Anglo-Chinese School

(Independent)



FINAL EXAMINATION 2015 YEAR THREE EXPRESS ADDITIONAL MATHEMATICS PAPER 2

5 October 2015

1 hour 30 minutes

Additional Materials: Answer Paper (8 sheets) Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in. Write in dark blue or black pen. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

Write your answers on the separate Answer Paper provided. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of a scientific calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 60.



This question paper consists of 4 printed pages. [Turn over...

Mathematical Formulae

ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^{n} = a^{n} + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{2} + \dots + \binom{n}{r}a^{n-r}b^{r} + \dots + b^{n},$$

where *n* is a positive integer and ${}^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

TRIGONOMETRY

Identities

$$\sin^{2} A + \cos^{2} A = 1$$

$$\sec^{2} A = 1 + \tan^{2} A$$

$$\cos ec^{2} A = 1 + \cot^{2} A$$

Formulae for
$$\triangle ABC$$

 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
 $a^2 = b^2 + c^2 - 2bc\cos A$

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

Answer all questions.

1 Solve the equation
$$2^{x+2}3^{x-4} = 5^{1-2x}$$
. [4]

2 A herd of 20 deer is introduced to an island as part of a conservation project. The population of deer is predicted to increase so that after a period of *t* years,

the population, D, is given by $D = \frac{k}{1 + 4e^{-0.14t}}$.

- (i) Find the value of k. [1]
 (ii) Find the predicted number of deer after 10 years. Give your answer correct to the nearest integer. [2]
 (iii) Find the number of years it will take for the population to reach 70. [3]
- 3 (i) Solve the equation |2-x| = 4x. [3]
 - (ii) On the same diagram, sketch the graphs of y = |2 x| and y = 4x, labelling the axes intercepts and their intersection point(s) clearly. [3]
 - (iii) Hence, or otherwise, solve |2 x| > 4x. [1]

4 (i) The line
$$\frac{x}{m-5} + \frac{y}{n+3} = 1$$
 cuts the x-axis and y-axis at P and Q. Find the coordinates
of the midpoint of PQ in terms of m and n. [3]

(ii) If
$$y = \frac{1}{2}x + 3$$
 is the perpendicular bisector of *PQ*, find the value of *m* and of *n*. [5]

5 (i) Prove the identity
$$\frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x} = 2 \sec x$$
. [3]

(ii) Prove that
$$(\sin x + 1)(\sin x - 1)(\sin^2 x + \tan^2 x + \cos^2 x) \equiv -1$$
. [3]

(iii) Hence, solve
$$\frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x} = 4(\sin x + 1)(\sin x - 1)(\sin^2 x + \tan^2 x + \cos^2 x)$$
 for x between 0° and 360°. [2]

6 (a) (i) Given that
$$x^4 + x^2 + x + 1 \equiv (x^2 + A)(x^2 - 1) + Bx + C$$
, determine the values of A, of B and of C. [3]

(ii) Hence, state the remainder when
$$x^4 + x^2 + x + 1$$
 is divided by $x^2 - 1$. [1]

(b) Express
$$\frac{x^3 + 7x + 2}{x^2 - 1}$$
 in partial fractions. [5]

7 (a) If
$$\tan x = a$$
, express $\frac{3+2\sin^2 x}{\cos^2 x}$ in terms of a . [3]

(b) Solve, for $0 \le x \le \pi$, the equation $2\cos x = \cot x$, leaving your answers in terms of π . [4]

8 Answer the whole of this question on a single sheet of graph paper.

The variables x and y are related by the equation $y = px^2 + q\sqrt{x}$, where p and q are constants. The table shows experimental values of the variables x and y.

x	1	2	3	4	5
у	5.5	12.2	19.2	38.0	56.7

(a) Plot the graph of
$$\frac{y}{\sqrt{x}}$$
 against $x\sqrt{x}$. [3]

- (**b**) Using your graph
 - (i) estimate the values of p and q, [3]
 - (ii) estimate the value of x when $2y = 35\sqrt{x}$,
 - (iii) identify the abnormal reading and estimate its correct value. [2]

(c) If instead, a graph of $\frac{y}{x^2}$ is plotted against $\frac{\sqrt{x}}{x^2}$, state the value of the gradient of this straight line. [1]

End of Paper

ANSWER KEY

 1.
 0.922

 2.
 (i) 100
 (ii) 50
 (iii) 16.0

 2
 2
 2
 2

3. (i)
$$\frac{2}{5}$$
 (iii) $x < \frac{2}{5}$

4. (i)
$$\left(\frac{m-5}{2}, \frac{n+3}{2}\right)$$
 (ii) $m = 9, n = 5$

6. (a)(i) A=2, B=1, C=3 (ii)
$$x+3$$
 (b) $x+\frac{5}{x-1}+\frac{3}{x+1}$

7. (a)
$$3+5a^2$$
 (b) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

8. (b) (i) $p \approx 2$, $q \approx 3$ (ii) $x \approx 3.73$ (iii) abnormal reading = (3, 19.2); correct reading (3, 23.6) (c) Gradient = $q (\approx 3)$

[2]