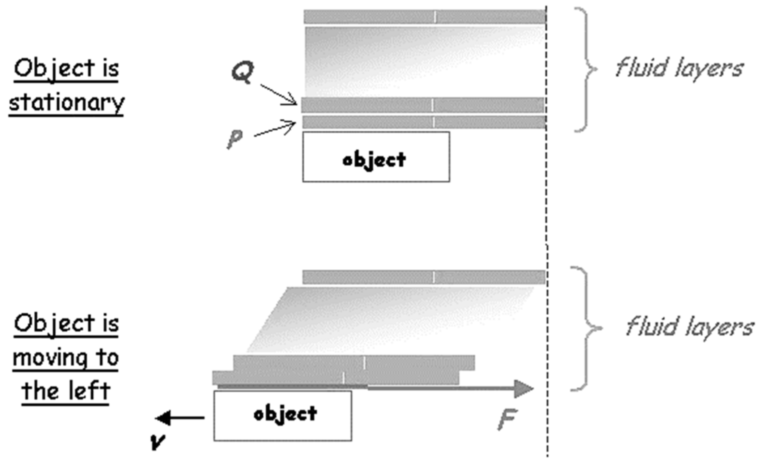
Static friction is the frictional force that acts when there is no relative motion between two surfaces. The magnitude of static friction is self-adjusting such that it is just sufficient to prevent motion, but only up to a maximum value known as the *limiting static friction*.

Kinetic friction is the frictional force that acts when two surfaces slide against each other. For the same two surfaces, the magnitude of kinetic friction is lower than that of the limiting static friction.



4.1.6.2 Viscous Force

In a viscous flow, fluid can be regarded as made up of a stack of very thin layers, each moving with different speeds due to internal friction between the layers. When an object moves relative to a fluid with velocity v, the layer of fluid P adjacent to it is dragged along by it. But the next layer Q slows layer P down. There is a gradual decrease in velocity for the layers with distance away from the object. The overall effect is represented by a net retarding force on the object. This retarding force, F (see diagram below) is acting to the right which opposes motion.

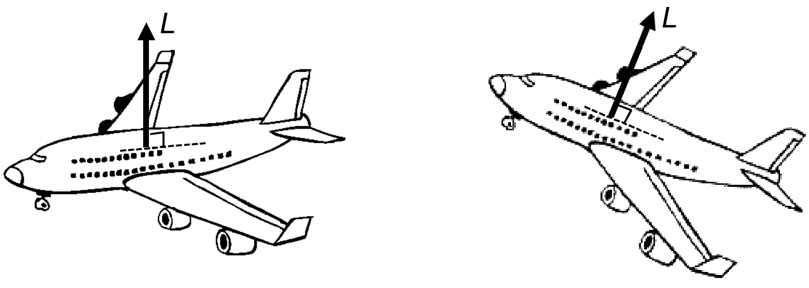


The magnitude of the viscous force increases with the speed of the object. At low speeds, the viscous force acting on the object is proportional to its speed. At high speeds, viscous force is proportional to the square of its speed. The exact relationship depends also on the shape, size, texture of surface and viscosity of fluid.

4.1.7 Aerodynamic Lift

Lift is the force that acts on a body, such as an airplane wing or a helicopter rotor, due to the motion of a body through a fluid. Lift acts in the direction perpendicular to the direction of the relative flow of the surrounding fluid.

When an airplane is flying horizontally, the lift L is drawn as a single vector on the plane in the upward direction. When an airplane is climbing, descending, or banking in a turn, the lift is tilted with respect to the vertical.

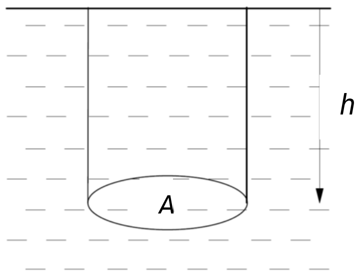


4.1.8 Upthrust in Fluids *(Not required for 8867 H1 Physics)

4.1.8.1 Fluid Pressure

Pressure is defined as the normal force acting per unit area and it is a scalar quantity.

Considering a cross-sectional area *A* in a fluid at a depth *h* below the surface of the fluid:



If the density of the fluid is ρ and the volume of the column of fluid above the area *A* is *V*, then the pressure *P* on *A* due to the fluid at this depth *h* is given by

$$
\begin{aligned}
P &= \frac{\text{Normal force exerted by fluid column on } A}{A} \\
&= \frac{\text{Weight of fluid column}}{A} \\
&= \frac{mg}{A} = \frac{(\rho V)g}{A} = \frac{\rho(Ah)g}{A} = h\rho g
\end{aligned}
$$

Hence fluid pressure *P* at a depth of *h* is given by

$$\boxed{P = h\,\rho\,g}$$

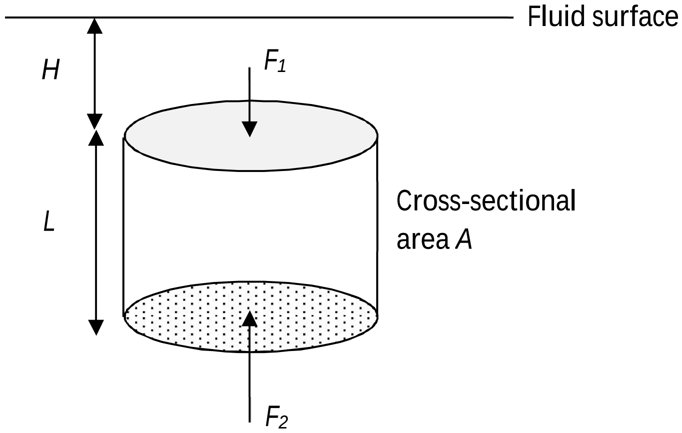
where ρ is the density of the fluid.

The absolute (total) pressure at this depth includes the over-bearing pressure acting on the open surface of the fluid. For example, if an atmospheric pressure P_o acts on the open surface of the fluid,

$$P_{absolute} = P_o + h\rho g$$

4.1.8.2 Upthrust

Consider an object of uniform cross-sectional area A and length L fully submerged in a fluid of density ρ_{fluid}, as shown.



Pressure exerted by fluid on upper surface of object, $P_1 = H\,\rho_{fluid}\,g$

Force exerted by fluid on upper surface of object, $F_1 = P_1 A$
$= (H\,\rho_{fluid}\,g)\,A$

Pressure exerted by fluid on lower surface of object, $P_2 = (H + L)\,\rho_{fluid}\,g$

Force exerted by fluid on lower surface of object, $F_2 = P_2 A$
$= (H + L)\,\rho_{fluid}\,g\,A$
$= F_1 + \rho_{fluid}\,L\,A\,g$

Upthrust exerted by fluid on object,

$$\begin{aligned} U &= F_2 - F_1 \\ &= \rho_{fluid}\,L\,A\,g \\ &= \rho_{fluid}\,V_{submerged}\,g\ , \end{aligned}$$

where $V_{submerged}$ is the volume of object submerged in the fluid, and is also equal to the volume of fluid displaced by the object V_{fluid}.

Since the mass of the fluid displaced by the object, $m_{fluid} = \rho_{fluid}\,V_{fluid}$,

$$\boxed{\text{upthrust, } U = \rho_{fluid}\,V_{fluid}\,g = m_{fluid}\,g}$$

The above result is known as the *Archimedes' Principle* which states that <u>when a body is immersed in a fluid, it is buoyed by an upthrust equal in magnitude to the weight of fluid displaced by the body</u>.

Upthrust is <u>the upward force exerted by a fluid on an object submerged fully or partially in the fluid due to the difference in pressure exerted by the fluid on the upper and lower surfaces of the object. It acts through the centre of mass of the displaced fluid.</u>

Example 1

(a) What is the upthrust on a human body of volume 7.4×10^{-2} m^3 when it is totally immersed in

(i) air of density 1.3 kg m^{-3}

(ii) sea water of density 1030 kg m^{-3} ?

Since the body is totally immersed in the fluid, the volume of body would equal the volume of fluid displaced.

(i) By Archimedes' Principle:

$$U = \rho_{fluid} g V_{fluid\ displaced}$$
$$= (1.3)(9.81)\left(7.4\times10^{-2}\right) = 0.94\ N$$

(ii) By Archimedes' Principle:

$$U = \rho_{fluid} g V_{fluid\ displaced}$$
$$= (1030)(9.81)\left(7.4\times10^{-2}\right) = 750\ N$$

(b) Hence explain why, the upthrust acting on a human body when in air is normally ignored.

The average person weighs about 600 N and the upthrust in air of about 1 N is less than 0.2 % of the weight of the person, making it negligible.

Example 2

A boat floating in fresh water displaces 35.6 kN of water.

(a) What weight of water would this boat displace if it were floating in salt water of density 1024 kg m^{-3}?

For the boat to float, the upthrust on the boat must be equal to the weight of boat W whether in salt or fresh water.
Hence weight of salt water displaced = 35.6 kN

(b) What is the volume of salt water displaced?

Upthrust = weight of salt water displaced.

$$\rho_s V_s g = 35.6\ \text{kN}$$
$$(1024)(V_s)(9.81) = 35.6 \times 10^3$$
$$V_s = 3.54\ \text{m}^3$$

An object floats because the upthrust acting on it is equal and opposite to the weight of the object. The object sinks when the upthrust acting on it is less than its weight. A ship made of steel can float because its internal hollow volume displaces a large amount of water and produces sufficient upthrust to keep the ship floating.

Example 3

A student rolled a lump of plasticene into the shape of an air-tight sphere. He dropped the plasticene into a bucket of water and observed that the plasticene sphere sank to the bottom of the water.

Show that the density of the plasticene must be greater than the density of water.

As the plasticene sinks, this implies that the weight of the plasticene is larger than the upthrust on it by the water.

$$W_{plasticene} > U$$

By Archimedes' Principle, $U = W_{fluid\ displaced}$

Therefore,

$$W_{plasticene} > W_{fluid\ displaced}$$
$$\rho_{plasticene} g V_{plasticene} > \rho_{water} g V_{water\ displaced}$$

Since when the plasticene is fully submerged, $V_{plasticene} = V_{water\ displaced}$

$$\rho_{plasticene} > \rho_{water}$$

4.2 Free-Body Diagrams

A *free-body diagram* of a body (or system) shows all external forces acting on the body (or system). In the consideration of both static and dynamic problems, it is necessary to draw the free-body diagram of the appropriate body or system that is being identified for analysis.

Besides the magnitude and direction of a force, it is also important that its point of exertion and line of action are also identified. The point of exertion of a force is the point at which the forces acting on a system seem to be concentrated, such that the forces can be replaced by a single force, equivalent to the sum of the forces, acting at that point. The line of action of a force is the line along which the force vector acts. Both are important in determining the equilibrium and stability of a body or system that the force exerts upon.

In drawing a free-body diagram, one should
1. identify the body (or system) that is being considered and draw a simple sketch representing the body (or system).
2. mark and label on the sketch from (1), all external forces acting on the body (or system), paying attention to the point of exertion and line of action of each force.
3. determine the magnitude of the forces (if necessary) that can be directly calculated via a formula.

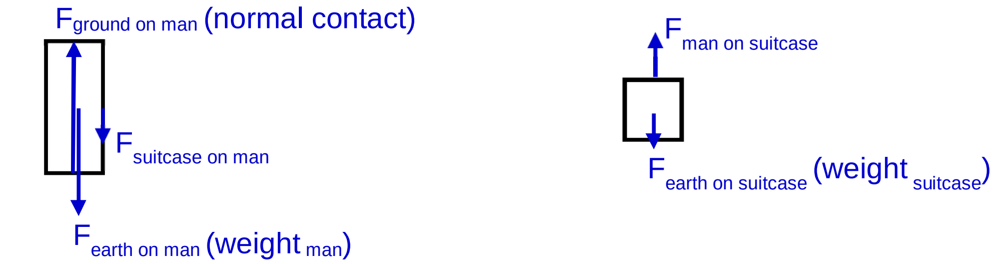
One salient point that helps in determining whether a force is to be included on a free-body diagram is to identify the origins of that force, i.e. the body exerting the force as well as the body experiencing the force. This is where the understanding of Newton's 3rd Law of Motion is especially important.

Newton's 3rd Law of Motion states that <u>when body A exerts a force on body B, body B will exert a force of equal magnitude but of opposite direction on body A</u>. One implication of this law is that the free-body diagram of body A (or that of body B) should not have both "action-reaction" forces drawn on it.

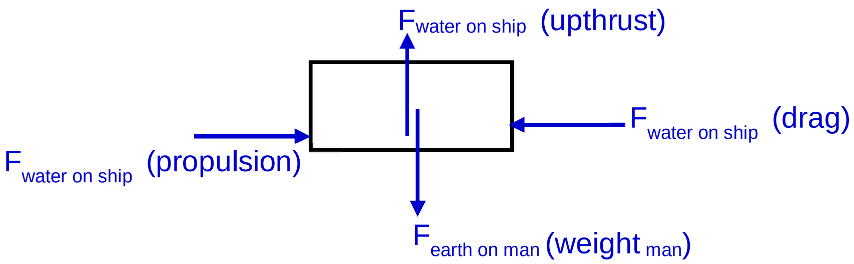
Practice Questions

1. In the following situations, sketch the free body diagrams for each of the underlined bodies. Label all forces in full.

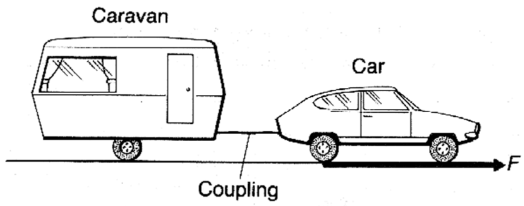
(a) A man standing stationary on the floor holding a suitcase.



(b) A ship moving through the sea at constant velocity.

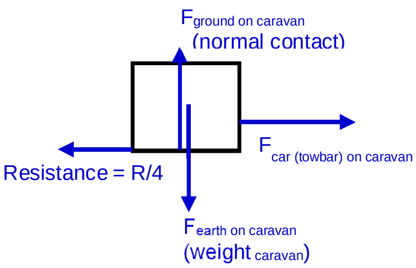


2. A car tows a caravan. The engine of the car provides a driving force F. The total resistance to motion has a constant value of R. A quarter of this resistance acts on the caravan.

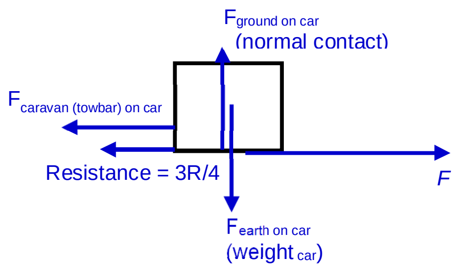


Sketch the free body diagrams, for

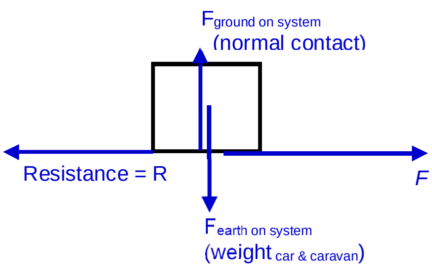
(a) the caravan



(b) the car

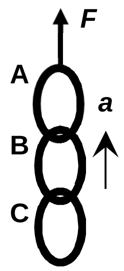


(c) the caravan and car as a single body.

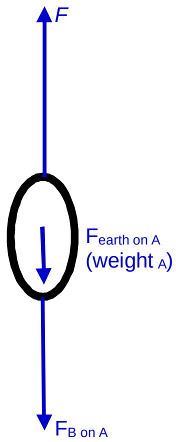


3. A chain with 3 identical chain links A, B and C is pulled upwards with a force *F* such that it produces an upward acceleration *a* as shown in the diagram on the right.

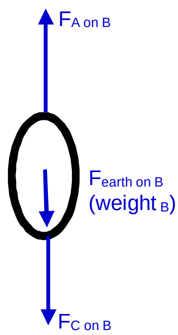
Label the forces on the free body for each of the chain links A, B and C



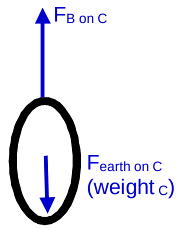
Chain Link A



Chain Link B

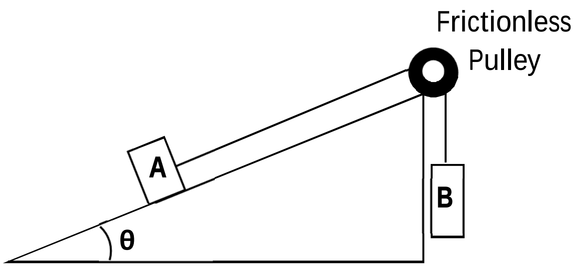


Chain Link C

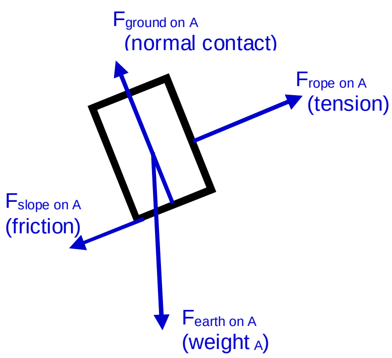


4. It is observed that the system shown on the right can be set in motion in such a way that the mass A slides upwards on a rough incline with a constant velocity.

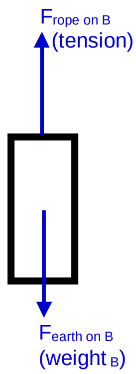
Identify and label all external forces acting each of the free-bodies below:



Mass A



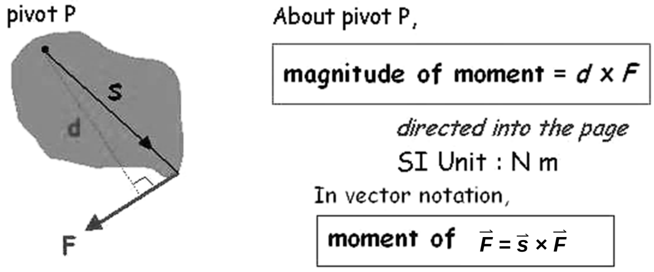
Mass B



4.3 Rotational Effects

4.3.1 Moment of a Force

The magnitude of the *moment of a force* about a point is defined as <u>the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force</u>.



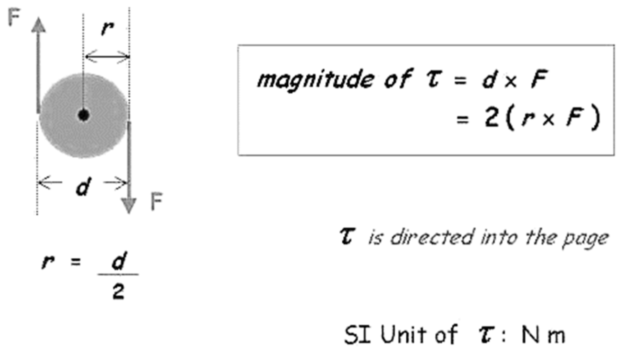
A moment supplied by a non-zero force may produce both a linear acceleration of its centre of mass as well as an angular acceleration.

4.3.2 Torque of a Couple

A couple is a pair of forces acting on the same body (or system) that
(i) are equal in magnitude and opposite in direction,
(ii) whose line of action do not coincide.

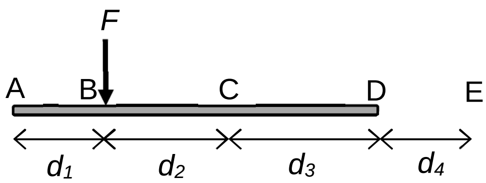
The magnitude of the *torque due a couple* is defined as the <u>product of the magnitude of one of the forces and the perpendicular distance between the lines of action of the forces</u>.

A torque supplied by a couple does not produce any linear acceleration, but only angular acceleration.



Conceptual Exercise

Considering a rod acted upon by a force *F*. Determine the moments about points A to E.



Taking moments about

A: $\sum M_A = Fd_1$, clockwise

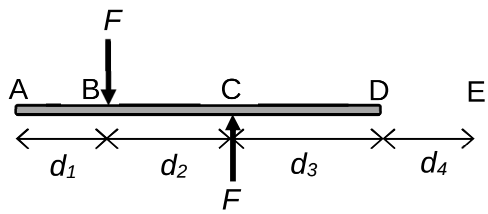
B: $\sum M_B = 0$

C: $\sum M_C = Fd_2$, anti-clockwise

D: $\sum M_D = F(d_2 + d_3)$, anti-clockwise

E: $\sum M_E = F(d_2 + d_3 + d_4)$, anti-clockwise

Considering a rod acted upon by a couple whose magnitude of one of the forces is F. Determine the moments about points A to E.



Taking moments about

A: $\sum M_A = Fd_2$, anti-clockwise

B: $\sum M_B = Fd_2$, anti-clockwise

C: $\sum M_C = Fd_2$, anti-clockwise

D: $\sum M_D = Fd_2$, anti-clockwise

E: $\sum M_E = Fd_2$, anti-clockwise

4.4 Conditions for Equilibrium

When a body is in equilibrium, the following 2 conditions must **both** be satisfied:

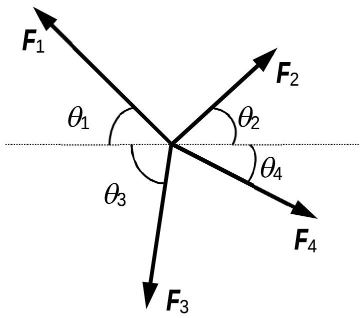
1. the vector sum of forces exerted on it must be zero

- there is no acceleration of its centre of mass and the object is said to be in *translational equilibrium*.

✓ *Method 1: Sum of force components in the x, y (and z) directions are zero*

If the forces can be resolved into components in two chosen perpendicular directions, say x and y-axis, then both $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Hence, if an object is in equilibrium under the influence of the 4 forces as shown, then



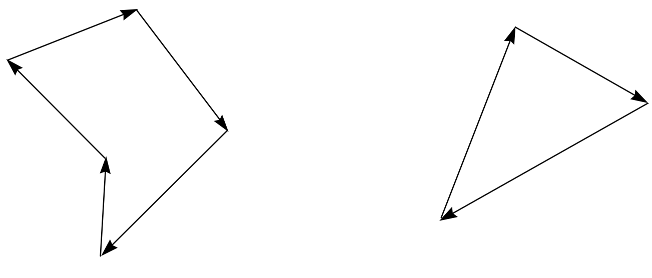
$$\Sigma F_x = 0$$
$$-F_1\cos\theta_1 + F_2\cos\theta_2 - F_3\cos\theta_3 + F_4\cos\theta_4 = 0$$

$$\Sigma F_y = 0$$
$$F_1\sin\theta_1 + F_2\sin\theta_2 - F_3\sin\theta_3 - F_4\sin\theta_4 = 0$$

These relationships can then be used to solve for unknown magnitudes and angles.

✓ *Method 2: Forces must form a closed polygon*

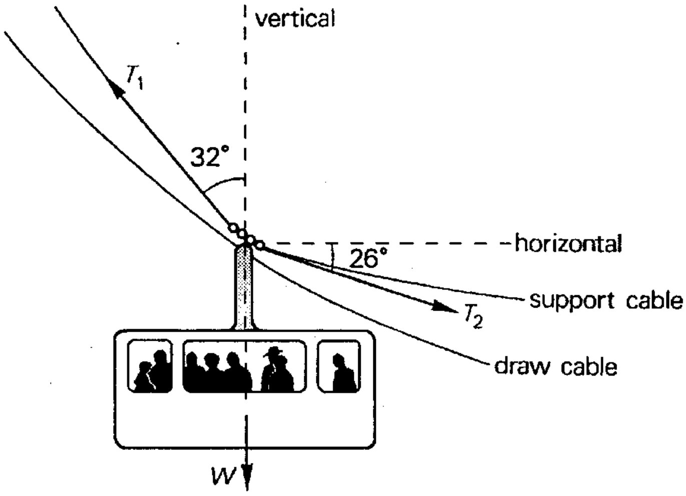
Since the vector sum of forces acting on the object is zero, the vector diagram showing the addition of all forces (placed head to tail) acting on the object will have to be a closed polygon.



Trigonometrical relationships can then be used to solve for the unknown magnitudes and angles. This method is recommended for situations involving only 3 forces due to the complexity of using it for polygons having more than 3 sides.

Example 4

A cable car travels along a fixed support cable and is pulled along this cable by a moving draw cable.



For the situation shown where the cable car can be considered to be stationary and the draw cable exerts negligible force on it, the weight W of the cable car and passengers is 8.0×10^4 N. Determine the magnitude of T_1.

Since the cable car can be considered to be stationary, it is in equilibrium.

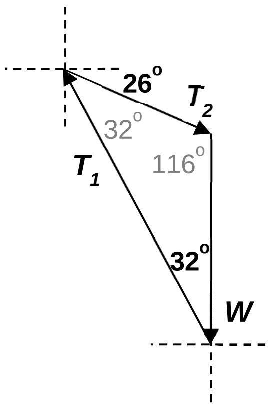
Method 1:

$$\Sigma F_x = 0$$
$$-T_1 \sin 32^\circ + T_2 \cos 26^\circ = 0$$

$$\Sigma F_y = 0$$
$$T_1 \cos 32^\circ - T_2 \sin 26^\circ - W = 0$$

Solving: $T_1 = 1.36 \times 10^5$ N

Method 2:
Considering the vector diagram of the 3 forces:



Using the sine rule:

$$\frac{T_1}{\sin 116^\circ} = \frac{W}{\sin 32^\circ}$$

$T_1 = 1.36 \times 10^5$ N

2. <u>the resultant torque on it must be zero about all axes</u>

- there is no angular acceleration of the object (i.e. it is not rotating at an increasing angular speed) and the object is said to be in *rotational equilibrium*.

  ✓ *Method: Principle of Moments*

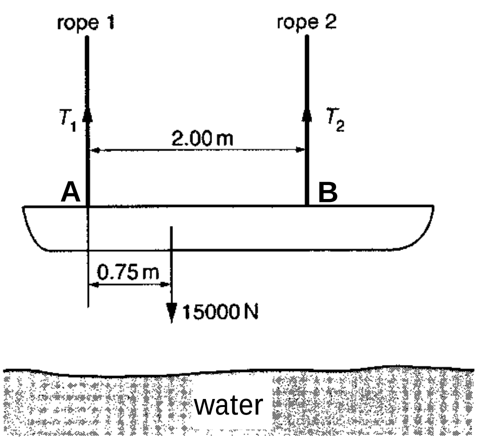
  The *Principle of Moments* states that <u>when a body is in equilibrium, the sum of clockwise moments about any point must be equal to the sum of anticlockwise moments about the same point</u>.

  Note: When the resultant torque of an object is zero about all axes, it further implies that the sum of moments of all forces acting on the object would be zero about any point. Hence, the principle of moments may be used deal with objects in rotational equilibrium.

It should be noted that in any given situation, one will require the same number of unique equations as there are unknowns in order for all unknowns to be evaluated mathematically.

Example 5

The force diagram below represents a boat that is being lifted by two ropes so that the boat remains horizontal and travels vertically upwards at a constant speed after leaving the water.



The weight of the boat is 15000 N. Determine the tensions T_1 and T_2 in the ropes 1 and 2 respectively.

Taking moments about B: $\sum M_B = 0$

$$T_1\,(2.00) - 15000\,(1.25) = 0$$

$$T_1 = 9.4 \times 10^3 \text{ N}$$

Taking moments about A: $\sum M_A = 0$

$$T_2\,(2.00) - 15000\,(0.75) = 0$$

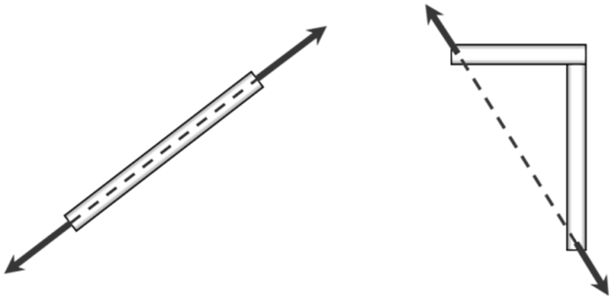
$$T_2 = 5.6 \times 10^3 \text{ N}$$

4.5 Special Considerations of Systems in Equilibrium

4.5.1 Systems involving Two Coplanar Forces

For a rigid body at equilibrium subjected to coplanar forces acting at only two points, the two forces must

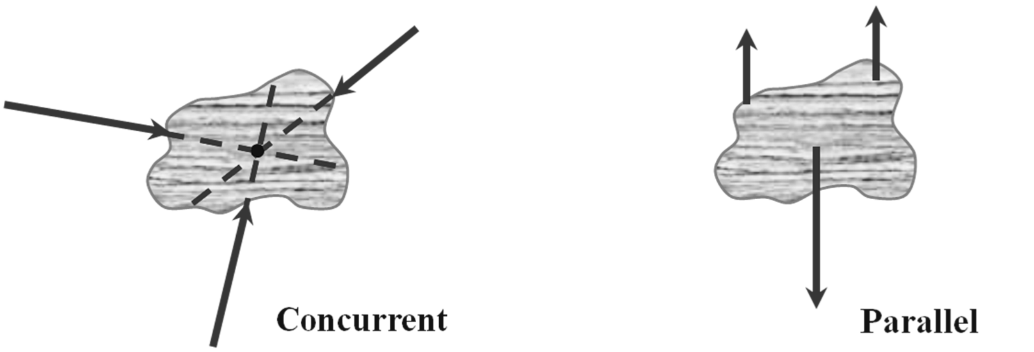
(i) have the same magnitude,
(ii) have the same line of action,
(iii) be opposite in direction.



4.5.2 Systems involving Three Coplanar Forces

For a rigid body at equilibrium subjected to coplanar forces acting at only three points, the lines of action of the three forces must **either** be

(i) concurrent (intersect at a point) **or**
(ii) parallel



Conceptual Question

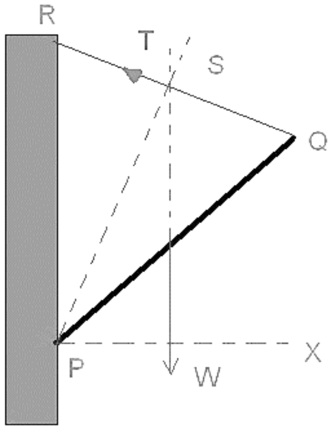
A body is acted upon by 3 coplanar forces.

If the lines of action of 3 forces are concurrent, the 3 forces must be able to maintain the body in equilibrium. Is this statement TRUE or **FALSE**?

If the lines of action of 3 forces are parallel, the 3 forces must be able to maintain the body in equilibrium. Is this statement TRUE or **FALSE**?

Example 6

The diagram below shows a heavy flagpole PQ hinged at a vertical wall at end P and held by a wire connected to the end Q and a point R on the wall. The weight of the flagpole is *W* and the tension in the wire is *T*.



What is the direction of the force exerted by the wall on the flagpole?

From P to S