SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

NA

Name: ______________________________( ) Class: __________

ADDITIONAL MATHEMATICS

4051/02

Paper 2

Monday 7 August 2023
1 hour 45 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

Questions	1	2	3	4	5	6	7	8	9	10
Marks										

This document consists of **14** printed pages.

Setter: Mdm Zoe Pow
Vetter: Mr Ang Hanping

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 Express $\dfrac{3x^4 + x^3 + 50x - 24}{x^2 - 2x + 6}$ in the form $ax^2 + bx + c$, where a, b and c are integers. [3]

2 The volume, V cm^3, of water in a tank may be modelled by the equation $V = \dfrac{1}{3}\pi x^3$, where x cm is the depth of the water in the tank. When the tap is turned on, the volume of water in the tank increases at a rate of 0.2 cm^3/min. Find the rate at which the depth of water in the tank is increasing when $x = 0.9$. [5]

3 **(i)** Express $x^2 - 6x + 14$ in the form $(x+a)^2 + b$ where a and b are constants. [3]

(ii) Hence state the coordinates of the vertex of the curve $y = x^2 - 6x + 14$. [2]

4 The equation of a curve is $y = x^3 + 4x^2 - 3x - 8$. Find the equation of the tangent to the curve at the point $x = -2$. [5]

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = ax^3 + 6x^2 + bx - 18$, where a and b are constants. It is given that $x+2$ is a factor of $\mathrm{p}(x)$ and when $\mathrm{p}(x)$ is divided by $x-3$ the remainder is 75.

(i) Show that $a = 2$ and find the value of b. [4]

(ii) Using the values from part (**i**), find the remainder when $\mathrm{p}(x)$ is divided by $2x-1$. [2]

6 **(a)** Given that $y = \dfrac{x^3 - 2}{1 + 3x}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Given that $y = 3x^2\left(2x+1\right)^4$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in the form $\left(2x+1\right)^3 \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic expression. [4]

7 It is given that $f(x) = \dfrac{5x}{(3x-2)(x+1)}$, where $x \neq \dfrac{2}{3}$ and $x \neq -1$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Hence find $f'(x)$. [3]

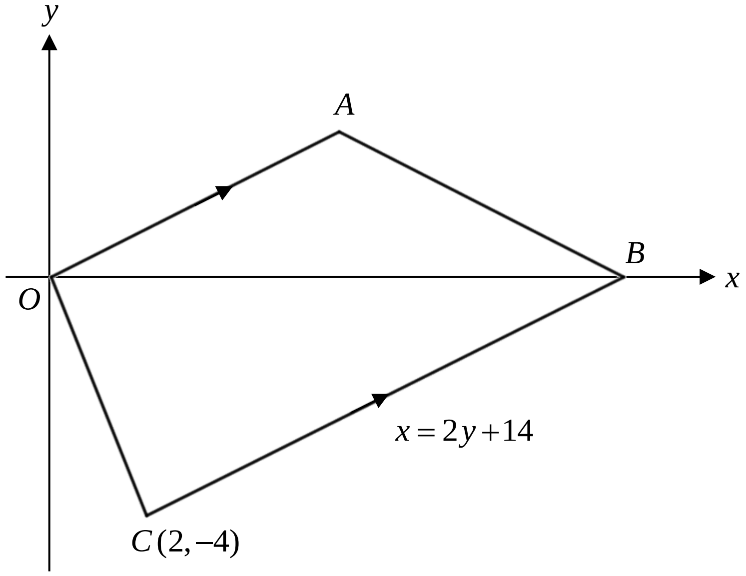
(iii) Given that a curve has equation $y = f(x)$, explain why the gradient of every point on the curve is negative. [2]

8 **(a)** Prove that $\dfrac{\cot^2\theta - 1}{\operatorname{cosec}^2\theta} = \cos 2\theta$. [3]

(b) **(i)** Write $5\cos x - 2\sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence, or otherwise, find the least value of $8+5\cos x-2\sin x$ and the smallest positive value of x for which this occurs. [4]

9 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a trapezium *OABC* in which *OA* is parallel to *CB*. The coordinates of the point *C* are $(2, -4)$. The equation of the line *BC* is $x = 2y + 14$.

(i) Given that the point *B* lies on the *x*-axis, find the coordinates of *B*. [1]

The point *X* lies on the line *CB* such that *CX* : *XB* is 3 : 1.

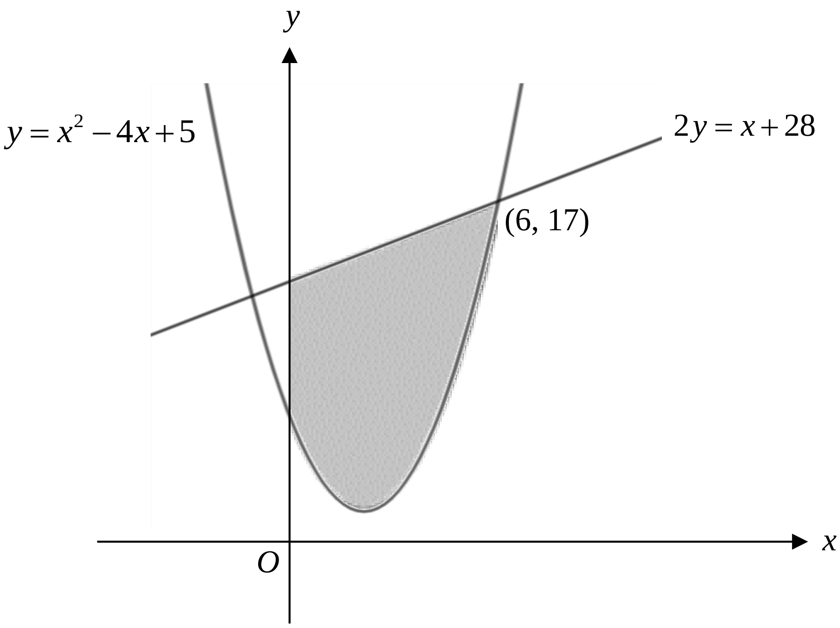
(ii) Find the coordinates of *X*. [2]

The line through X, perpendicular to CB, passes through the point A.

(iii) Find the coordinates of A. [6]

(iv) Calculate the area of the trapezium $OABC$. [3]

10



The diagram shows the curve $y = x^2 - 4x + 5$ and the line $2y = x + 28$. The line and the curve intersect at the point (6, 17). Find the area of the shaded region. [7]

End of Paper