Anglo-Chinese School
(Independent)

THE BEST IS YET TO BE

# FINAL EXAMINATIONS 2020

# YEAR 3 INTEGRATED PROGRAMME
# ADVANCED MATHEMATICS
# PAPER 2

**Thursday** **8 October 2020** **1 hour and 30 minutes**

**Additional Materials:**
Writing paper (6 sheets)

**INSTRUCTIONS TO STUDENTS**

Do not open this examination paper until instructed to do so.
A graphic display calculator may be used in this paper.
Answer all questions on the answer sheets provided.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

**INFORMATION FOR STUDENTS**
The maximum mark for this paper is 80.

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**This question paper consists of 4 printed pages.**

**[Turn over]**

*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.*

1. *[Maximum marks: 8]*

The polynomial $f(x) = 2x^3 + (7-2k)x^2 + (3-7k)x - 3k$, leaves a remainder of $-12$ when divided by $(x-1)$.

(a) Find the value of *k*. [2 marks]

(b) Hence solve $f(\sin x) = 0$ for $0 \le x \le 360°$. [6 marks]

2. *[Maximum marks: 13]*

(a) Express $\dfrac{2x^2+3x+7}{(x+1)(x^2+2)}$ as partial fractions. [5 marks]

(b) The volume of a rectangular freight container, $V$ m$^3$, is given by the equation,

$$V = x^3 - x^2 - 6x.$$

(i) Write down the dimension of the rectangular freight container in terms of *x*. [2 marks]

(ii) Find the dimensions of the rectangular freight container with a volume of 24 m$^3$, showing clearly there is only one real answer. [6 marks]

3. *[Maximum marks: 8]*

At the clinical development stage, a vaccine was tested for its effectiveness in treating a strain of bacteria.

The number of bacteria cells, *N*, after *t* number of days, may be estimated by using the formula, $N = 12000\,e^{-kt}$, where $k > 0$.

(a) State the initial number of bacteria cells. [1 mark]

(b) Given that the number of bacteria cells decreases to 7352 after a week, find the value of *k*. [3 marks]

For the new vaccine to be certified effective, it must be able to reduce the number of bacteria cells to 5% of the initial cell count or lower within 14 days.

(c) Justify if this vaccine is effective. [4 marks]

4. *[Maximum marks: 9]*

(a) Sketch the graph of $f(x)=e^{4x}-8e^{2x}+15$ for $-4\le x\le 2$, stating the equation of the asymptote, the coordinates of the turning point and the points of intersection with the axes. [4 marks]

(b) On the same axes, sketch the graph of $g(x)=3+\dfrac{2}{(x-2)^2}$ for $-4\le x\le 2$, stating the equation(s) of the asymptote(s), and the points of intersection between $f(x)$ and $g(x)$. [4 marks]

(c) Solve $f(x)<g(x)$. [1 mark]

5. *[Maximum marks: 8]*

(a) Describe the transformations necessary to transform $f(x)=\sqrt{x}$ into $g(x)=4\sqrt{x}-1$. [2 marks]

(b) (i) State the equation of the asymptote and the *x*-intercept of $y=3\ln x$. [2 marks]

(ii) The graph of $y=3\ln x$ undergoes a translation of 2 units along the negative *x*-direction followed by a stretch of scale factor of 2 along the *y*-direction.

State the equation of the transformed graph and the corresponding equation of the asymptote and axes intercepts. [4 marks]

6. *[Maximum marks: 8]*

Given that angle $A$ is obtuse such that $\cos A=-\dfrac{1}{3}$, find each of the following, leaving your answers in surd form where applicable.

(a) $\sin A$, [2 marks]

(b) $\dfrac{\tan(90°-A)}{\tan A}$, [3 marks]

(c) $\dfrac{3\tan 30°}{\cos(180°-A)}$. [3 marks]

7. *[Maximum marks: 9]*

The height of the tide, $h(t)$ can be modelled by the equation $h(t) = A\sin\left(\frac{\pi}{B}t\right) + C$ where $t$ is the number of hours after midnight.

The height first reaches a maximum of 3.2 m at 5 am and a minimum of 2.2 m at 3 pm.

(a) Find the values of $A$, $B$, and $C$. [3 marks]

(b) Hence, sketch the graph of $h(t) = A\sin\left(\frac{\pi}{B}t\right) + C$ from midnight to 8 pm, clearly indicating the coordinates of the turning points and the point of intersection with the $y$-axis. [4 marks]

(c) State the time, correct to the nearest minute, during which the height is above 3 m. [2 marks]

8. *[Maximum marks: 17]*

The functions, $f$, $g$ and $h$, are defined as follows.

$$f : x \mapsto 2 + \frac{1}{x} \quad , \; x \neq 0 ,$$

$$g : x \mapsto \frac{x}{2x+1} , \; x \neq k , \text{ and}$$

$$h : x \mapsto 2x - 3 .$$

(a) State the value of $k$. [1 mark]

(b) Express the following in similar form

(i) $f^{-1}$, [2 marks]

(ii) $hg$. [3 marks]

(c) Find the value of $p$ such that $g^{-1} h^{-1} (p) = 2$. [2 marks]

(d) Evaluate $fgh(2)$. [3 marks]

(e) Find $g^2(x)$ and $g^3(x)$. Hence deduce $g^n(x)$. [6 marks]

**End of Paper 2**

**Answers**

1. (a) $\therefore k = 2$ (b) $\therefore x = 210^\circ, 330^\circ$

2. (a) $\therefore \dfrac{2}{(x+1)} + \dfrac{3}{(x^2+2)}$ (b) $\therefore (x+2)$ by $(x-3)$ by $x$ , $\therefore x = 4$

3. (a) $N = 12000$ (b) $\therefore k = 0.070$ (c) $\therefore t = 42$, Not effective

4. (c) $\therefore 0.303 < x < 0.928$ & $\therefore 1.97 < x < 2$

5. (a) A stretch of scale factor of 4 parallel to the *y*-axis, followed by a translation of 1 unit in the negative *y* – direction.
   (b) Asymptote: $x = 0$ , *x* – intercept: $(1,0)$
   $y = 6\ln(x+2)$, Asymptote: $x = -2$ , *x*–intercept: $(-1,0)$,
   *y*–intercept: $y = (0, 6\ln 2) = (0, 4.16)$

6. (a) $\dfrac{2\sqrt{2}}{3}$ (b) $\dfrac{1}{8}$ (c) $3\sqrt{3}$

7. (a) $\therefore A = 0.5$, $\therefore B = 10$, $\therefore C = 2.7$
   (c) $\therefore t = 2.05,\ 7.95$ Required duration: 2.03am<t<7.57am

8. (a) $k = -\dfrac{1}{2}$
   (b) (i) $\dfrac{1}{x-2}$ , $x \neq 2$ (ii) $\dfrac{-4x-3}{2x+1}$ , $x \neq -\dfrac{1}{2}$
   (c) $-\dfrac{11}{5}$ (d) 5
   (e) $g^2(x) = \dfrac{x}{4x+1}$ , $x \neq -\dfrac{1}{4}$, $g^3(x) = \dfrac{x}{6x+1}$, $x \neq -\dfrac{1}{6}$
   $g^n(x) = \dfrac{x}{2nx+1}$, $x \neq -\dfrac{1}{2n}$