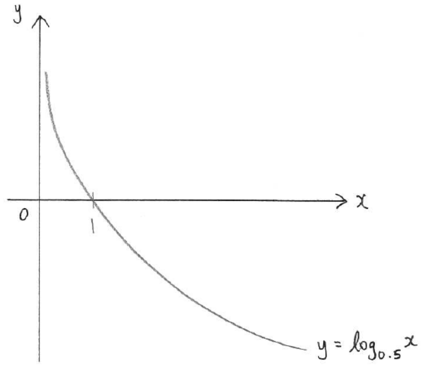
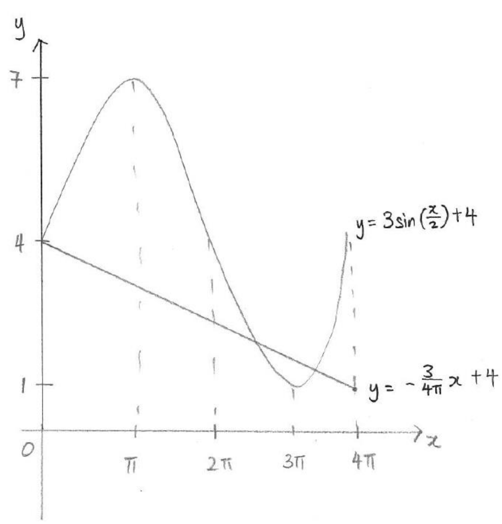
Additional Mathematics (90 marks)

Qn. #	Solution	Mark Allocation
1	$\frac{dy}{dx} = 9x^2 + 2ax$ $9x^2 + 2ax > 0$ $x(9x+2a) > 0$ $x < -\frac{2}{9}a$ or $x > 0$	**M1** (Find $\frac{dy}{dx}$) **M1** ($\frac{dy}{dx} > 0$) **A1**
2	$\left(5-\sqrt{2}\right)\left(a+4\sqrt{2}\right) = 7 + b\sqrt{2}$ $5a + 20\sqrt{2} - a\sqrt{2} - 8 = 7 + b\sqrt{2}$ $(5a-8) + (20-a)\sqrt{2} = 7 + b\sqrt{2}$ $5a - 8 = 7$ and $20 - a = b$ $a = 3, b = 17$	**M1** (expansion) **M1** (compare coefficient) **A2**
3(a)	$2x^2 + 12x + 11 = 2(x^2 + 6x) + 11$ $= 2\left[(x+3)^2 - 3^2\right] + 11$ $= 2(x+3)^2 - 7$	**B1** (either $(x+3)^2$ or -7 correct) **B2** (all correct)
3(b)	$2x^2 + 12x + 11 = px + 11$ $2x^2 + (12-p)x = 0$ $(12-p)^2 - 4(2)(0) > 0$ $(12-p)^2 > 0$ $p \neq 12$	**M1** (sim eqn) **M1** (Find discriminant) **A1**
4(a)	Let $\angle ABC = x$. $AB = AC$ (tangents from external point) $\angle ACB = \angle ABC = x$ (base angles of isosceles triangle) $\angle CEB = \angle ACB = x$ (alternate segment theorem) $\angle CED = 180^\circ - \angle ACB$ (adj. angles on straight line) $= 180^\circ - x$ $\angle ABC + \angle CED = x + (180^\circ - x)$ $= 180^\circ$	**M1** ($\angle ACB = \angle ABC$) **M1** ($\angle CEB = \angle ACB$) Note: If first M1 not awarded, maximum 2 out of 3 marks **A1**
4(b)	Suppose there exists a circle that passes through *A*, *B*, *E* and *C*. $\angle BAC = 180^\circ - \angle ABC - \angle ACB$ (sum of angles of triangle) $= 180^\circ - 2x$ $\angle BAC = 180^\circ - \angle CEB$ (opp angles of cyclic quad) $= 180^\circ - x$	**M1** (opp angles of cyclic quad)

<table>
<tr><th>Qn. #</th><th>Solution</th><th>Mark Allocation</th></tr>
<tr><td></td><td>For $x \neq 0$, $180^\circ - 2x \neq 180^\circ - x$
Hence, there is no circle that passes through A, B, E and C.</td><td>A1 (contradiction)</td></tr>
<tr><td>5(a)</td><td>$\log_5 x + 2 = 3\log_x 5$
$\log_5 x + 2 = \frac{3}{\log_5 x}$
Let $u = \log_5 x$
$u + 2 = \frac{3}{u}$
$u^2 + 2u - 3 = 0$
$(u-1)(u+3) = 0$
$u = 1$ or $u = -3$
$\log_5 x = 1$ or $\log_5 x = -3$
$x = 5$ or $x = 5^{-3}$
$x = \frac{1}{125}$</td><td>M1 (change of base)
M1 (form quad eqn)
M1 (solve quad eqn)
A2</td></tr>
<tr><td>5(b)</td><td>
</td><td>B1 (shape)
B1 (x-int and y-axis asymptote)</td></tr>
<tr><td>6(a)</td><td>Least value $= 1$
Greatest value $= 7$</td><td>B1
B1</td></tr>
<tr><td>6(b)</td><td>Period $= 4\pi$ or 720°</td><td>B1</td></tr>
<tr><td>6(c)</td><td>
</td><td>B1 (shape + correct number of cycles)

B1 (coordinates of start/end point + max/min points)</td></tr>
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Qn. #	Solution	Mark Allocation
6(d)	$\sin\left(\frac{x}{2}\right)=-\frac{x}{4\pi}$ $3\sin\left(\frac{x}{2}\right)=-\frac{3}{4\pi}x$ $3\sin\left(\frac{x}{2}\right)+4=-\frac{3}{4\pi}x+4$ $y=-\frac{3}{4\pi}x+4$ After drawing line: Number of solutions $=3$	**M1** (find eqn of line) **A1** (draw line + number of solutions)

7	$\frac{dy}{dx}=\int 3e^{-2x}+\cos 2x \, dx$
	$=-\frac{3}{2}e^{-2x}+\frac{1}{2}\sin 2x+c$
	Sub $x=0$, $\frac{dy}{dx}=5$
	$5=-\frac{3}{2}+c$
	$c=\frac{13}{2}$
	$\frac{dy}{dx}=-\frac{3}{2}e^{-2x}+\frac{1}{2}\sin 2x+\frac{13}{2}$
	$y=\int -\frac{3}{2}e^{-2x}+\frac{1}{2}\sin 2x+\frac{13}{2} \, dx$
	$=\frac{3}{4}e^{-2x}-\frac{1}{4}\cos 2x+\frac{13}{2}x+c_1$
	Sub (0, 3)
	$3=\frac{3}{4}-\frac{1}{4}+c_1$
	$c_1=\frac{5}{2}$
	$y=\frac{3}{4}e^{-2x}-\frac{1}{4}\cos 2x+\frac{13}{2}x+\frac{5}{2}$
		M1 (integrate $3e^{-2x}$)
		M1 (integrate $\cos 2x$)
		M1 (find c)
		M1 (integrate $-\frac{3}{2}e^{-2x}$)
		M1 (integrate $\frac{1}{2}\sin 2x$)
		M1 (integrate $\frac{13}{2}$)
		A1

8(a)	$T_{r+1}=\binom{n}{r}(3x)^{n-r}\left(-\frac{2}{x^2}\right)^r$ $=\binom{n}{r}3^{n-r}x^{n-r}(-2)^r(x^{-2})^r$ $=\binom{n}{r}3^{n-r}(-2)^r x^{n-3r}$	**M1** (general term) **M1** (simplification)

Qn. #	Solution	Mark Allocation
	$n-3r=0$ $n=3r$ where r is a positive integer Thus n is a multiple of 3.	**A1** (explanation)
8(b)	Term independent of x: $9=3r$ $r=3$ $T_4=\binom{9}{3}3^6(-2)^3$ $=-489888$ For $\frac{1}{x^6}$ term: $9-3r=-6$ $r=5$ $T_6=\binom{9}{5}3^4(-2)^5x^{-6}$ $=-\frac{326592}{x^6}$ $\frac{\text{coefficient of term independent of } x}{\text{coefficient of } \frac{1}{x^6}}=\frac{-489888}{-326592}$ $=\frac{3}{2}$	**B1** (Obtain -489888) **M1** (Find r for $\frac{1}{x^6}$ term) **M1** (Find $\frac{1}{x^6}$ term) **A1**
9(a)	$\frac{9x^2-4x+8}{(x-2)(x+1)^2}=\frac{A}{x-2}+\frac{B}{(x+1)^2}+\frac{C}{x+1}$ $9x^2-4x+8=A(x+1)^2+B(x-2)+C(x-2)(x+1)$ Sub $x=-1$ $9(-1)^2-4(-1)+8=B(-1-2)$ $B=-7$ Sub $x=2$ $9(2)^2-4(2)+8=A(2+1)^2$ $A=4$ Sub $x=0$ $9(0)^2-4(0)+8=4(1)^2-7(-2)+C(-2)(1)$ $C=5$ $\frac{9x^2-4x+8}{(x-2)(x+1)^2}=\frac{4}{x-2}-\frac{7}{(x+1)^2}+\frac{5}{x+1}$	**M1** (form 3 fractions) **M1** (form identity) **M2** (A, B, C correct) **M1** (1 of 3 constants correct) **A1**
9(b)	$\int\frac{9x^2-4x+8}{(x-2)(x+1)^2}\,\mathrm{d}x=\int\frac{4}{x-2}-\frac{7}{(x+1)^2}+\frac{5}{x+1}\,\mathrm{d}x$ $=4\ln(x-2)+\frac{7}{x+1}+5\ln(x+1)+c$	**B3** (**B1** for each term) Note: Subtract 1 mark if there is no "+ c"

Qn. #	Solution	Mark Allocation
10(a)	$v = \frac{ds}{dt}$ $v = -3 + \frac{1}{2}e^{\frac{t}{2}}$ $0 = -3 + \frac{1}{2}e^{\frac{t}{2}}$ $6 = e^{\frac{t}{2}}$ $\ln 6 = \frac{t}{2}$ $t = 2\ln 6$	**M1** (find v) **M1** ($v = 0$) **A1**
10(b)	At $t = 0$, $s = 1$ At $t = 1$, $s = -1.35$ (3sf) Since displacement changes from positive to negative, the particle passes through $s = 0$ some time between $t = 0$ and $t = 1$. Hence particle passes through O in first second.	**M1 (**both values of s) **A1** (explanation)
10(c)	At $t = 2\ln 6$, $s = -4.7506$ At $t = 4$, $s = -4.6109$ Total distance $= (1 + 4.7506) + (4.7506 - 4.6109)$ $= 5.89$ cm (3sf)	**M1** (both values of s) **M1** (sum of distances) **A1**
11(a)	Refer to attached graph	**B1** (table of values) **B1** (plot points) **B1** (draw line)
11(b)	Using points (0, 4.17) and (2, 3.78), Gradient $= \frac{4.17 - 3.78}{0 - 2}$ $= -0.195$ (accept -0.225 to -0.165) $C = Ae^{-kt} + 15$ $\ln(C - 15) = \ln A - kt$ $k = 0.195$ (3 s.f.) (accept 0.165 to 0.225) $\ln A = 4.17$ (accept 4.14 to 4.2) $A = 64.7$ (3 s.f.) (accept 62.8 to 66.7) $C = 64.7e^{-0.195t} + 15$ OR $\ln(C - 15) = -0.195t + 4.17$ $C - 15 = e^{-0.195t + 4.17}$ $C - 15 = e^{-0.195t} \times e^{4.17}$ $C = 64.7e^{-0.195t} + 15$	**B1** (Gradient) **M1** (Form linear eqn) **A1** (Find A) **A1** **M1** (remove ln) **A2** (**A1** to find A, **A1** for eqn)
11(c)	$64.7155e^{-0.195t} + 15 < 35$	**M1** (accept $= 35$)

Qn. #	Solution	Mark Allocation
	$e^{-0.195t} < \frac{20}{64.7155}$ $-0.195t < \ln\left(\frac{20}{64.7155}\right)$ $t > 6.02$ Year 2030	**M1** (apply ln) **A1** (Year)
12(a)	Let $B\left(x, \frac{1}{2}x+1\right)$ $(x+2)^2 + \left(\frac{1}{2}x+1\right)^2 = \left(5\sqrt{5}\right)^2$ $x^2+4x+4+\frac{1}{4}x^2+x+1=125$ $\frac{5}{4}x^2+5x-120=0$ $x^2+4x-96=0$ $(x-8)(x+12)=0$ $x=8$ or $x=-12$ (rej) $y=5$ $B(8,5)$	**M1** (form eqn using length) **M1** (simplification) **M1** (solve quad eqn) **A1**
12(b)	Gradient of $BC = \frac{7-5}{7-8}$ $= -2$ Gradient of $AB \times$ Gradient of $BC = \frac{1}{2} \times -2$ $= -1$ Therefore $\angle ABC = 90^\circ$ Since *ABCD* is a parallelogram with int angle $= 90^\circ$, *ABCD* is a rectangle.	**M1** (Gradient of *BC*) **M1** (Show right angle) **A1** (explanation)
12(c)	Length $BC = \sqrt{(8-7)^2+(5-7)^2}$ $= \sqrt{5}$ units Area of $ABCD = 5\sqrt{5} \times \sqrt{5}$ $= 25$ units^2	**M1** (Find *BC*) **A1**
13(a)	$\frac{dy}{dx} = 6(-3)(2x-5)^{-4}(2)$ $= -\frac{36}{(2x-5)^4}$ For $x > 2.5$, since numerator of $\frac{dy}{dx} \neq 0$, $\frac{dy}{dx} \neq 0$ Therefore there are no stationary points.	**M1** ($\frac{dy}{dx}$ without $\times 2$) **M2** (correct $\frac{dy}{dx}$) **A1** (with explanation)

Qn. #	Solution	Mark Allocation
13(b)	At $x=1$, $y=-\frac{2}{9}$	**B1** (y – coordinate)
	At $x=1$, $\frac{dy}{dx}=-\frac{4}{9}$	
	Gradient of normal $=\frac{9}{4}$	**M1** (gradient of normal)
	Eqn of normal: $y+\frac{2}{9}=\frac{9}{4}(x-1)$	**M1** (form eqn of normal)
	$y=\frac{9}{4}x-\frac{89}{36}$	
	$36y=81x-89$	
	Points of intersection: $x^2+90x-78=81x-89$	**M1** (sim eqn)
	$x^2+9x+11=0$	
	$x=\frac{-9\pm\sqrt{9^2-4(1)(11)}}{2(1)}$	**M1** (quad formula)
	$=-\frac{9}{2}\pm\frac{\sqrt{37}}{2}$	
	Difference between x-coordinates	
	$=-\frac{9}{2}+\frac{\sqrt{37}}{2}-\left(-\frac{9}{2}-\frac{\sqrt{37}}{2}\right)$	**M1** (difference)
	$=\sqrt{37}$	**A1**