Additional Mathematics Paper 2 (70 marks)

Qn. #	Solution	Mark Allocation
1	Long division: $x^2-2x+6 \overline{)\,3x^4+x^3+50x-24}$, quotient $3x^2+7x-4$ $-(3x^4-6x^3+18x^2)$ $7x^3-18x^2+50x$ $-(7x^3-14x^2+42x)$ $-4x^2+8x-24$ $-(-4x^2+8x-24)$ 0 $\frac{3x^4+x^3+50x-24}{x^2-2x+6}=3x^2+7x-4$	M1: Attempt to do long division A1: Obtain $3x^2+7x$ in the quotient A1: Obtain completely correct solution
2	$V=\frac{1}{3}\pi x^3$ $\frac{dV}{dx}=\pi x^2$ When $x=0.9$, $\frac{dV}{dx}=\pi(0.9)^2$ $=0.81\pi$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ $0.2=0.81\pi\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{0.2}{0.81\pi}$ $=0.0786$ The depth of water in the tank is increasing at a rate of 0.0786 cm/min.	B1: $\frac{dV}{dx}=\pi x^2$ M1: Find required $\frac{dV}{dx}$ B1: Used $\frac{dV}{dt}=0.2$ M1: Chain rule used correctly A1 (with units)
3i	$x^2-6x+14=\left(x-\frac{6}{2}\right)^2+14-\left(-\frac{6}{2}\right)^2$ $=(x-3)^2+5$	B1: Obtain correct value for a M1: Show correct process for completing square A1: Obtain correct answer
3ii	(3, 5)	B1 for each coordinate
4	$y=x^3+4x^2-3x-8$	

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	$\frac{dy}{dx} = 3x^2 + 8x - 3$ At $x = -2$, $\frac{dy}{dx} = 3(-2)^2 + 8(-2) - 3$ $= -7$ $y = (-2)^3 + 4(-2)^2 - 3(-2) - 8$ $= 6$ At $(-2, 6)$, $6 = (-7)(-2) + c$ $c = -8$ Equation of tangent is $y = -7x - 8$	M1: Attempt to differentiate A1: All correct A1: $\frac{dy}{dx} = -7$ and $y = 6$ M1: Substitute into $y = mx + c$ A1
5i	$p(x) = ax^3 + 6x^2 + bx - 18$ $p(-2) = 0$ $a(-2)^3 + 6(-2)^2 + b(-2) - 18 = 0$ $-8a - 2b + 6 = 0$ $4a + b = 3$ ------ (1) $p(3) = 75$ $a(3)^3 + 6(3)^2 + b(3) - 18 = 75$ $27a + 3b = 39$ $9a + b = 13$ ------ (2) (2) − (1): $5a = 10$ $a = 2$ (shown) Sub $a = 2$ into (1): $4(2) + b = 3$ $b = -5$	M1: Any attempt at using $p(-2) = 0$ AND $p(3) = 75$ to form two equations A1: Both equations correct, need not be simplified DM1: Solving *their* simultaneous equations, must be 2 unknowns in each A1: Obtain correct answers
5ii	$p(x) = 2x^3 + 6x^2 - 5x - 18$ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 6\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) - 18$ $= -18\frac{3}{4}$	M1: Attempt to find $p\left(\frac{1}{2}\right)$ for *their* cubic A1: Obtain correct answer
6a	$y = \frac{x^3 - 2}{1 + 3x}$ $\frac{dy}{dx} = \frac{(1 + 3x)(3x^2) - (x^3 - 2)(3)}{(1 + 3x)^2}$ $= \frac{6x^3 + 3x^2 + 6}{(1 + 3x)^2}$ $= \frac{3(2x^3 + x^2 + 2)}{(1 + 3x)^2}$	M1: Attempt to differentiate a quotient or appropriate product A1: Correct denominator A1: Correct numerator

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6b	$y=3x^2(2x+1)^4$ $\frac{dy}{dx}=3x^2[4(2x+1)^3(2)]+(2x+1)^4(6x)$ $=6x(2x+1)^3[4x+(2x+1)]$ $=(2x+1)^3(36x^2+6x)$	M1: Attempt to differentiate using the product rule A2: A1 for each term B1: Manipulate the answer to the correct format
7i	$\text{f}(x)=\frac{5x}{(3x-2)(x+1)}$ $\frac{5x}{(3x-2)(x+1)}=\frac{A}{3x-2}+\frac{B}{x+1}$ $5x=A(x+1)+B(3x-2)$ Sub $x=-1$: $-5=-5B$ $B=1$ Sub $x=\frac{2}{3}$: $5\left(\frac{2}{3}\right)=\frac{5}{3}A$ $A=2$ $\frac{5x}{(3x-2)(x+1)}=\frac{2}{3x-2}+\frac{1}{x+1}$	B1: Correct partial fraction seen M1: Attempt to find A and B A2: Obtain correct answers
7ii	$f'(x)=-\frac{2(3)}{(3x-2)^2}-\frac{1}{(x+1)^2}$ $=-\frac{6}{(3x-2)^2}-\frac{1}{(x+1)^2}$	M1: Attempt chain rule for either term A2: Obtain correct answers
7iii	Since $(3x-2)^2>0$, $-\frac{6}{(3x-2)^2}<0$ and $(x+1)^2>0$, $-\frac{1}{(x+1)^2}<0$, $f'(x)=-\frac{6}{(3x-2)^2}-\frac{1}{(x+1)^2}<0$, therefore the gradient of every point on the curve is negative.	B1 B1
8a	$LHS=\frac{\cot^2\theta-1}{\text{cosec}^2\theta}$ $=\frac{\text{cosec}^2\theta-1-1}{\text{cosec}^2\theta}$ $=1-\frac{2}{\text{cosec}^2\theta}$ $=1-2\sin^2\theta$ $=\cos 2\theta$	B1: Use $\cot^2\theta=\text{cosec}^2\theta-1$ B1: Use $\frac{1}{\text{cosec}^2\theta}=\sin^2\theta$ B1: Use $\cos 2\theta=1-2\sin^2\theta$
8bi	Let $5\cos x-2\sin x=R\cos(x+\alpha)$	

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	$5\cos x - 2\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ $R\cos\alpha = 5$ and $R\sin\alpha = 2$ $R = \sqrt{5^2 + 2^2}$ $= \sqrt{29}$ $\alpha = \tan^{-1}\left(\frac{2}{5}\right)$ $= 21.801°$ $5\cos x - 2\sin x = \sqrt{29}\cos(x + 21.8°)$	M1 M1: Attempt to solve 2 equations in R and α A2: Obtain correct answers
8bii	Least value of $8 + 5\cos x - 2\sin x = 8 - \sqrt{29}$ $= 2.61$ Least value occurs when $\cos(x + 21.8°) = -1$ $x + 21.801° = 180°$ $x = 158.2°$	M1: $-\sqrt{29}$ A1: Obtain correct answer M1: $x + 21.801° = 180°$ A1: Obtain correct answer
9i	Point B lies on the x-axis → $y = 0$ $x = 2y + 14$ $x = 2(0) + 14$ $= 14$ Coordinates of B are (14, 0)	B1
9ii	x-coordinate of $X = 2 + \frac{14-2}{4} \times 3$ $= 11$ y-coordinate of $X = -4 + \frac{0-(-4)}{4} \times 3$ $= -1$ Coordinates of X are (11, −1)	M1 A1
9iii	Gradient of $AX = -1 \div \frac{1}{2}$ $= -2$ At $X(11, -1)$, $-1 = (-2)(11) + c$ $c = 21$ Equation of AX is $y = -2x + 21$ ------- (1) Gradient of $OA = \frac{1}{2}$ Equation of OA is $y = \frac{1}{2}x$ ------- (2) Sub (1) into (2): $\frac{1}{2}x = -2x + 21$ $2\frac{1}{2}x = 21$ $x = 8\frac{2}{5}$	M1 M1 M1 M1 M1

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	Sub $x=8\frac{2}{5}$ into (2): $y=\frac{1}{2}\left(8\frac{2}{5}\right)$ $=4\frac{1}{5}$ Coordinates of A are $(8\frac{2}{5}, 4\frac{1}{5})$	A1
9iv	Area of the trapezium $OABC$= $\frac{1}{2}\begin{vmatrix}0 & 2 & 14 & 8\frac{2}{5} & 0\\ 0 & -4 & 0 & 4\frac{1}{5} & 0\end{vmatrix}$ $=\frac{1}{2}\left\|14\times 4\frac{1}{5}-14\times(-4)\right\|$ $=\frac{1}{2}\left\|114\frac{4}{5}\right\|$ $=57\frac{2}{5}$ units^2	M1 M1 A1
10	Area of shaded region $=\int_0^6 \left(\frac{1}{2}x+14\right)-(x^2-4x+5)\,\mathrm{d}x$ $=\int_0^6 \left(-x^2+4\frac{1}{2}x+9\right)\,\mathrm{d}x$ $=\left[-\frac{x^3}{3}+4\frac{1}{2}\left(\frac{x^2}{2}\right)+9x\right]_0^6$ $=\left[-\frac{6^3}{3}+4\frac{1}{2}\left(\frac{6^2}{2}\right)+9(6)\right]$ $=63$ units^2	M1: Attempt to integrate $\int_0^6((x^2-4x+5))\,\mathrm{d}x$ (at least one term correct) A1: Correct result M1: Correct use of limits in *their* expression A1: 30 units^2 B1: When $x=0, y=5$ or $\int_0^6\left(\frac{1}{2}x+14\right)\,\mathrm{d}x$ B1: Area of trapezium or substituting limits and getting 93 A1: 63 units^2